

Operations on Whole Numbers

Chapter 2

Operations on whole no.

Whole no. \Rightarrow 0, 1, 2, 3, ... ∞

Properties of Addition ✓

① Closure Property:

If a & b whole no.

then, $a+b$ is also a whole no.

Eg: 5 and 6

$$\begin{array}{ccc} 5 & + & 6 & = & 11 \\ \hline & & & & \downarrow \\ & & & & \text{Whole no.} \end{array}$$

eg: 12, 0 \Rightarrow whole no.

$$\begin{array}{ccc} 12 & + & 0 & = & 12 \\ & & & & \downarrow \\ & & & & \text{whole no.} \end{array}$$

\Rightarrow whole nos. are closed under addition.

\Rightarrow closure property holds for add. of whole nos.

② Commutative Property for addition

a, b are whole nos.

$$a + b = b + a$$

eg. 5 and 7 are whole nos.

$$5 + 7 = 12$$

$$7 + 5 = 12$$

$$5 + 7 = 7 + 5$$

retains its identity.

$$5 + 0 = 5$$

↑
Additive identity.

⇒ Zero is additive identity.

⇒ Commutative property holds for addition of whole nos.

③ Associative Property

If a , b and c are three whole nos., then

$$\boxed{(a+b)+c = a+(b+c)}$$

eg. 5 , 2 , 7 are whole nos.

$$\left. \begin{array}{l} (5+2)+7 = 14 \\ 5+(2+7) = 14 \end{array} \right\}$$

\Rightarrow Associative property holds for addition of whole nos.

Ex. $1 + 2 + 3 + 4 + 996 + 997 + 998 + 999 = \underline{\underline{4000}}$

Ex. 2 $\underline{2062} + \underline{353} + \underline{1438} + \underline{547}$

$\Rightarrow (\underline{2062} + \underline{1438}) + (\underline{353} + \underline{547})$

$= 3500 + 900$

$= \underline{\underline{4400}}$

$$\begin{array}{r} 353 = 300 + \underline{53} \\ + 547 = 500 + \underline{47} \\ \hline = 800 + 100 \\ = \underline{\underline{900}} \end{array}$$

Properties of Subtraction

① Closure Property

↳ does not hold for subtraction of whole nos.

eg.
$$\begin{array}{ccc} 5 - 3 = 2 \\ \uparrow \quad \uparrow \quad \uparrow \\ W \quad W \quad W \end{array}$$

$$\begin{array}{ccc} 3 - 5 = -2 \\ \uparrow \quad \uparrow \quad \uparrow \\ W \quad W \quad \underline{\underline{\text{Not } W}} \end{array}$$

②

Commutative

$$5 - 3 = \underline{\underline{2}}$$

$$3 - 5 = \underline{\underline{-2}}$$

$$5 - 3 \neq 3 - 5$$

\Rightarrow Commutative property does not hold for subtraction of whole no.

(iii) Associative

⇒

$$5 - (3 - 1) = 3$$

$$(5 - 3) - 1 = 1$$

$$5 - (3 - 1) \neq (5 - 3) - 1$$

⇒ Subtraction of whole no. is not associative.

(iv) If a, b and c are 3 whole nos. such that,

$a - b = c$, then $b + c = a$

for ex.

$5 - 3 = 2$
 $\Downarrow \quad \Downarrow \quad \Downarrow$
 $a \quad b \quad c$

$3 + 2 = 5$

Properties of multiplication

① Closure Property

↓

It holds for multiplication

$$\begin{array}{ccccccc} \text{eg} & & 7 & \times & 2 & = & 14 \\ & & \uparrow & & \uparrow & & \uparrow \\ & & \mathbb{N} & & \mathbb{N} & & \mathbb{N} \end{array}$$

* 1 is called multiplicative identity

$$\begin{array}{l} 13 \times 1 = 13 \\ 7 \times 1 = 7 \end{array}$$

ii) Commutative

$$2 \times 3 = 3 \times 2$$

\Rightarrow multiplication of whole no. is commutative

iii) Associative Property. $a, b \text{ \& } c$

$$(a \times b) \times c = a \times (b \times c)$$

$$(2 \times 3) \times 4 = 24$$

$$2 \times (3 \times 4) = 24$$

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

\Rightarrow multiplication of whole nos. is associative.

(iv) Distributive property

→ Distributivity of multiplication over addition / subtraction

(i) a, b & c are whole nos.

$$a \times (b + c) = a \times b + a \times c$$

$$\begin{aligned} 2 \times (5 + 3) &= 2 \times 5 + 2 \times 3 \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

$$\underline{\underline{\text{eg.}}} \quad 3 \times (7 + 15) = 3 \times 22 = 66$$

$$\begin{aligned} 3 \times (7 + 15) &= 3 \times 7 + 3 \times 15 \\ &= 21 + 45 \\ &= \underline{\underline{66}} \end{aligned}$$

$$\underline{\underline{\text{eg.}}} \quad 5 \times (19 - 2) = 5 \times 17 = \underline{\underline{85}}$$

$$\begin{aligned} \checkmark \quad 5 \times (19 - 2) &= 5 \times 19 - 5 \times 2 \\ &= 95 - 10 \\ &= \underline{\underline{85}} \end{aligned}$$

Q. Find the products:

$$\begin{aligned} \text{(i)} \quad 4 \times 4957 \times 25 &= 4957 \times 25 \times 4 \\ &= 4957 \times 100 \\ &= 495700 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 37256 \times 25 \times 9 \times 40 &= (37256 \times 9) \times (25 \times 40) \\ &= \underline{335304} \times 1000 \\ &= 335304000 \end{aligned}$$

eg. multiply : 475 by 64 using distributive property.

Multiply 5217 by 325 using distributive property

$$5217 = (5000 + 200 + 10 + 7) \times 325$$

$$= 5000 \times 325 + 200 \times 325 + 10 \times 325 + 7 \times 325$$

$$= 162500 + 65000 + 3250 + 2275$$

$$= \underline{1695525}$$

$$72398 \times 416$$

$$= 30117568$$

$$\begin{aligned} \textcircled{1}0000 \times 416 &= 4160000 \\ \textcircled{2}0000 \times \textcircled{4}06 &= 832000 \end{aligned}$$

$$200 \times 42 = 8400$$

$$= \textcircled{7}0000 \times 416 + 2000 \times 416 + 300 \times 416 + 90 \times 416 + 8 \times 416$$

$$= 2912000 + 832000 + 124800 + 37440 + \textcircled{70000 \times 416}$$

$$7 \times \textcircled{4}16 = \textcircled{2}912000$$

$$\begin{array}{r} 416 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 70000 \\ \times 416 \\ \hline \end{array}$$

$$\underline{\underline{736 \times 103}}$$

$$736 \times (100 + 3)$$

$$736 \times 100 \oplus \boxed{736 \times 3}$$

$$\Rightarrow \underline{\underline{73600}} + \underline{\underline{2208}}$$

$$\Rightarrow 75808$$

$$\underline{1245} + \underline{4960}$$

$$\boxed{6205}$$

$$\underline{816} \times 745$$

$$816 \times (700 + 40 + 5)$$

$$\begin{array}{r} 46 \\ 1 \times 1 \\ \times 32 \\ \hline \end{array}$$

12 27 12

1472

$$\begin{array}{r} 18 \\ + 8 \\ \hline 26 \end{array}$$

$$\checkmark \left(\underline{800} + \underline{10} + 6 \right) \times \begin{array}{r} 34 \\ \underline{745} \\ \underline{23} \end{array}$$

$$596000 + 7450 + 4470$$

$$\Rightarrow \underline{607920}$$

$$\begin{array}{r} 19 \\ 1 \uparrow \\ 24 \\ \hline \end{array} \quad \text{456}$$

2 2 5 3 6

Properties of division of whole nos.

$$\boxed{20 \div 5}$$

$$\begin{array}{r} 4 \\ 5 \overline{) 20} \end{array}$$

① Division is repeated subtraction

$$\begin{array}{r} 20 - 5 = 15 \checkmark \\ 15 - 5 = 10 \checkmark \\ 10 - 5 = 5 \checkmark \\ 5 - 5 = 0 \checkmark \end{array}$$

5 is subtracted 4 times
from 20
Hence 4 is the quotient.

Division is inverse of multiplication

eg. $12 \div (3) = (4)$ | $4 \times 3 = 12$

(i) Closure property

$$\begin{array}{ccc} 12 \div 5 & = & \underline{2.4} \\ \uparrow & & \downarrow \\ W & & \text{not a whole no.} \end{array}$$

\therefore Division of whole nos. is not closed.

(ii) Commutative Property

$$5 \div 2 \neq 2 \div 5$$

Not Commutative

(iii) Associative

$$(15 \div 3) \div 2 \neq \underline{15 \div (3 \div 2)}$$

Not associative

(iv) $0 \div (\text{whole no.}) = 0$

(v) $(\text{Whole no.}) \div 0 = \text{Can't be done (Not defined)}$

$\frac{5}{0}$ (Not defined)

$\frac{0}{5} = 0$ defined

Division Algorithm

⇒ If a whole no. 'a' is divided by a non-zero whole no. 'b', then there exists whole nos. q and r such that

$$\boxed{a = bq + r}, \text{ where}$$

$$\underline{r = 0} \text{ or } \underline{r < b}.$$

Expression:

$$\begin{array}{r} q \\ b \overline{) a} \\ \underline{-bq} \\ r \end{array}$$

Divisor

eg1

$$\begin{array}{r} \text{Quotient} \\ 2 \\ 3 \overline{) 7} \\ \underline{-6} \\ 1 \end{array}$$

Dividend

Remainder

$$\boxed{\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}}$$

eg2

$$\begin{array}{r} 2 \\ 3 \overline{) 6} \\ \underline{-6} \\ 0 \end{array}$$

Q. Divide 46087 by 356 and check the result by division algorithm.

Sl:

$$\begin{array}{r} 129 \leftarrow q \\ b \rightarrow 356 \overline{) 46087 \leftarrow a} \\ \underline{-356} \\ 1048 \\ \underline{-712} \\ 3367 \\ \underline{-3204} \\ 163 \rightarrow r \end{array}$$

$$356 \times q$$

$$\begin{array}{r} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} 4 \end{array}$$

Check: Dividend = 46087

$$\begin{aligned} \boxed{\text{Quotient} \times \text{divisor} + \text{Remainder}} &= \underline{129 \times 356} + 163 \\ &= 45926 + 163 \\ &= 46087 \Rightarrow \underline{\text{Dividend}} \end{aligned}$$

Q2: Find the number which when divided by 46 gives a quotient 11 and remainder 18.

Sol:

$$a = bq + r$$

$$a = 46 \times 11 + 18$$

$$\begin{aligned} a &= 506 + 18 \\ &= \underline{\underline{524}} \end{aligned}$$

$$\underline{\underline{a = ?}} \} \text{ To find.}$$

$$\left. \begin{aligned} b &= 46 \\ q &= 11 \\ r &= 18 \end{aligned} \right\} \text{ Given}$$

Q3 :→ The the product of two numbers is 504347. If one of the numbers is 1591, find the other.

Sol:

$$\begin{array}{r} \textcircled{317} \\ \hline \textcircled{1591} \overline{)504347} \\ \underline{-4773} \\ 2704 \\ \underline{-1591} \\ 11137 \\ \underline{-11137} \\ \boxed{0} \end{array}$$

So the other no. is 317.

Q. On dividing $\underline{55390}$ by $\underline{299}$, the remainder is $\underline{75}$. Find the quotient.

Sol: Given: (a) dividend = 55390
(b) divisor = 299
(r) remainder = 75

To find: (q) quotient = ?

Division algorithm, $\boxed{a = bq + r}$

$$55390 = 299 \times \textcircled{q} + 75$$

Subtract 75 from both side,

$$55390 - 75 = 299 \times q \quad \boxed{+75-75}$$

$$55390 - 75 = 299 \times q + 0$$

$$\frac{55315}{299} = \frac{299 \times q}{\cancel{299}}$$

$$\boxed{\frac{55315}{299} = q}$$

$$\boxed{q = \frac{55315}{299}} \quad \boxed{\text{H.W}}$$