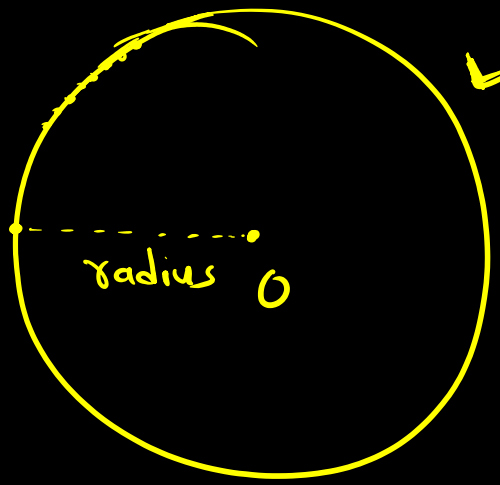
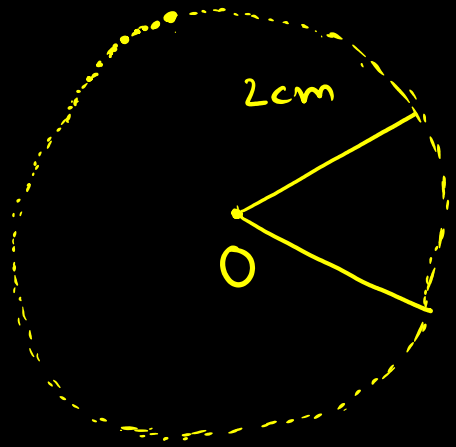
Geometry

Circle

Area of Circle



final image.



skeleton

⇒ Collection of points which are equidistant from a fixed central point

↓

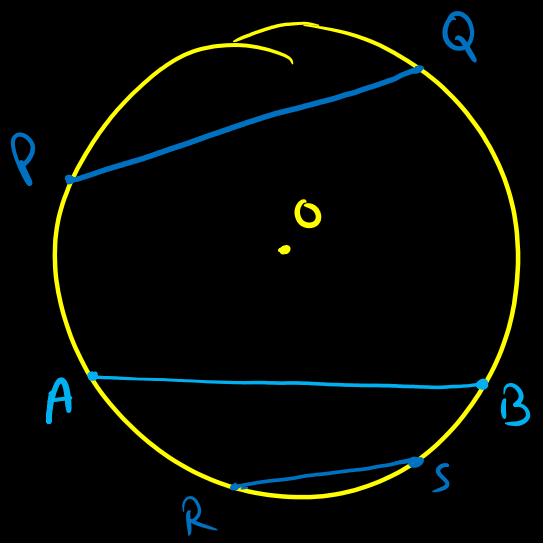
Centre of circle

↓

Radius of circle.

Chord

a line segment which joins any two points on a circle.



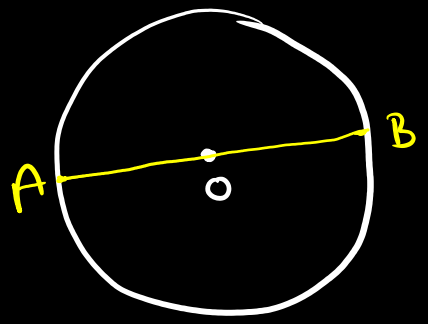
Here line segment AB is a chord of circle.

$\overline{PQ}$ & $\overline{RS}$ are also chords.

Diameter

⇒ A chord which passes through the centre of a circle

⇒ AB is a chord which passes through centre of circle hence AB is also diameter of circle.



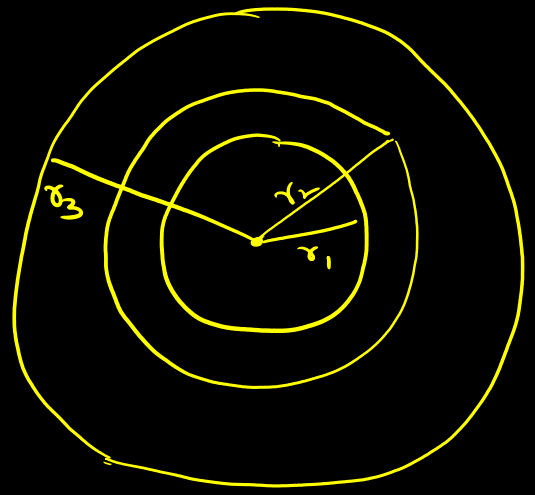
OA ⇒ Radius

OB ⇒ Radius.

Diameter = Radius + Radius

Diameter = 2 x Radius

Concentric Circle :



$r_1 = 3$ cm

$r_2 = 4$ cm

$r_3 = 5$ cm.

Two or more circles with same centre but different radii.

L2

$\angle 1 = (7x - 8)^\circ$

$+$

$\angle 2 = (2x + 17)^\circ$

Co-interior

$\parallel$

180°

$$(7x - 8) + (2x + 17) = 180^\circ$$

$$7x - 8 + 2x + 17 = 180^\circ$$

$$7x + 2x - 8 + 17 = 180^\circ$$

$$9x + 9 = 180$$

$$9x = 180 - 9$$

$$9x = 171$$

$$x = \frac{171}{9} = 19^\circ$$

Circumference of circle = $2\pi r$

π → irrational no.

π = constant

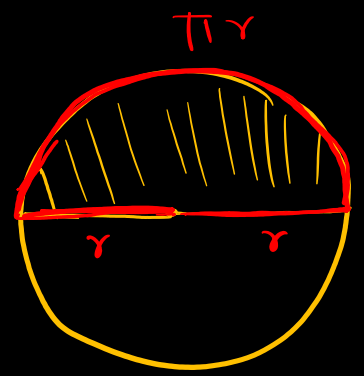
$\simeq$ 3.14 or $\frac{22}{7}$

~~(Circumference)~~ of semi-circle

$+$

(Perimeter)

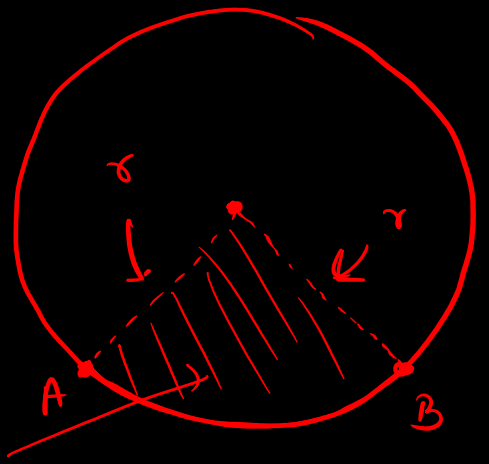
$= \pi r + 2r$



Sector of a circle :

↓

Region of circle bound by an arc and two radii.



Sector of a circle.

arc of a circle

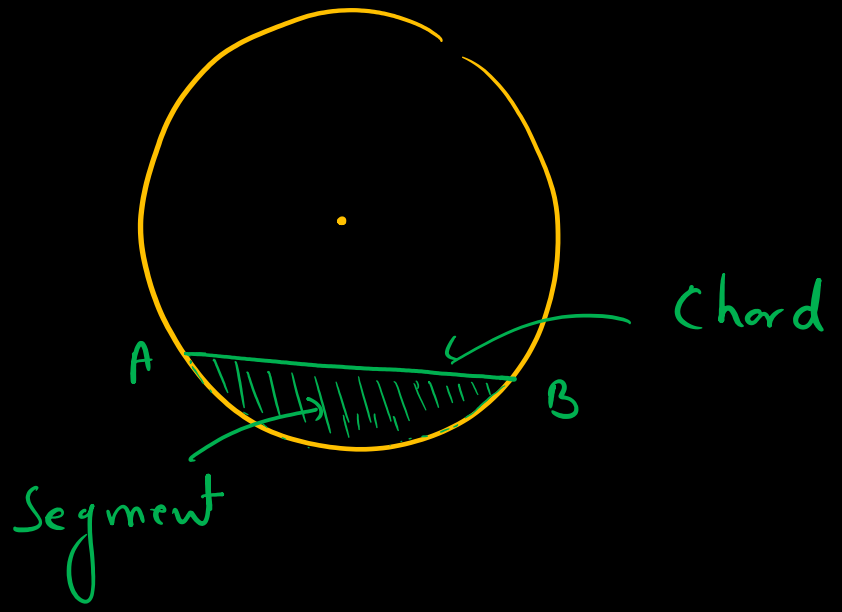
↓

Part of circumference of circle.

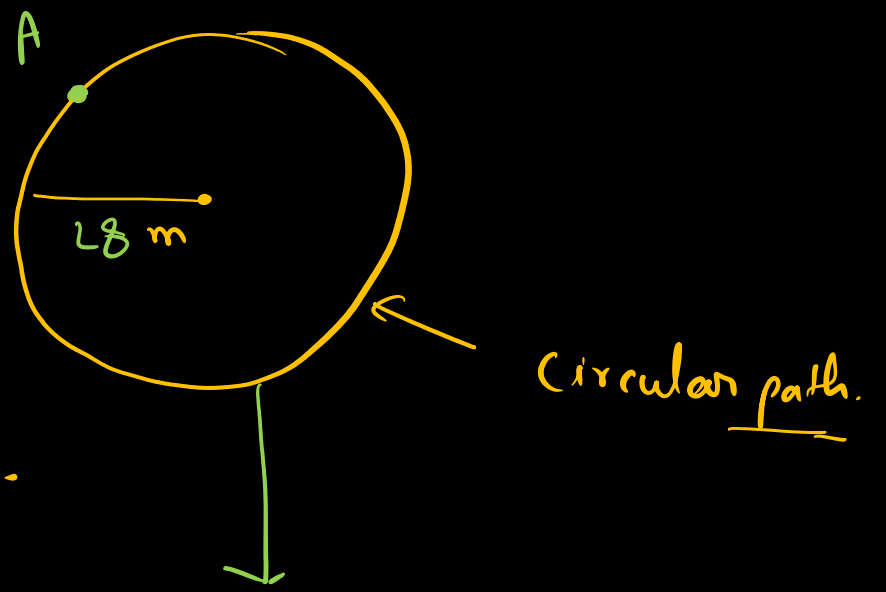
Segment of Circle

↓

Region of circle bound by a chord and an arc.



Q. $r = 28\ m$



Person ⇒ 5 complete rounds

⇒ Distance covered the person in 5 rounds.

Length of 5 rounds = 5 × circumference

= 5 × $(2\pi r)$

= 5 $\left(2 \times \frac{22}{7} \times 28\ m\right)$

= 5 $(44 \times 4)\ m$ = 5 $(176)\ m$

= 880 m

Length of one round = Circumference of circle.

Q. Find the circumference of a circle whose radius is 49 m.

Sol. Circumference of circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 49^{7}$ m

$= 44 \times 7$ m

$= 308$ m

$154 \times 49 = 7546$

Area of circle $= \pi r^2 = \frac{22}{7} \times 49^{7} \times 49$ m

$= 22 \times 7 \times 49$

$= 154 \times 49 = 7546$ m^2

Q. Find area of a circle whose diameter is 28 m.

$r = \frac{28}{2}\,m = 14\,m$

$A = 616\ m$

$A = \pi r^2$

$= \frac{22}{\cancel{7}} \times \overset{2m}{\cancel{14m}} \times 14m$

$= 44m \times 14\,m$

$= 616\ m^2$

Q. Radius of a wheel of a bike is 49 cm. If the wheel of bike rotates 100 times on the road, find the distance covered by the bike.

Distance = $100 \times 2\pi r$

$= 100 \times 2 \times \frac{22}{7} \times 49$

$= 100 \times 308$

$= 30800$ cm

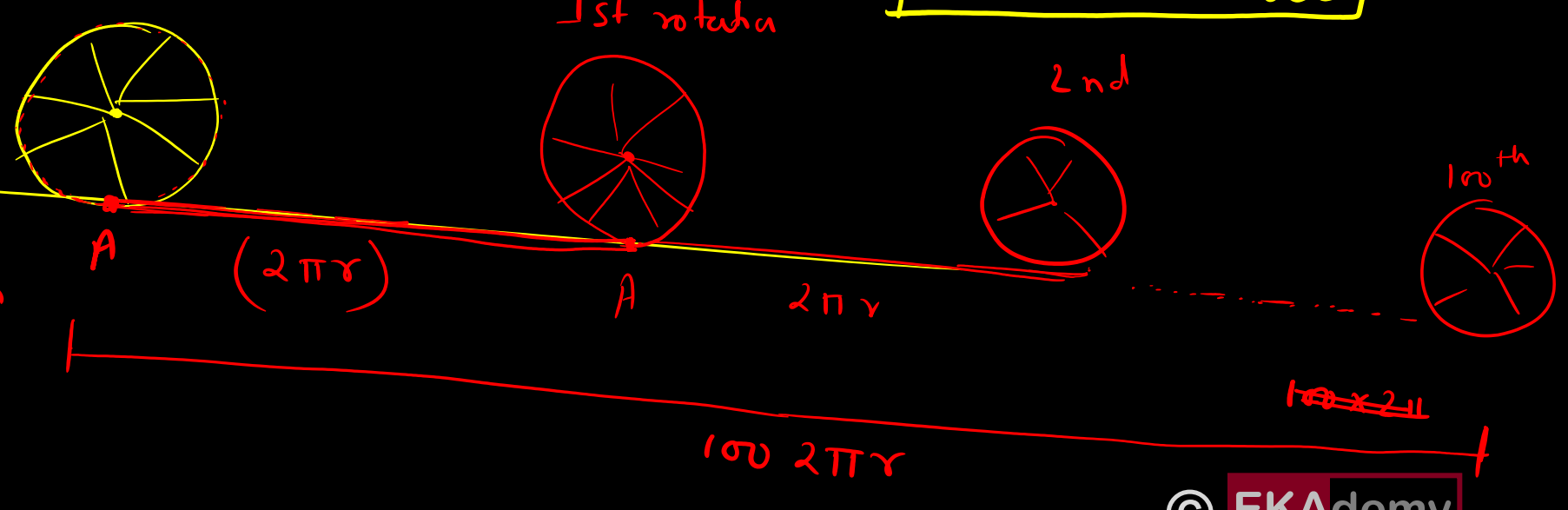
$= \frac{308}{1000}$ m

$= 0.308$ km

Circumference = $2\pi r$

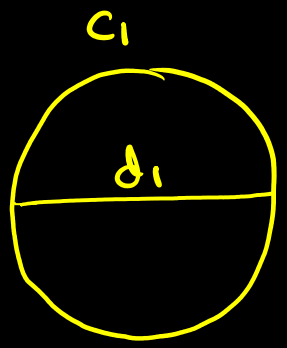
$= \pi (2r)$

Circumference = $\pi \cdot d$

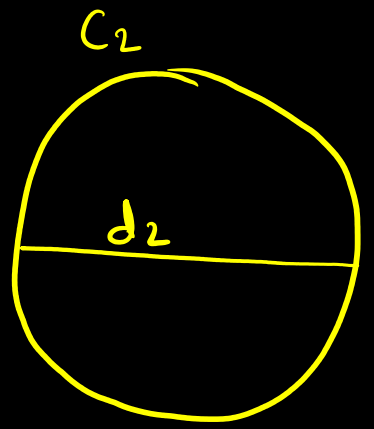


Circumference of circle = $2\pi r$

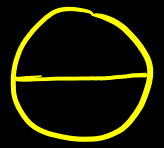
$\pi = 3.14 = \frac{22}{7}$



$\frac{C_1}{d_1} = 3.14....$



$\frac{C_2}{d_2} = 3.14....$



$\frac{C_3}{d_3} = 3.14....$

$\frac{C}{d} = \pi$

$\pi \rightarrow$ Ratio of circumference to the diameter of a given circle.

$$\frac{C}{d} = \pi$$

$$C = \pi \cdot d$$

$$C = \pi (2r)$$

$$\boxed{C = 2\pi r}$$

Q. Find the diameter of a circle whose circumference is 15.7 cm.

Sol. Given, $C = 15.7$ cm

Diameter = ? → $d = 2 \cdot r$

formula : $C = 2\pi r$

$C = \pi \cdot d$

$d = \frac{C}{\pi} = \frac{15.7 \text{ cm}}{\left(\frac{22}{7}\right)} = 15.7 \div \frac{22}{7}$

$= 15.7 \times \frac{7}{22}$

$= \frac{109.9}{22}$ cm $= \sim 5$ cm

$15.7 \times \frac{7}{22} = \frac{157}{10} \times \frac{7}{22} = \frac{1099}{220}$

$15.7 \times \frac{35}{22} \Rightarrow \frac{157}{10_2} \times \frac{35}{22} 7$

15.7×35

$= 549.5$

15.7×7

$= 109.9$

109.9

1099

Q. The ratio of the radii of two circles is 2 : 5. What is the ratio of their circumferences?

$r_1 : r_2 = 2 : 5$

$\boxed{\frac{r_1}{r_2} = \frac{2}{5}}$

$\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{2}{5}$

$\frac{C_1}{C_2} = \frac{r_1}{r_2}$

$\boxed{\frac{C_1}{C_2} = \frac{2}{5}}$

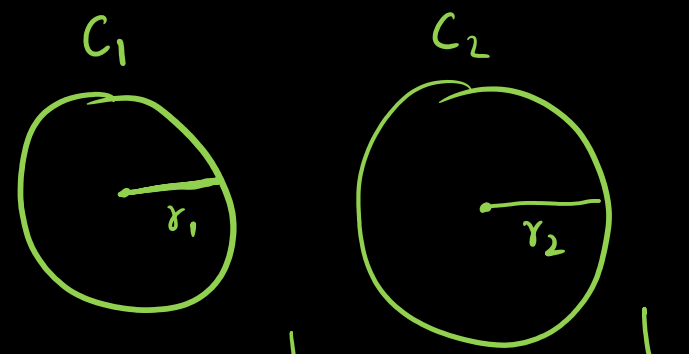
method II

$\boxed{\frac{2\pi(2x)}{2\pi(5x)} = \frac{2}{5}}$

$\boxed{C_1 : C_2 = 2 : 5}$

2 : 5.

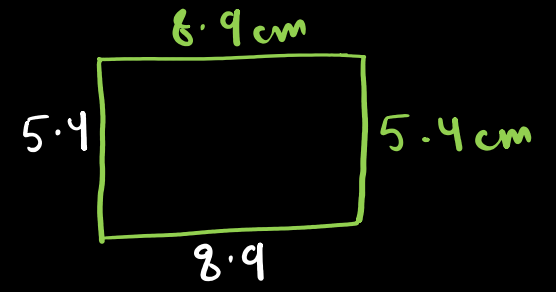
$2x$ $5x$



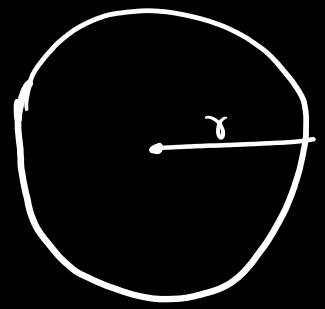
$r_1 = 2x$ $r_2 = 5x$

$\frac{r_1}{r_2} = \frac{2x}{5x} = \frac{2}{5}$

Q. A piece of wire in the form of a rectangle 8.9 cm long and 5.4 cm broad is reshaped and bent into the form of a circle. Find the radius of the circle.



$\Rightarrow$



Perimeter of rectangle $= 2(8.9 + 5.4)$

$= 2 \times (14.3)$

$= 28.6$ cm

↑ length of wire

Circumference of circle $= 28.6$

$2\pi r = 28.6$

$r = \frac{28.6}{2\pi} = \frac{14.3}{\pi}$

$= 14.3 \times \frac{7}{22} = \frac{100.1}{22}$

$= 4.5\bar{5}$ cm

14.3
x 7

Q. The circumference of a circle exceeds the diameter by 30 cm. Find the radius of the circle.

Sl.

$$C = (d + 30) \text{ cm}$$

$$2\pi r = 2r + 30$$

$$2\pi r - 2r \times 1 = 30$$

$$2r(\pi - 1) = 30$$

$2r\pi - 2r$

$$2r\left(\frac{22}{7} - 1\right) = 30$$

$$2r \times \left(\frac{15}{7}\right) = 30$$

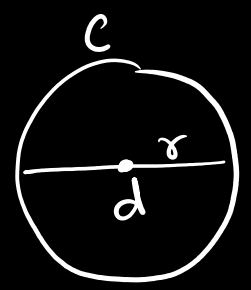
$$2r = 30 \div \frac{15}{7}$$

$$2r = \overset{2}{\cancel{30}} \times \frac{7}{\cancel{15}}$$

$$2r = 2 \times 7$$

$$2r = 14$$

$$r = 7 \text{ cm}$$



$$\frac{22}{7} - \frac{1}{1} \times \frac{7}{7}$$

$$\frac{22}{7} - \frac{7}{7}$$

$$\frac{22-7}{7}$$

$$= \frac{15}{7}$$

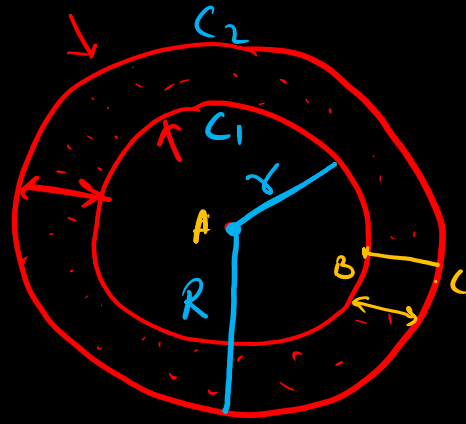
$2a + 2b$

$2(a + b)$

$2a + 2 \times 1$

$2(a + 1)$

Q. A race track is in the form of a ring whose inner circumference is 352 m and the outer circumference is 396 m. Find the width of the track.



$\Rightarrow$ Width of the track = $R - r$

$C_1 = 2\pi r$

$2\pi r = 352$

$r = \frac{352}{2\pi}$

$r = 176 \times \frac{7}{22}$

$\boxed{r = 56\ m}$

$C_2 = 2\pi R$

$2\pi R = 396$

$R = \frac{396}{2\pi} = \frac{396}{2} \times \frac{7}{22}$

$\boxed{R = 63\ m}$

Width $= R - r$

$= 63 - 56$

$= 7\ m$

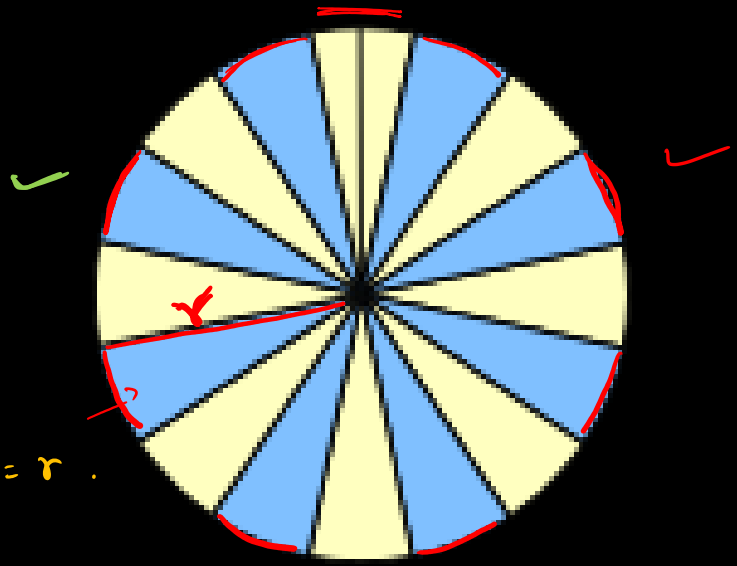
Area of Circle = πr^2

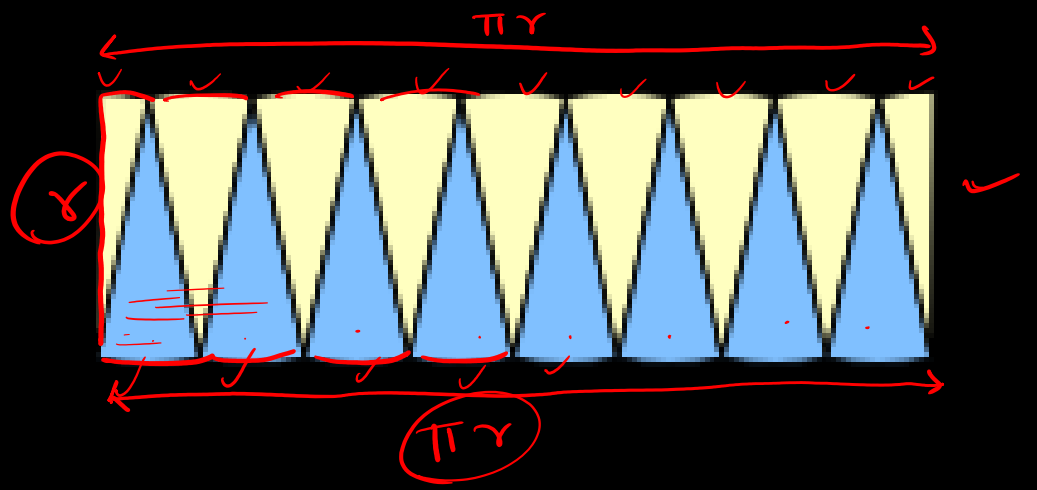
Area of circle = Area of 16 pieces

Area of 16 pieces = area of rectangle with $l = \pi r$ & $w = r$.

area of rectangle = $l \times w = \pi r \times r = \pi r^2$

Area of circle = πr^2





Relation between circumference and area of a circle.

$$C = 2\pi r \qquad A = \pi r^2$$

$$\frac{C}{2} \times r = \frac{2\pi r}{2} \times r = \pi r^2 = A$$

$$\boxed{A = \frac{C \times r}{2}}$$

$$A = \pi r^2$$

$$r^2 = \frac{A}{\pi}$$

$$\boxed{r = \sqrt{\frac{A}{\pi}}}$$

$$A = \pi r^2$$

Ar. Semi-circle $= \frac{\pi r^2}{2}$

Ar. of quadrant $= \frac{\pi r^2}{4}$

Q. The area of a circle is $616\ cm^2$. Find the radius of the circle.

Sol. Given; Area $= 616\ cm^2$

Let the radius of circle $= r$

$\therefore$ Area $= \pi r^2$

$\pi r^2 = 616$

$r^2 = \frac{616}{\pi} = 616 \div \pi = 616 \div \frac{22}{7} = {}^{28}\cancel{616} \times \frac{7}{\cancel{22}}$

$r^2 = 28 \times 7 = 196$

$r = \sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7} = \sqrt{14^2}$

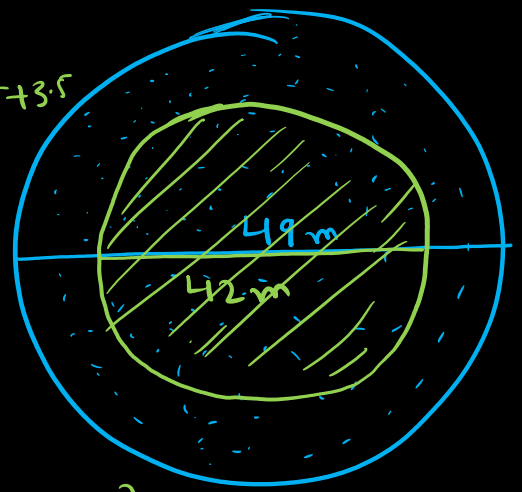
$= 2 \times 7 = 14\ cm$

$C = 2\pi r$

$r = \frac{C}{2\pi}$

2	196
2	98
7	49
7	7
	1

Q. A circular grassy plot of land, 42 m in diameter, has a path 3.5m wide running round it on the outside. find the cost of gravelling the path at $4 per square metre.



Diameter of outer circle = 42 + 3.5 + 3.5
= 49m

radius of outer circle = 24.5m

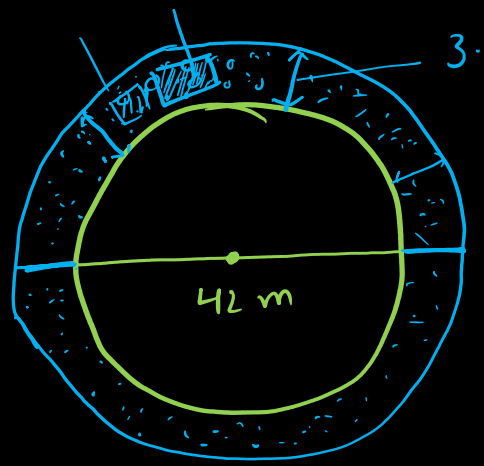
Area of outer circle = $\pi (24.5)^2$

Area of inner circle = $\pi (21)^2$

Area of path = Area of outer circle − Area of inner circle

$= \pi (24.5)^2 - \pi (21)^2$

$= 1886.5 - 1386 = 500.5\ m^2$



Diameter of inner circle = 42m

radius of inner circle = $\frac{42}{2} = 21m$

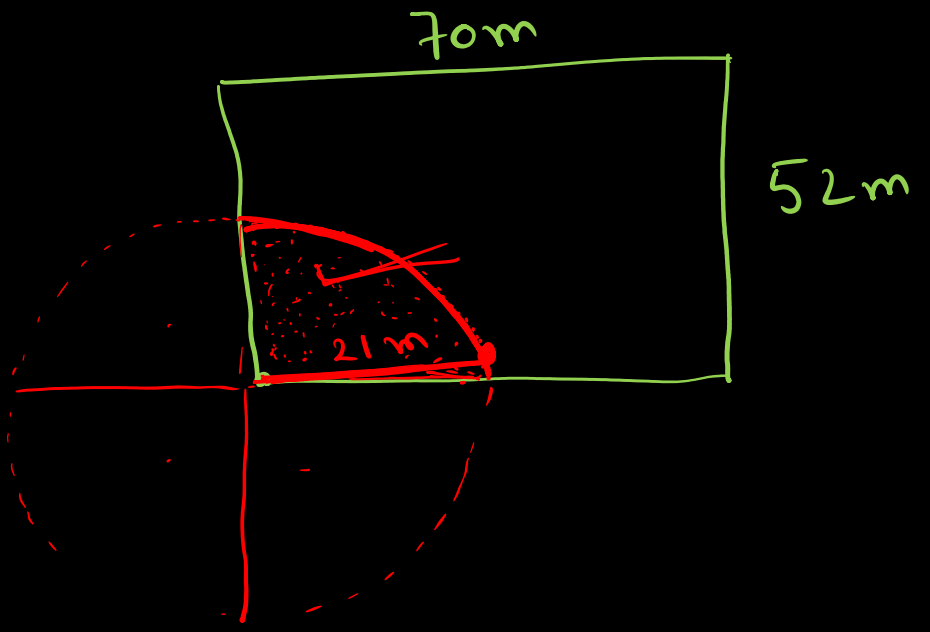
Cost of gravelling = $ 4 × 500.5

= $ 2002

Q. A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze?

= Quadrant

$r = 21$ m



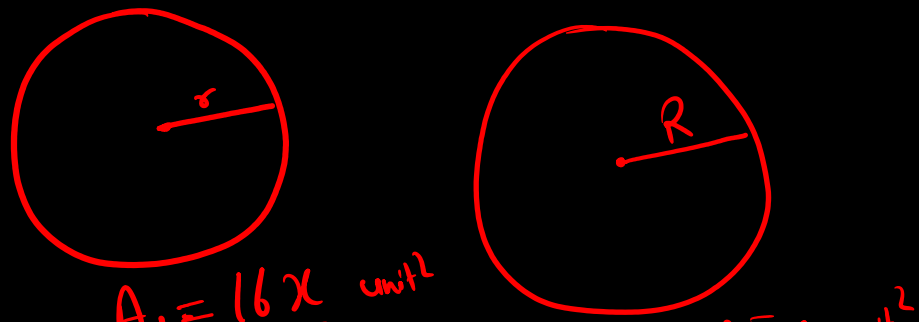
Area horse can graze $= \frac{1}{4}\pi r^2$

$= \frac{1}{4_2} \times \frac{22}{7} \times 3 \times 21$

$= \frac{33 \times 21}{2} = \frac{693}{2} = 346.5 \text{ m}^2$

Q. The areas of two circles are in the ratio 16 : 25. Find the ratio of their circumferences.

Sol: Let r and R be the radii of two circles and A_1 and A_2 be their areas respectively.



$A_1 = 16x$ unit^2

$A_2 = 25x$ unit^2

$A_1 = \pi r^2$

$A_2 = \pi R^2$

$\because \quad A_1 : A_2 = 16 : 25$

$\Rightarrow \quad \frac{A_1}{A_2} = \frac{16}{25}$

$\Rightarrow \quad \frac{\pi r^2}{\pi R^2} = \frac{16}{25}$

$\Rightarrow \quad \frac{r^2}{R^2} = \frac{4^2}{5^2}$

$\Rightarrow \quad \boxed{\frac{r}{R} = \frac{4}{5}}$

Let C_1 and C_2 be the circumferences.

$C_1 = 2\pi r, \quad C_2 = 2\pi R$

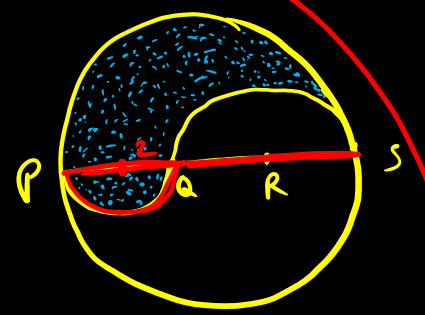
$\therefore \quad \frac{C_1}{C_2} = \frac{2\pi r}{2\pi R} = \frac{r}{R} = \frac{4}{5}$

$\Rightarrow \quad \frac{C_1}{C_2} = \frac{4}{5}$ or $\boxed{C_1 : C_2 = 4 : 5}$

$\therefore$ ⇒ Therefore

$\because$ ⇒ Since.

Q. PQRS is a diameter of a circle of radius is 6 cm. The length PQ, QR, and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in the figure. Find the area of the shaded region.



Sol: $r = 6\text{cm}$ (given)

Length $\frac{PQRS}{PS}$ (diameter) $= 2r = 12\text{cm}$.

$\because PQ = QR = RS = \frac{1}{3}PS$

$\therefore PQ = 4\text{ cm}$, $QR = 4\text{cm}$, $RS = 4\text{cm}$

Required area $= [$(Area of semi circle on PS $-$ Area of semicircle on QS) $+$ Area of semicircle on PQ$]$

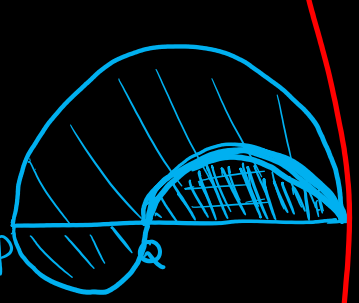
$= \left[\left(\frac{1}{2}\pi(6)^2 - \frac{1}{2}\pi(4)^2\right) + \frac{1}{2}\pi(2)^2\right]$

$= \frac{1}{2}\times\pi\left[6^2 - 4^2 + 2^2\right]$

$= \frac{1}{2}\times\frac{22}{7}(36-16+4)$

$= \frac{11}{7}\times(24)$

$= \frac{264}{7} = 37.71\text{cm}^2$



$\frac{1}{2}\pi r^2$

Area of semicircle $= \frac{1}{2}\pi r^2$

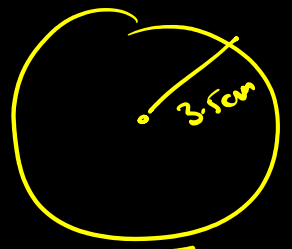
$C = 2\pi r$

$A = \pi r^2$

$r = ?$

$\pi = 3.14$ or $\frac{22}{7}$

$C = 2\pi r$



$A = ?$ $\quad A = \pi r^2$

given $C = 22\text{ cm}$

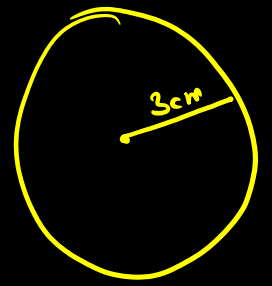
$C = 2\pi r$

$2\pi r = 22$

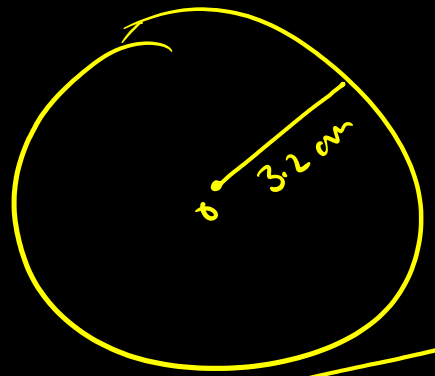
$\pi r = \frac{22}{2} = 11$

$r = \frac{11}{\pi} = \frac{11}{1} \div \frac{22}{7}$

$= \frac{11}{1} \times \frac{7}{22} = 3.5\text{ cm}$



$A = \pi r^2$



Area = 32 cm^2 given

Circumference = ?

$C = 2\pi r$

$C = 2 \times \frac{22}{7} \times 3.2$

C = ——— cm

$A = \pi r^2$

$\pi r^2 = 32$

$r^2 = \frac{32}{\pi}$

$r^2 = \frac{\cancel{32}^{16}}{1} \times \frac{7}{\cancel{22}_{11}} = \frac{112}{11}$

$r^2 = \frac{112}{11}$

$r = \sqrt{\frac{112}{11}} = \sim 3.2$ cm

$r = 3.2$ cm

Topic Completed