Area and Volume

Rectangles, Triangles, Prism

Rectilinear Figures

1. Closed fig. bound by 3 or more straight lines

eg: . triangle 

. Rectangle 

. Parallelogram 

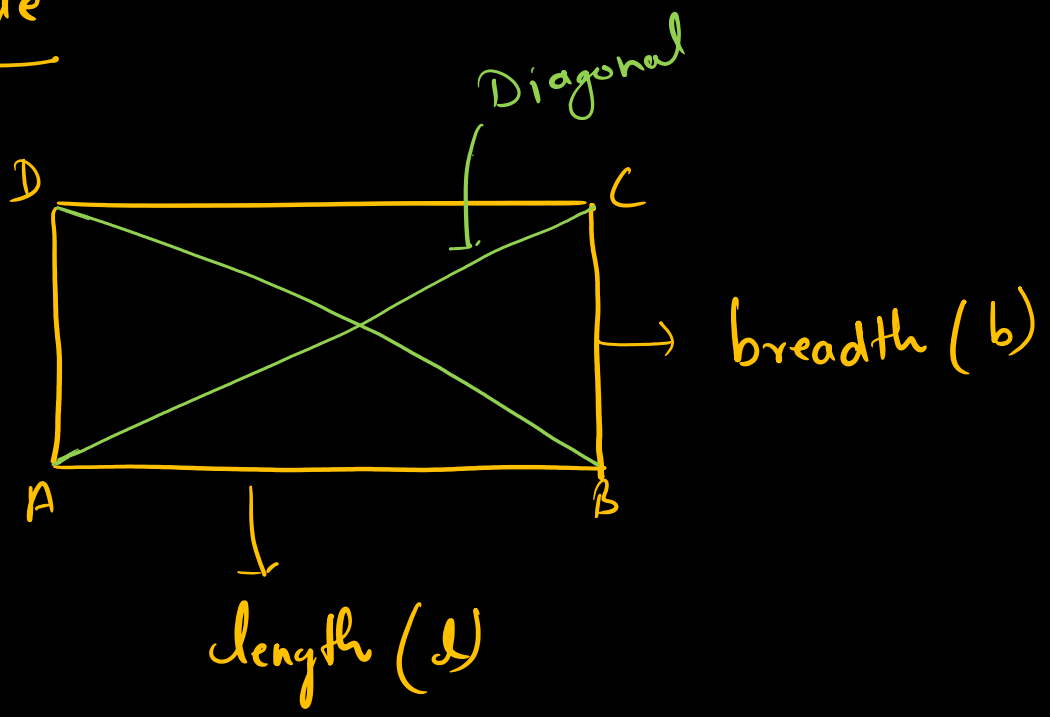
. Trapezium 

Perimeter and Area of Rectangle

⇒ A rectangle ABCD.

⇒ opposite sides are equal

⇒ two diagonals are of same length.

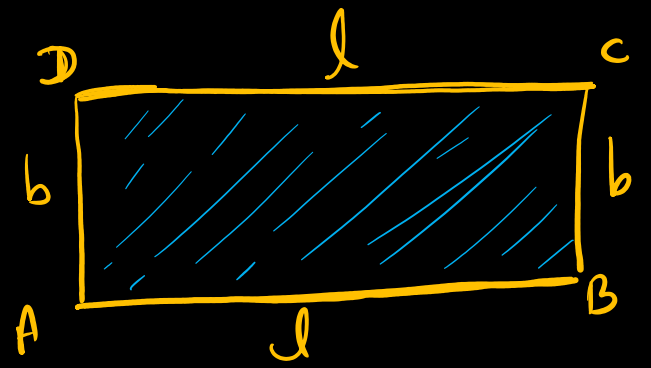


Perimeter of Rectangle = $l + b + l + b$

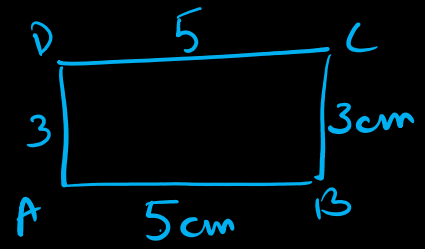
$= 2l + 2b$

$$\text{Perimeter} = 2(l + b)$$

$$\text{Area of rectangle} = l \times b$$



Eg.



Perimeter = $2(5+3)$ cm

$= 2 \times 8$

$= 16$ cm

Area = $5\,\underline{cm} \times 3\,\underline{cm}$

$= 15\ \underline{\underline{cm^2}}$

Add/sub of units.

unit + unit = unit

unit − unit = unit

multiplication of unit

unit × unit = $unit^2$

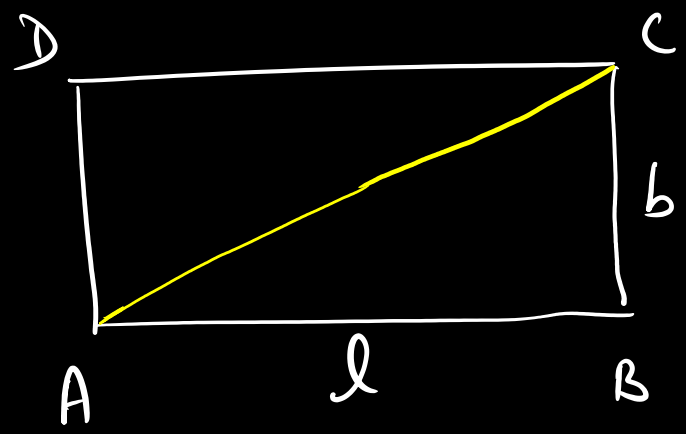
unit × unit × unit = $unit^3$

Division of units

$\frac{unit}{unit} = 1$ unit less (no unit)

$\frac{5\,cm + 3\,cm}{2\,cm} = \frac{8\,cm}{2\,cm} = 4$

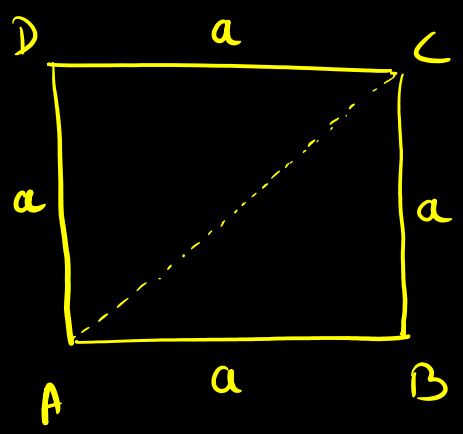
Diagonal of rectangle



length of diagonal AC

$$\text{Diagonal} = \sqrt{l^2 + b^2}$$

Perimeter and Area of Square



$$\text{Perimeter} = 4a$$

$$\text{Area} = a \times a = a^2$$

$$\text{Area} = a^2 \Rightarrow a = \sqrt{\text{Area}}$$

$$\sqrt{a^2} = a$$

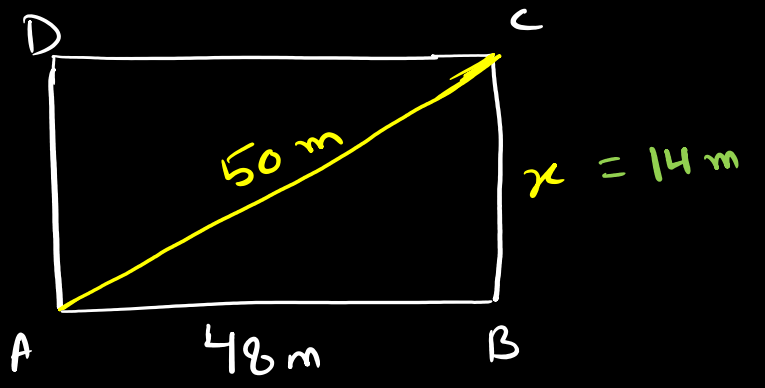
$$\text{Diagonal} = \sqrt{a^2 + a^2}$$

$$= \sqrt{2a^2}$$

$$\text{Diagonal} = a\sqrt{2}$$

Q. Find the area of a rectangular plot one side of which is 48 m and its diagonal 50 m.

Sol: Let us assume that width of rectangle is x m



$$\text{Diagonal} = \sqrt{l^2 + b^2}$$

$$50 = \sqrt{(48)^2 + x^2}$$

$$\Rightarrow 50^2 = 48^2 + x^2$$

$$\Rightarrow x^2 = 50^2 - 48^2$$

$$\Rightarrow x^2 = 2500 - 2304 = 196$$

$$x^2 = 196$$

$$x = \sqrt{196}$$

$$x = \sqrt{14^2}$$

$$\boxed{x = 14}$$

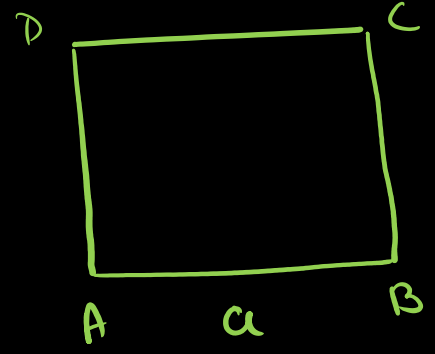
$\therefore$ area $= 48\text{m} \times 14\text{m}$

$$\boxed{\text{area} = 672\ \text{m}^2}$$

Q. Find the area of a square park whose perimeter is 320 m.

Sol.

Area of square = a^2



Given,

Perimeter = 320 m

$4a = 320$

$a = \frac{320}{4}\ m = 80\ m$

Area = 80^2

Area = 6400 m^2

Q. The perimeter of a rectangle is 100 cm. If the length is 35 cm, find its breadth and also find its area.

Sol: Given, Perimeter = 100 cm

$2(l+b) = 100$ cm

$l+b = \frac{100}{2}$ cm

$l+b = 50$ cm

$35+b = 50$ cm

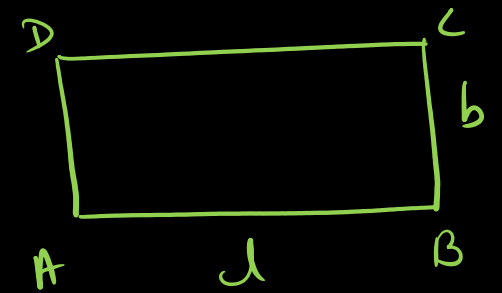
$b = 50-35$ cm

$\boxed{b = 15 \text{ cm}}$

Area $= l \times b$

$= 35 \times 15$

Area $= 525 \text{ cm}^2$



$l = 35$ cm

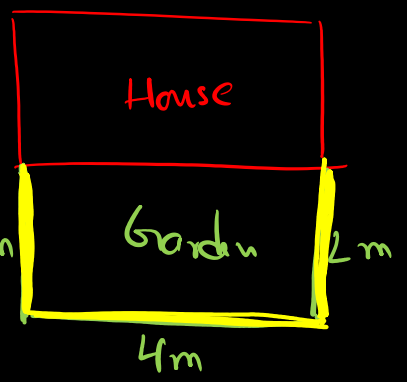
$$\sqrt{2a^2} = \sqrt{2 \times a^2}$$

$$= \sqrt{2 \times a \times a}$$

$$= a$$

$$= a\sqrt{2}$$

Find the cost of fencing this garden at the rate of ₹ 150 per metre.



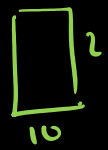
Length of required fence = 2m + 4m + 2m

= 8m

Cost of fencing : ₹ (150 × 8)

= ₹ 1200

Q. A wall of length 4.84 m and height 3.1 m is covered with rectangular tiles of size 22 cm by 10 cm. Find the total cost of the tiles at the rate of ₹ 1.50 per tile.



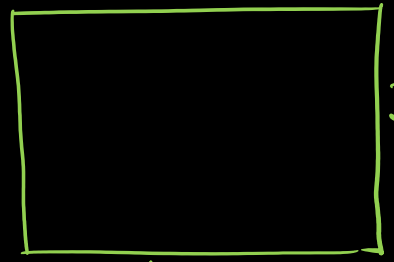
area of tile = 22 cm × 10 cm
= 220 cm²

No. of tiles = $\frac{150040\ \text{cm}^2}{220\ \text{cm}^2}$

= 682

Cost of one tile = ₹ 1.50

Total cost = ₹ 1.50 × 682 = ₹ 1023



3.1 m ⇒ 3.1 × 100 cm

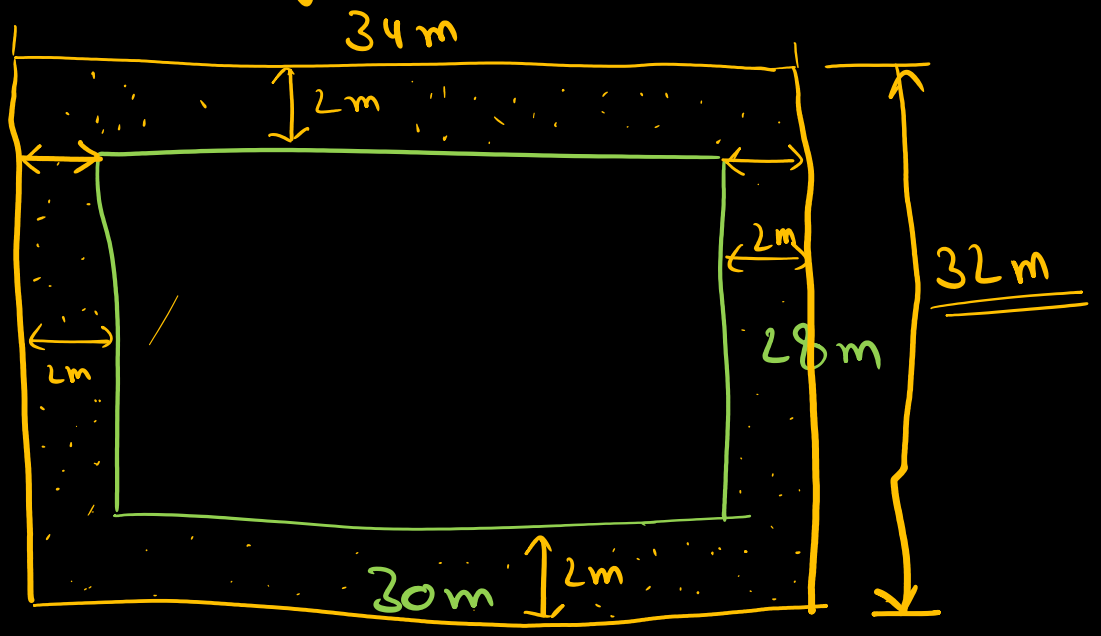
4.84 × 100 cm

Area of wall = 3.1 × 4.84 × 100 × 100 cm²

= 15.004 × 10000

= 150040 cm²

Q. A rectangular lawn 30m by 28m is to be surrounded by a path 2m wide. Find the cost of leveling the path at the rate of ₹ 15 per square meter.



Area of path = Area of outer rectangle − Area of inner rectangle

$= (34m \times 32m) - (30m \times 28m)$

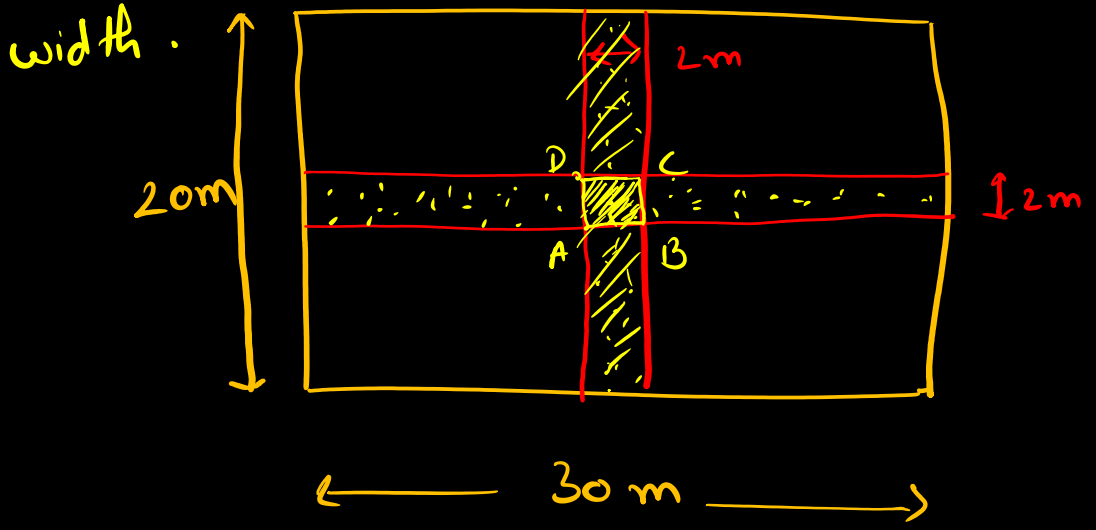
$= 1088 m^2 - 840 m^2$

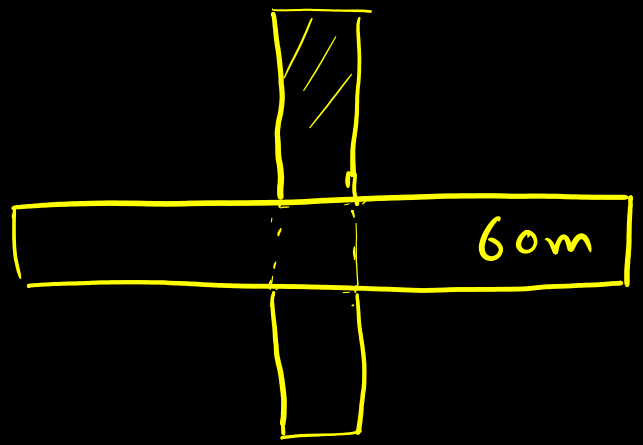
$= 248 m^2$

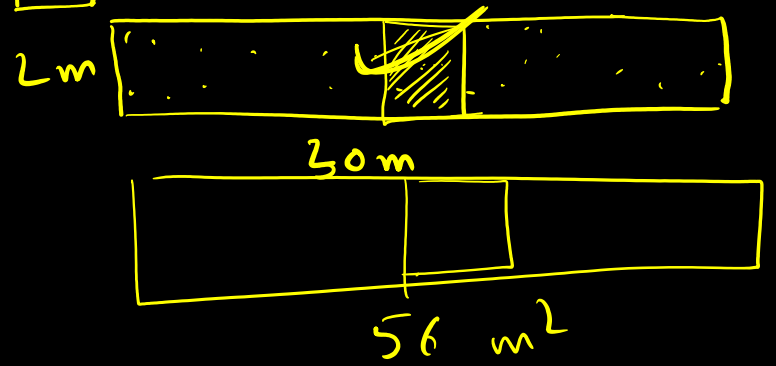
Cost of traveling the path = ₹ (248 × 15)

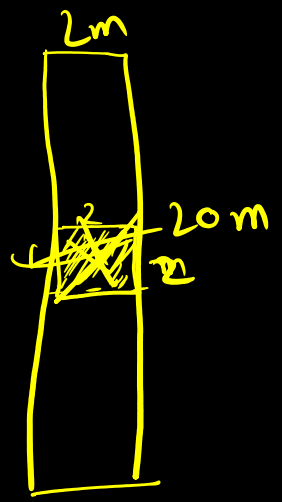
= ₹ 3720

Q. A rectangular lawn is 30 m by 20 m. It has two roads each 2m wide running in the middle of it, one parallel to the length and the other parallel to the width. Find the area of the roads

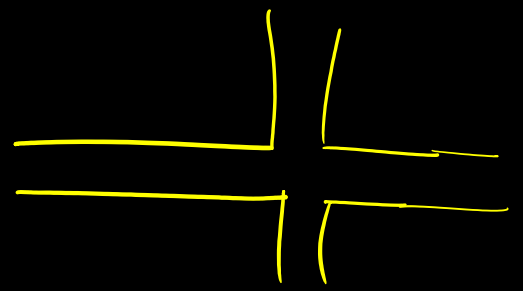








40 m²



Area of long road = $2m \times 30m = 60m^2$

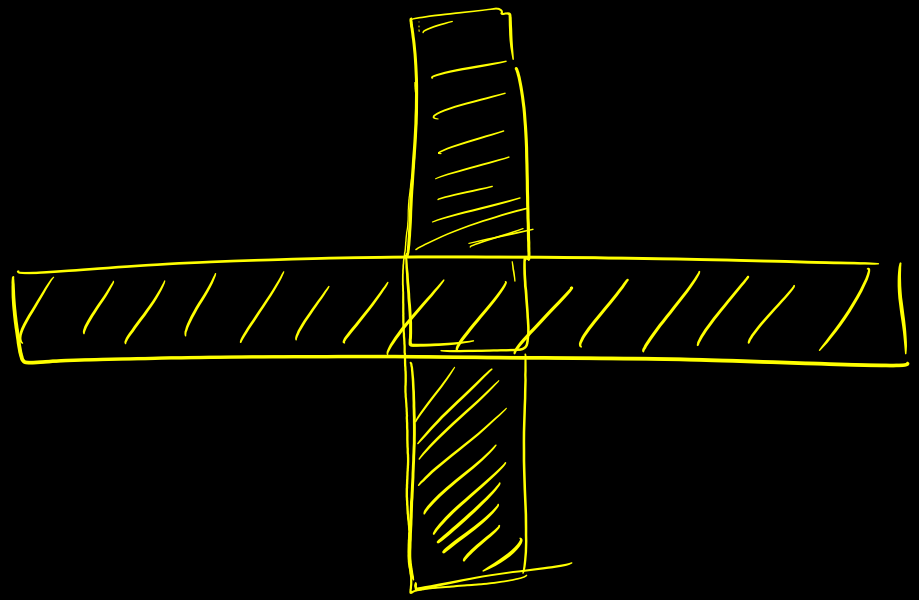
Area of short road = $2m \times 20m = 40m^2$

Area of square ABCD = $2m \times 2m = \underline{4m^2}$

Area of roads = $40m^2 + 60m^2 - 4m^2$

$= 100m^2 - 4m^2$

$= \underline{\underline{96m^2}}$

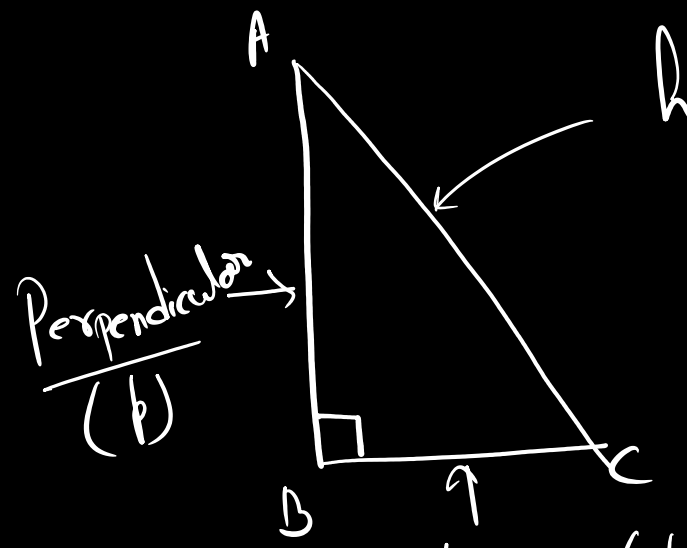


Area of Triangle

Pythagoras Theorem

Right angle triangle

$$(AC)^2 = (AB)^2 + (BC)^2$$



$$h^2 = p^2 + b^2$$

By Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

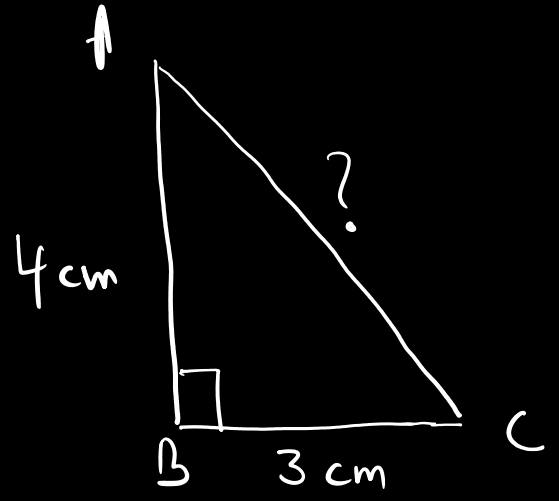
$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$\boxed{AC = 5 \text{ cm}}$$



Imp:

- Hypotenuse is the longest side in right triangle.
- Hypotenuse is always less than the sum of perpendicular and base.

Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

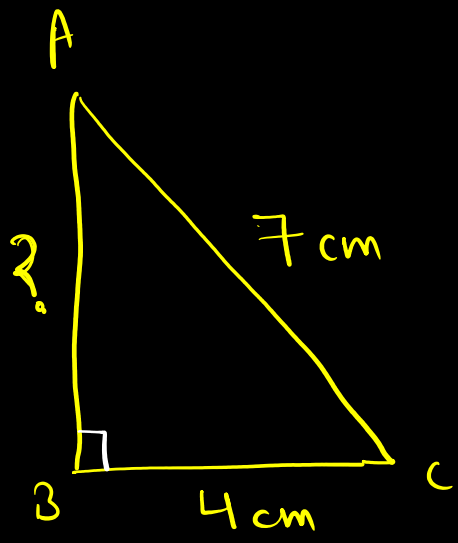
$$AB^2 = AC^2 - BC^2$$

$$AB^2 = 7^2 - 4^2$$

$$AB^2 = 49 - 16$$

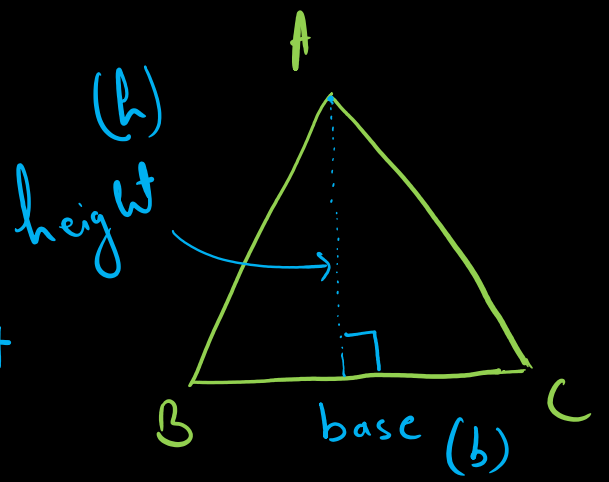
$$AB^2 = 33$$

$$AB = \sqrt{33} \text{ cm}$$

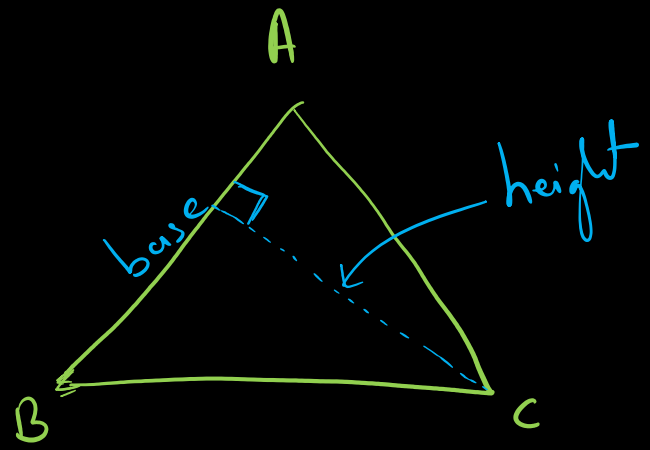


Area of triangle.

Area of triangle ABC = $ar(\Delta ABC) = \frac{1}{2} \times \text{Base} \times \text{height}$



$$ar(\Delta ABC) = \frac{1}{2} \times b \times h$$



height ≡ altitude

Q. Find the area of triangle whose base and altitude (height) are: 15 cm and 8 cm respectively.

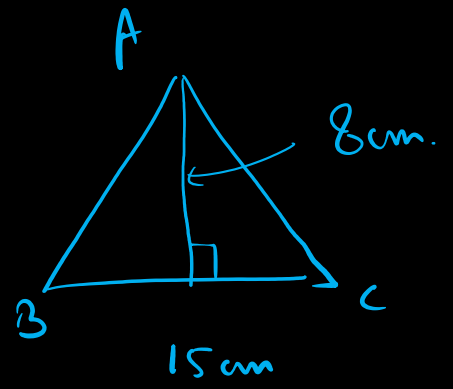
Sol: Given height = 8cm, base = 15cm

$$ar(\triangle ABC) = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 15\ cm \times \cancel{8\ cm}\ 4\ cm$$

$$= 15\ cm \times 4\ cm$$

$$\boxed{ar(\triangle ABC) = 60\ cm^2}$$

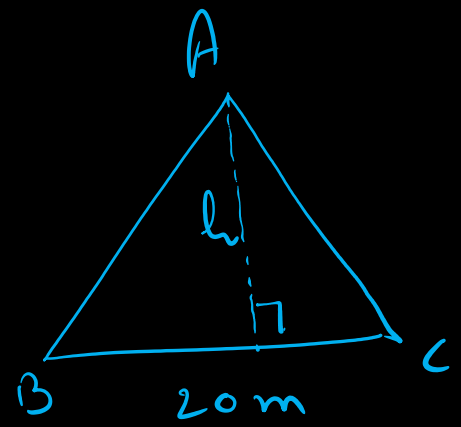


Q: Area of a triangle is $50\ m^2$. If one of its side is 20m, find the hight of triangle from this side.

Sol: **Given:** $ar = 50\ m^2$

$base = 20\ m$

$h = ?$



$$ar(\Delta ABC) = \frac{1}{2} \times b \times h$$

$$b \times h = 2 \times ar(\Delta ABC)$$

$$h = \frac{2 \times ar(\Delta ABC)}{b}$$

$$= \frac{2 \times 50\ m^2}{20}$$

$$h = 5\ m$$

$2x = 4$

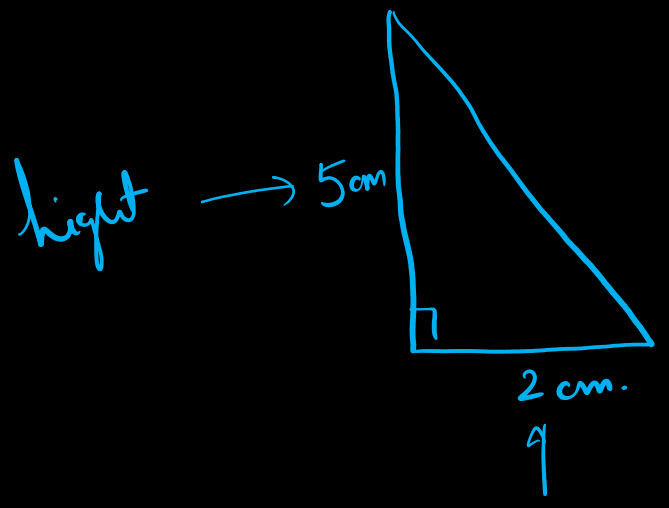
$x = \frac{4}{2}$

$x = \frac{4}{2} \Rightarrow 2x = 4$

Q. Area of triangle is $65\ cm^2$. If height of triangle is 7 cm, find its base.

Ans: ~ 18.5 cm

Q. Find the area of right angle triangle as described in the figure.



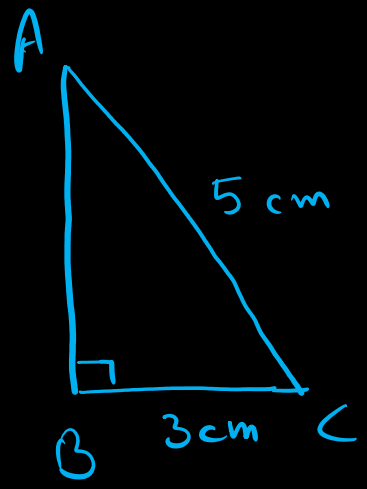
Sol.

$$ar(\text{rt.}\Delta) = \frac{1}{2} \times 5\,\text{cm} \times 2\,\text{cm}$$

$$= 5\,\text{cm}^2$$

{hight, base} legs of right triangle.

Q. Find the area of right angle triangle as described in the figure!



Sol:

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2$$

$$= 25 - 9$$

$$AB^2 = 16 \text{ cm}$$

$$AB = \sqrt{16} \text{ cm}$$

$$\boxed{AB = 4 \text{ cm}}$$

$$\text{ar}(\Delta) = \frac{1}{2} \times \overset{2\text{cm}}{\cancel{4\text{cm}}} \times 3\text{cm} = 2\text{cm} \times 3\text{cm} = \underline{6\text{cm}^2}.$$