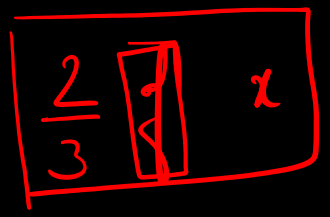
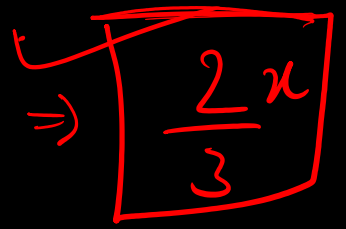
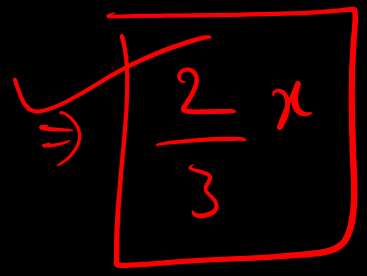
Introduction to Algebra

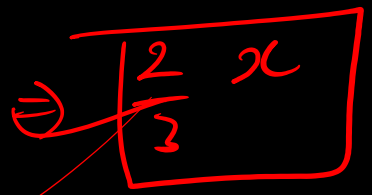




⇒ Yacht ✓

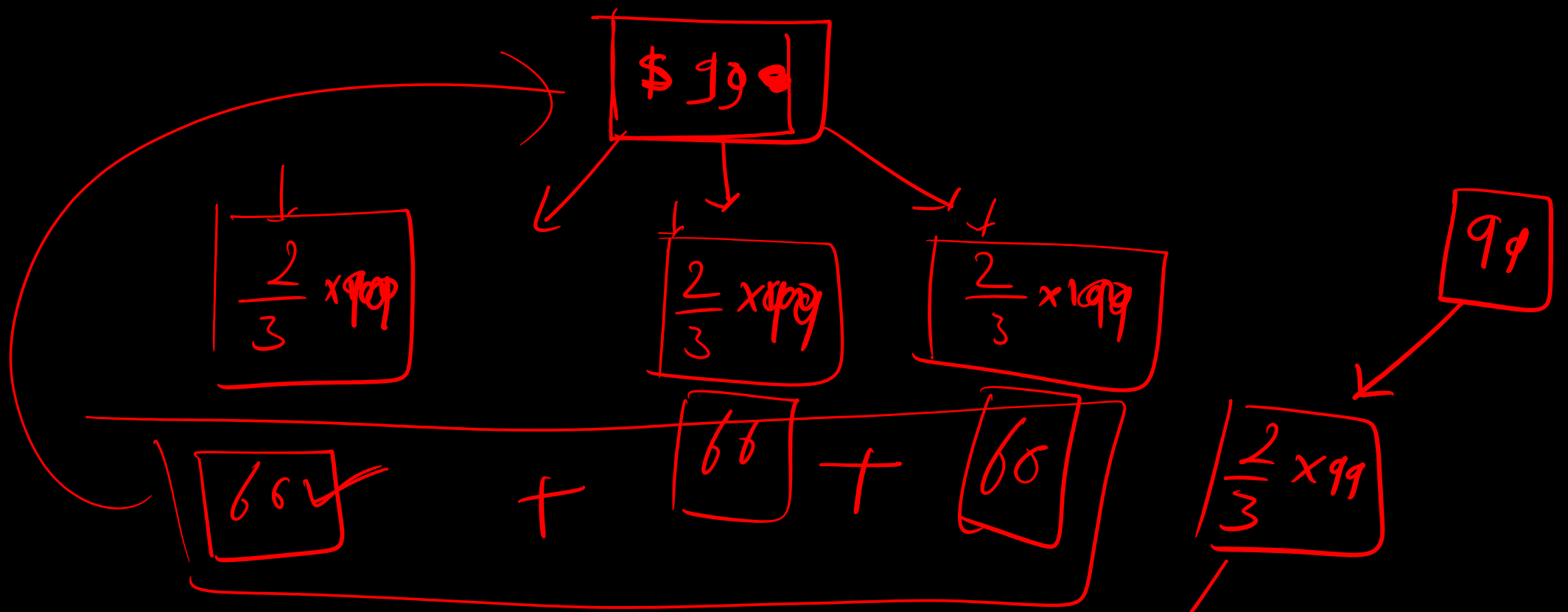


⇒ mansion. ✓



⇒ Car ✓

⇒ P. Jet. ✓

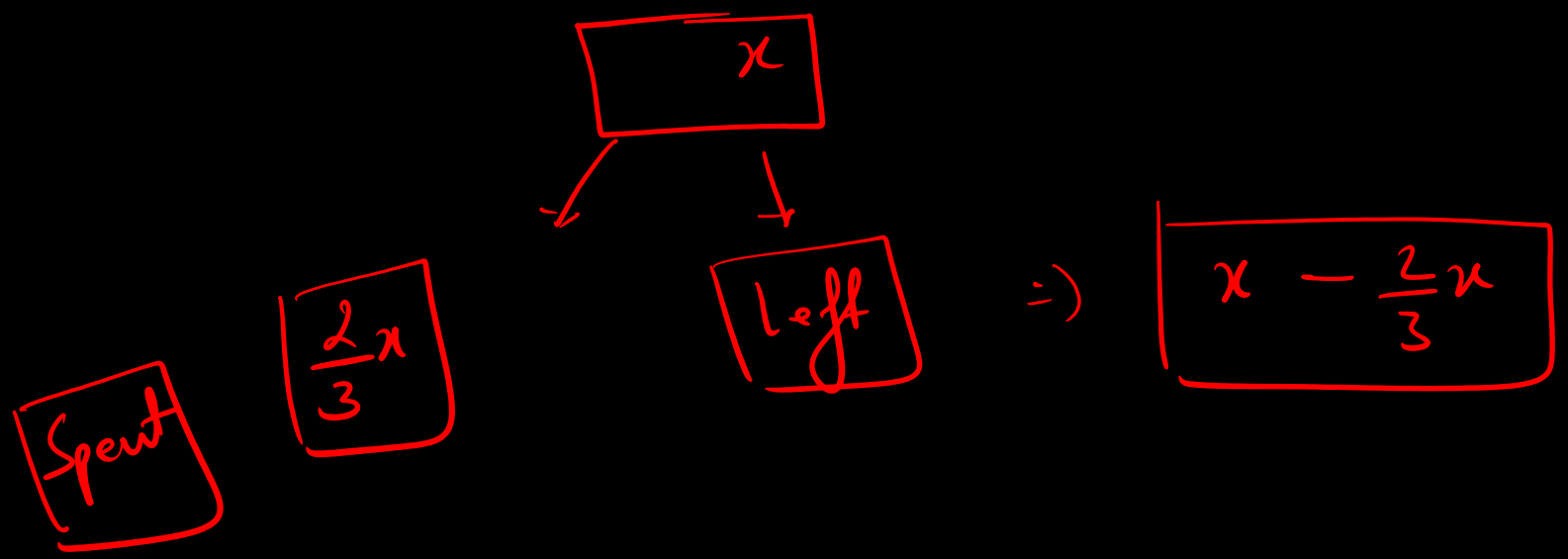


3.66

$\frac{2}{3}(11)$

$\sim 2 \times 3.67$

~ 7.34



1. Tanuj walked 8.62 km on monday, 7.05 km on tuesday and some distance on Wednesday. If he walked 21.01 km in three days, how much distance did he walk on Wednesday?

2. Mr. Sharma has 24 apples. He uses $\frac{1}{4}$ of them. How many apples does he use? How many does he have left?

Apples used $= \frac{1}{4}$ of 24

$= \frac{1}{\cancel{4}_{1}} \times \cancel{24}^{6}$

$= \underline{\underline{6}}$

Apples left $= 24 - \left(\frac{1}{4} \text{ of } 24\right)$

$= 24 - 6$

$= 18$

3. Yashi has a packet of 20 biscuits. She gives $\frac{1}{2}$ of them to Yana and $\frac{1}{4}$ of them to Jaya. The rest she keeps.

i) How many biscuits does Yana get?

ii) How many biscuits does Jaya get?

iii) How many biscuits does Yashi keep?

$= 20 - (10 + 5)$

$= 20 - 15$

$= 5$

4. Reduce $\frac{289}{391}$ to lowest term.

$\frac{\cancel{25}^{5}}{\cancel{10}_{2}} = \frac{5}{2}$

Step Division method

$$\frac{289}{391} = \frac{289 \div 17}{391 \div 17}$$

$$= \frac{17}{23}$$

$$\begin{array}{r} 1 \\ 289\overline{)391} \\ -\,289 \\ \hline 102\overline{)289}\,(2 \\ -\,204 \\ \hline 85\overline{)102}\,(1 \\ 85 \\ \hline 17\overline{)85}\,(5 \\ -\,85 \\ \hline 0 \end{array}$$

Reduce $\frac{296}{481}$ to lowest term.

Click has a camera that takes film that allows 24 exposures, whereas Snapp has a camera that allows 36 exposures. Both of them want to be able to take the same number of photographs and complete their rolls of film. How many rolls should each buy?

$\Rightarrow$ LCM of 24 and 36 will give the out of same no. of pictures.

Therefore LCM of 24 and 36 = $\underline{\underline{72}}$

$\Rightarrow$ ~~72~~ They can click 72 pictures using their roll/card so that the cards/rolls are full.

$\Rightarrow$ No. of cards needed by click = $\frac{72}{24} = \underline{\underline{3}}$.

No. of cards needed by Snapp = $\frac{72}{36} = \underline{\underline{2}}$.

Q Four bells toll at intervals of 4, 7, 12 and 84 seconds. The bells toll together at 5 o'clock. When will they toll together again? How many times will they do it in 28 minutes?

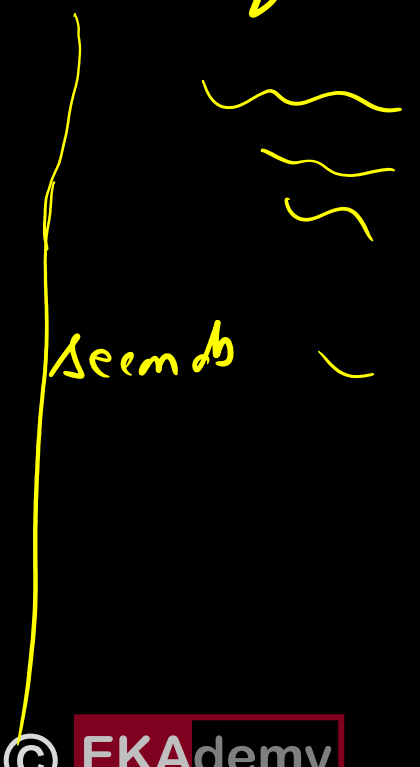
⇒ Bells will toll together at a time which is a multiple of 4, 7, 12 and 84 seconds.

&, LCM of 4, 7, 12, 84 = 84 seconds.

∴ Thus, the bell will toll together after every 84 seconds

i.e 1 minute and 24 seconds.

∴ Bells will toll again at 5 : 01 : 24.



No. of times they will fall together is 28 mins = $\frac{28 \times 60 \text{ seconds}}{84 \text{ seconds}}$

$= \frac{1680}{84}$

$= 20$ times.

Introduction to Algebra

Arithmetic Symbols (Numerals)

0, 1, 2, 3, ..., 9

Operators

+ , − , × , ÷

also use Letter Symbols in algebra.
↳ English, Greek, Latin

$1 + 2 = 3$

$2 \times 0 = 0$

Operations on numerals or numbers.

→ Algebra

↳ letter symbols

↳ used to represent unknown quantities.

↓

hence aka Variables

↳ ① Basic operations on letter symbols.

↳ ⑪ idea of algebraic expression, terms, and its evaluation.

$7 \times 0 = 0$

$x \times 0 = 0$ ✓

x

x

x x

$a \times 0 = 0$

$a \Rightarrow$ represent any number

0, 9 3 1

09

$1 \times 5 = 5$

$1 \times a = a$

$1 \times x = x$

↳ represent any number

$4 \times 5 = 20$

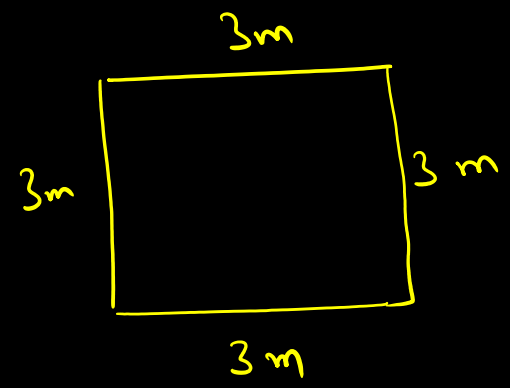
$4 \times a = \boxed{4a}$ four times a or $4a$

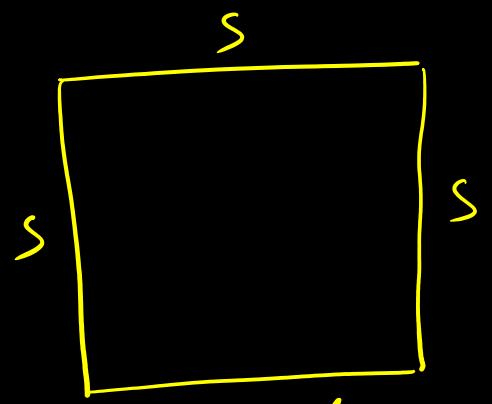
$4 \times x =$

Perimeter of a square of side 3 m = 3+3+3+3

= 4 x 3

= 12 m





Perimeter of a square of side S m = S+S+S+S

= 4 x S

= 4S

4 x S = 4 x 6 = 24m

Literals

↳ These are the letter symbols or variables which are used to represent or replace any number.

A.K.A Literal numbers or simply literals.

for eg: In $4x$, x is a literal ⇒ x is a variable

∴ or

x represents any number

or

x is a number

In $5a$, a is a literal.

↓

'a' is a number

Basic Operations on Literals and Numbers

① Addition of Literals

Sum of 3 and 5 = $\boxed{3+5}$ = 8

Sum of x and 7 = $\boxed{x+7}$ ⇒ x plus 7

or → 7 more than x

or → increase x by 7

} Read as

eg: Increase y by x ⇒ $\boxed{y+x}$

z more than a ⇒ $a+z$

Q. Sum of literals x and y is added to literal Z.

$$Z + (x + y)$$

Add 2 in $\bar{5}$.

$$5 + 2$$

$$2 + \bar{5}$$

Properties of Addition of Literals and numbers.

(i) Commutative property:

$$x + y = y + x$$

$$7 + x = x + 7$$

$$a + b = b + a$$

(ii) Associative property:

$$a + (b + c) = (a + b) + c$$

$$(a + 7) + x = a + (7 + x)$$

(iii) Identity

$$x + 0 = x$$

0 : additive identity.

$$0 + x = x$$

0 : additive identity.

Q. Write below phrases using numbers, literals and basic operators.

(i) The sum of x and 3 $\Rightarrow 3+x$

(ii) 3 more than a number x. $\Rightarrow x+3$

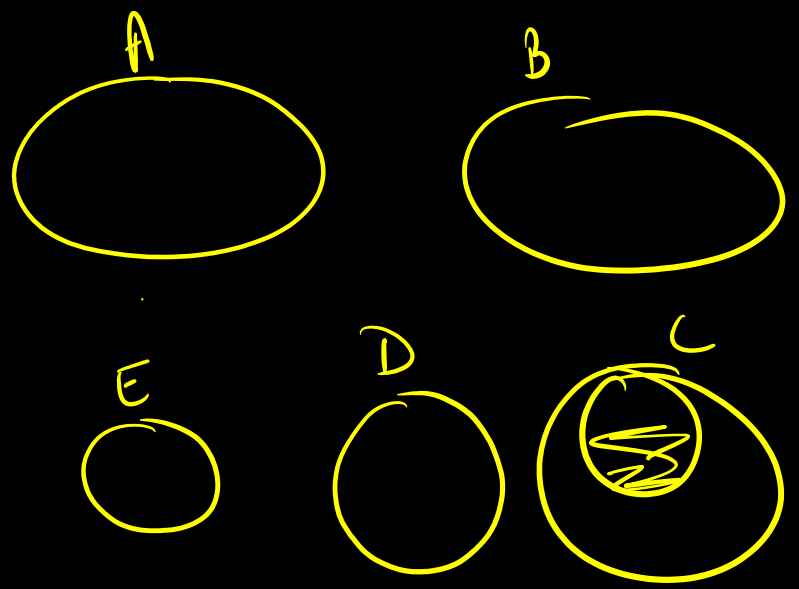
(iii) y added to 9 $\Rightarrow 9+y$

(iv) Increase x by 4 $\Rightarrow \underline{x+4}$

(v) The sum of x and 5 added to y $\Rightarrow y+(5+x)$

(vi) x added to the sum of y and 4 $\Rightarrow (4+y)+x$

Subtraction of Literals



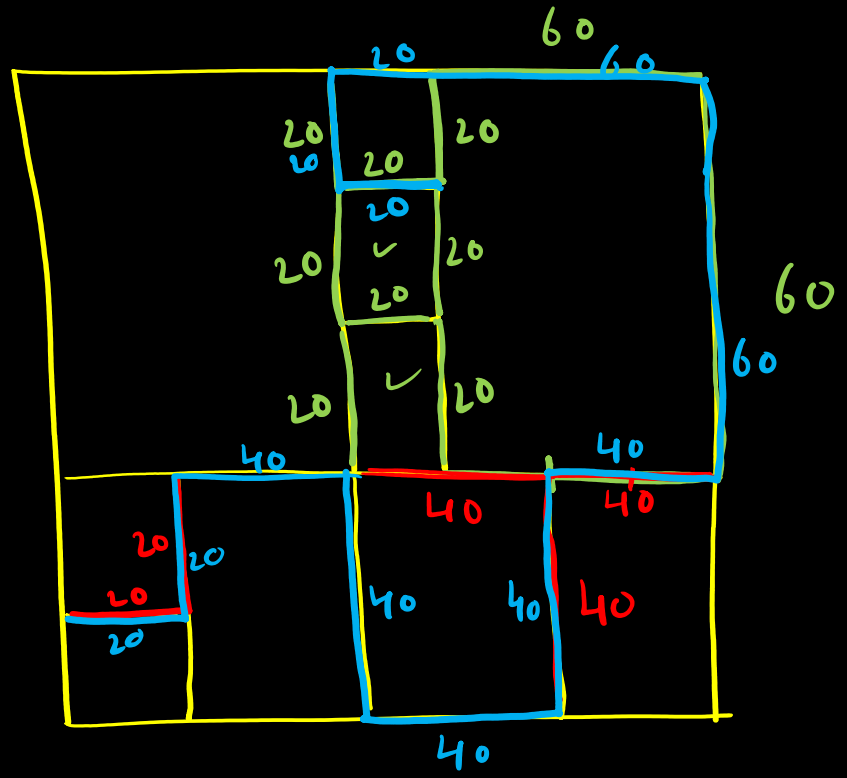
D C B A

E C B A

E D B A

E D C A

E D C B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40

$$\begin{cases} G \to 20 - 1 + 11 \Rightarrow 30 \\ W \to 40 - 11 + 1 \Rightarrow 30 \end{cases}$$

$G \to 20 - 1 + 11 = 30$

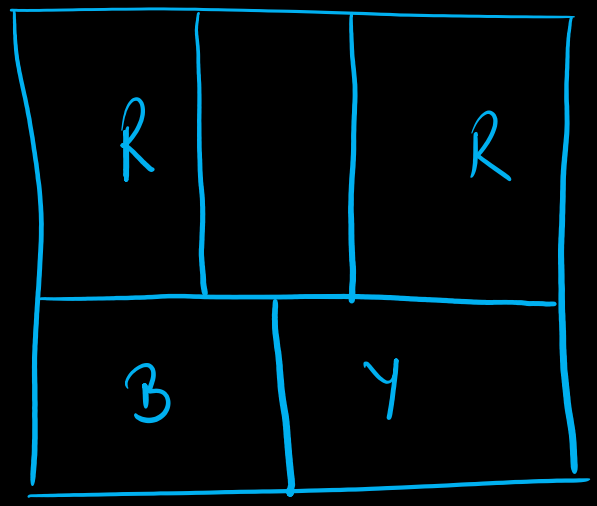
$W \Rightarrow 40 - 11 + 1 = 30$

(A) (1) (11)

(3) 2/8

$G \to 20 - 2 + 8 = ₹26$

$W \to 40 - 8 + 2 = 34$



B
Y
R

R R

Y B

Y Y
R B

B B
R Y

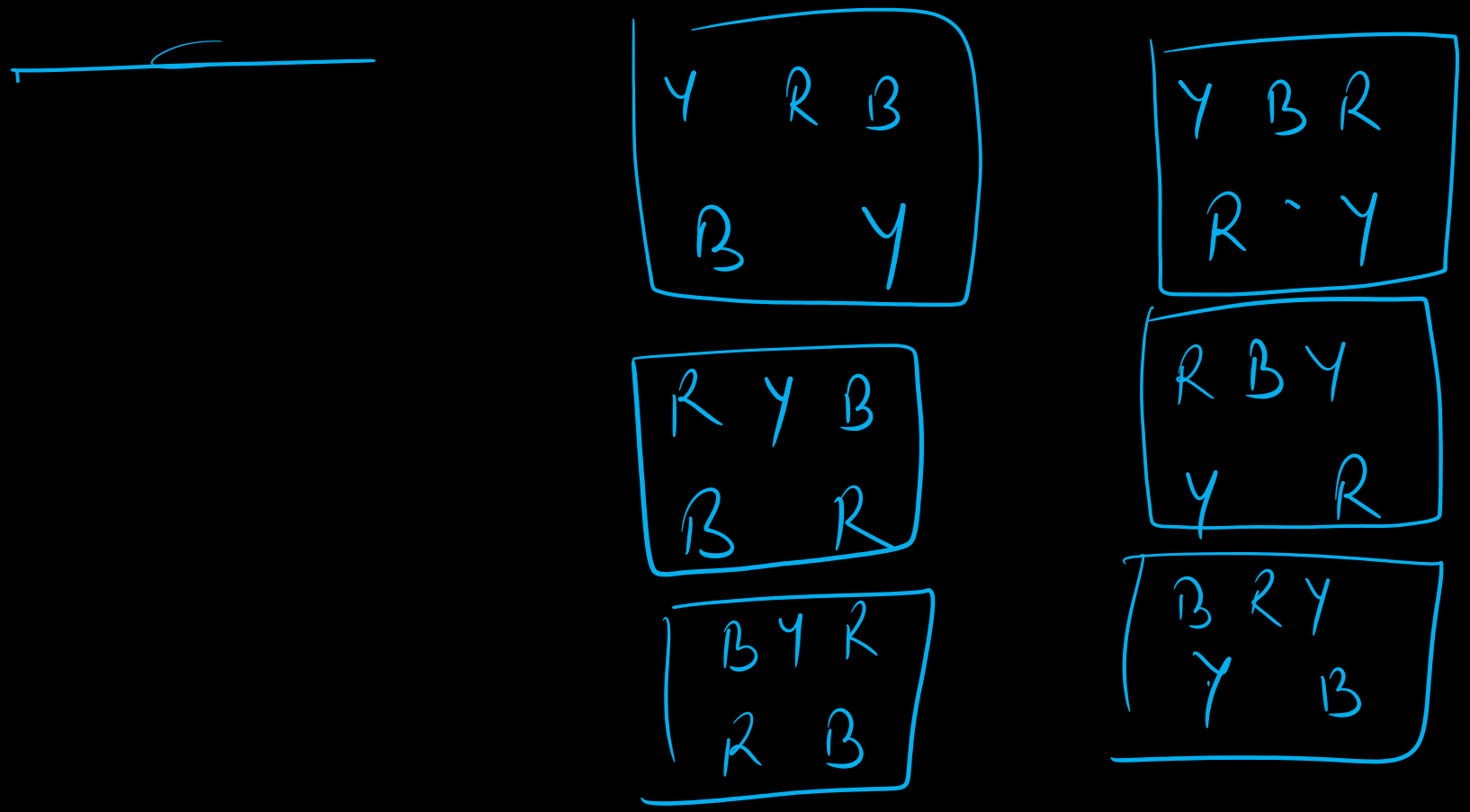
Eg

Y O Y

B R

B B

Y R



Subtraction

Subtract 5 from 7 ⇒ 7 − 5 = 2

Subtract 3 from a ⇒ a − 3 =

Subtract 5a from 7a ⇒ 7a − 5a = 2a

aa aa aa − a a a a a

$9x - 2x = 7x$

$\Rightarrow (9-2)x$

$\Rightarrow 7x$

$9x - 2a = 5x - 2a$

$x \quad x \quad x \quad x \quad x \qquad a \quad a$

(1) x less than the sum of $(y$ and $7)$

$(y+7) - x$

$x - (y+7)$ ✗

⇒ (2) less than the sum of 5 & 7

$(5+7) - 2$

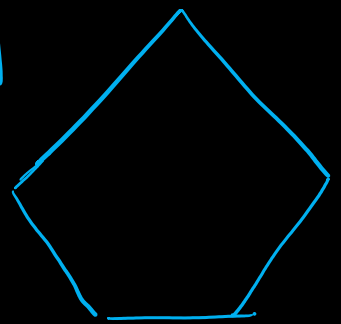
$2 - (5+7)$

Total spoon: x →

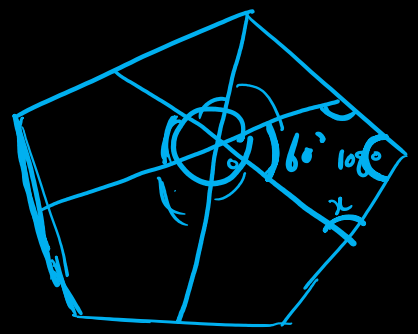
Left spoon → $x - \left(\frac{x}{3} + 8\right)$

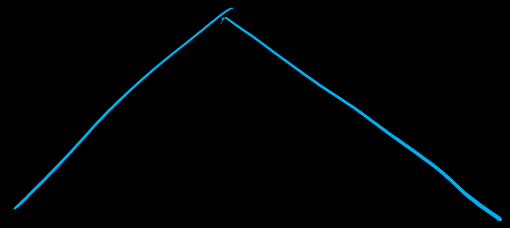
Daughter ⇒ $\left(\frac{1}{3}x + 8\right)$

$$\frac{(n-2) \times 180^\circ}{n}$$



$$\frac{(n-2) \times 180^\circ}{n}$$



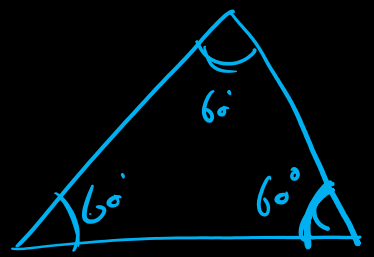


Complete angle $= 360^\circ$

one angle $= \frac{360^\circ}{6} = 60^\circ$

108°

each angle of a regular pentagon



$$= \frac{(n-2) \times 180^\circ}{n}$$

$n \Rightarrow$ no. of sides

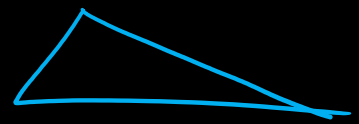
triangle : $\boxed{\frac{(3-2) \times 180^\circ}{3}}$

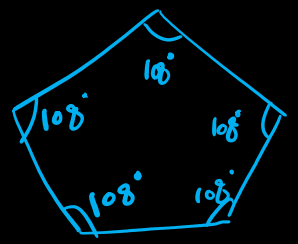
Regular	triangle	$\Rightarrow$ 3 sides

$\hookrightarrow n = 3$

$$= \frac{180^\circ}{3}$$

$$= 60^\circ$$





$n = 5$

$$\text{measure of each angle} = \frac{(n-2) \times 180^\circ}{n}$$

$$= \frac{(5-2) \times 180^\circ}{5}$$

$$= \frac{3 \times 180^\circ}{5}$$

$$= \frac{540^\circ}{5} = 108^\circ$$

(11) decrease the sum of x and y by z

$$(x+y)-z$$

Multiplication of Literals

→ Repeated addition

$3+3+3 = 9$

$3 \times 3 = 9$

$ab = a \times b$

$4 \times a = a + a + a + a = 4a$

$x + x + x + x + x = 5 \times x$ ✓

$= 5x$ ✓

(x)

multiply x and y $= x \times y = xy$

$4 \times a \Rightarrow 4a$

$a \times 4 = a4$ we don't write like this.

$\Rightarrow$ Number is written first followed by literal.

2×3
3×2

$\therefore$ we write $4a$ instead of $a4$

$\Rightarrow$ It is a convention not a rule.

Properties

$\boxed{x}$ x

① Commutative

$3 \times 2 = 2 \times 3$

↳ multiplication of literal is also commutative.

$ab = ba$

$4a = a4$

② Associative Property

$(2 \times 3)4 = 2(3 \times 4)$

↳ multiplication of literals is associative.

$(ab)c = a(bc)$

(iii) Identity.

1 is multiplicative identity.

$$a \times 1 = 1a = a$$

$$1 \times b = 1b = b$$

(iv) Distributive property

⇒ multiplication is distributive over addition & subtraction.

$$a(b+c) = ab + ac$$

$$a(x+y+z) = ax + ay + az$$

$$a \times (b-c) = ab - ac$$

$$2(3+4) = 2\times3 + 2\times4$$
$$= 6 + 8$$
$$= 14$$
$$= 14$$

(i) 4 times the sum of x and y

$4 \times (x+y)$

$4 \times (\;) \; x+y$ (closing)

(opening)

$\Rightarrow$ $4(x+y)$ ← preferred.

$(x+y)4$ ←

(4) 3 times x is subtracted from y.

$$y - (3x)$$

$$y - 3x$$

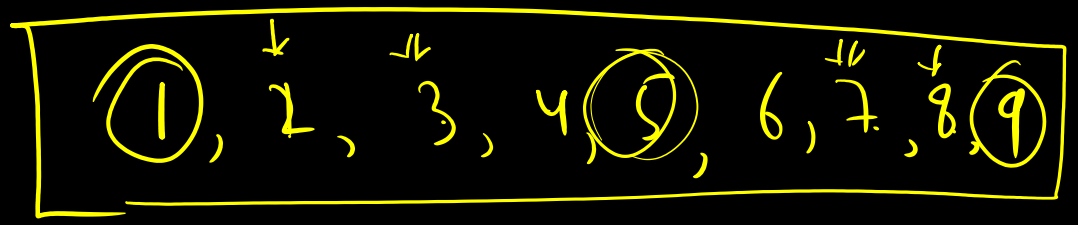
Division of Literals

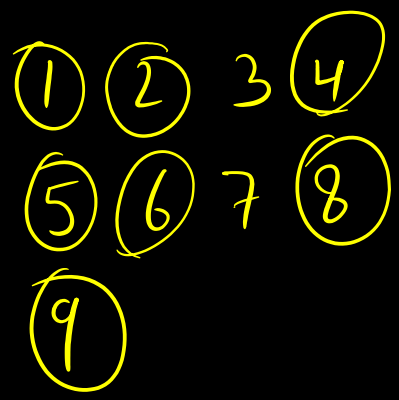
(i) divide 20 by x $\Rightarrow$ $\frac{20}{x}$

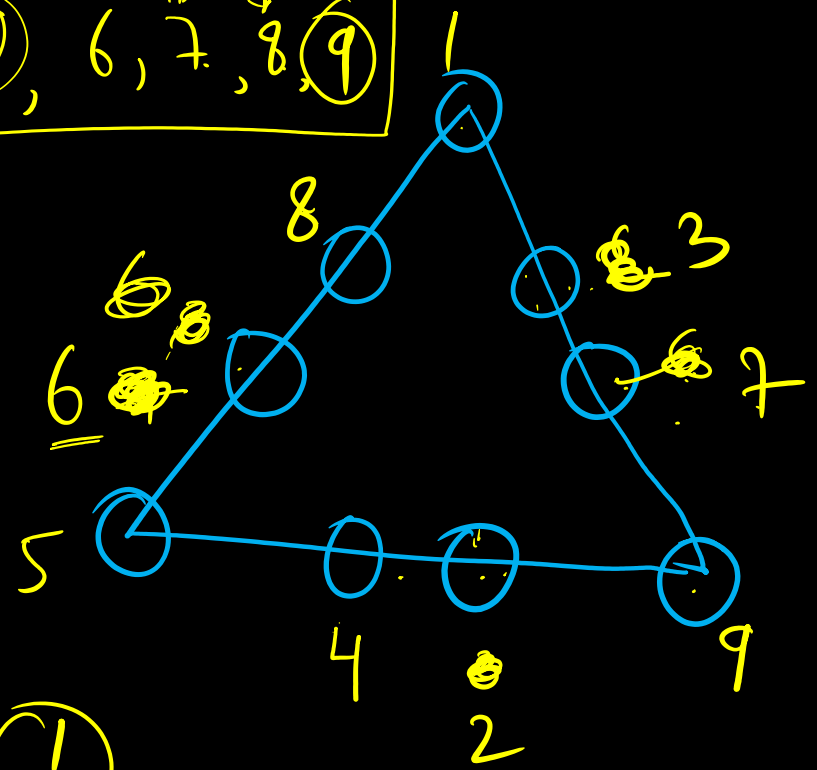
(ii) divide x by 3 $\Rightarrow$ $\frac{x}{3}$

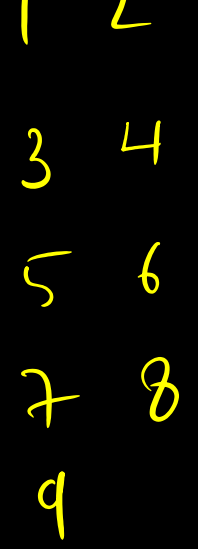
(iii) $\frac{1}{3}$ rd of x $\Rightarrow$ $\frac{1}{3}$ of x $\Rightarrow$ $\frac{1}{3} \times x$ = $\boxed{\frac{x}{3}}$

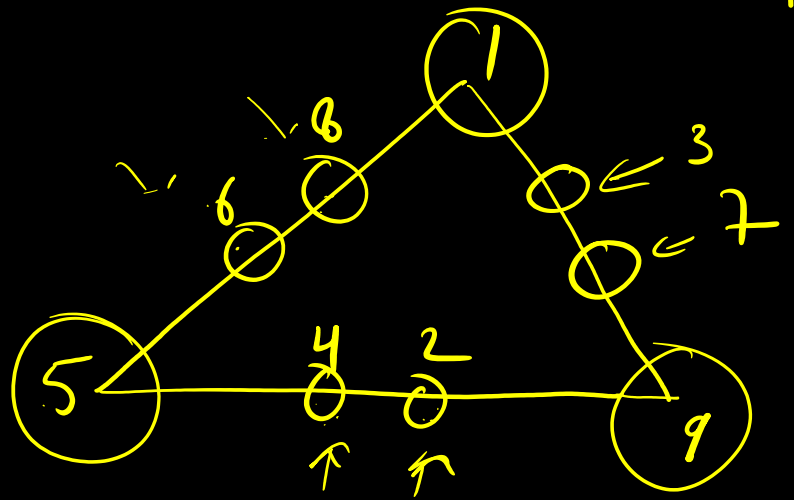
Quotient of x by 3











① Quotient of z by 6 multiplied by y.

(Quotient of z by 6) $= \frac{z}{6}$

multiply $\left(\frac{z}{6}\right)$ by $y = \frac{z}{6} \times y$

$= \frac{z}{6} \times \frac{y}{1}$

$= \frac{zy}{6}$

$\frac{y}{1}$

$$\frac{2}{3} \times \frac{4}{5} =$$

(11) Quotient of x by y added to the product of x and y.

$\frac{x}{y}$ $+$ $xy = xy$

$$\Rightarrow \frac{x}{y} + xy$$

(iii) 3 take away from the quotient of x by $2y$:

3 take away → -3

quotient of x by $2y$ → $\frac{x}{2y}$

$\Rightarrow \boxed{\frac{x}{2y} - 3} \Rightarrow$ Expression

(iv) Eight times a number p is x less than a number y.

Eight times a number p → $8 \times p \Rightarrow \boxed{8p}$

is → $=$

x less than a number y → $\boxed{(y - x)}$

$$\boxed{8p = y - x} \Rightarrow \text{equation}$$

Q. Rohan spends $x daily and saves $y per week. What is his income after 3 weeks?

Sol: Income = Expenditure + Saving.

1 week = 7 days.

3 weeks = 7 × 3 = 21 days.

∵ Rohan spends $x daily

∴ amount spent by Rohan in 3 weeks = $21x.

∵ Rohan saves $y ~~day~~ per week

∴ amount saved by Rohan in 3 weeks = $3y

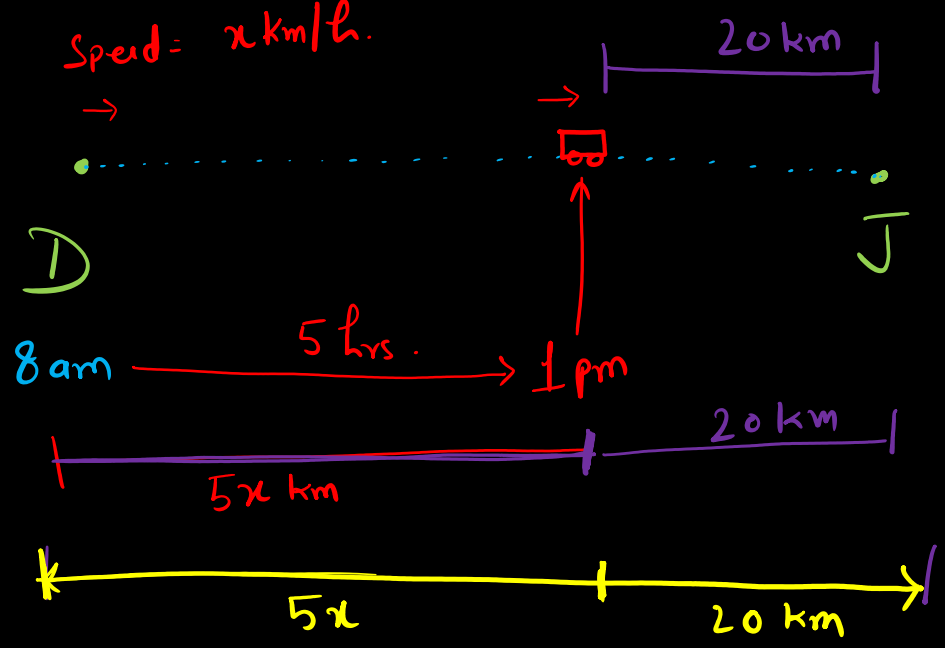
Hence, Rohan's income after 3 week = $(21x + 3y)$

∴ is read as Since.

∴ is ~~the~~ read as therefore

∵

Q. Ahmed starts from Delhi at 8 am to Jaipur. If his car is running at the speed of x km/h and at 1 pm he observes that he is 20 km away from Jaipur. Find the distance between Delhi and Jaipur.



Total Distance between D & J = $(5x + 20)$ km

Speed of car = x km/h.

Time taken from 8 am to 1 pm = 5 hrs.

$\therefore$ distance covered in 5 hr (i.e. till one 1 Pm) = $5x$ km.

Remaining distance to Jaipur at 1 pm = 20 km.

$\therefore$ Total distance between Delhi and Jaipur = $(5x + 20)$ km.

Powers of Literals

Power
Base

$3 \times 3 = 3^{2}$ → Power

Base

3 raised to the power 2

$3 \times 3 \times 3 = 3^{3}$ ⟶ 3 raised to the power 3

$3 \times 3 \times 3 \times 3 \times 3 = 3^{5}$ ⇒ 3 raised to the power 5

$a \times a = a^{2}$ a raised to the power 2

$3 \times 3 \times 3 \times 3 \times a \times a \times a = 3^{4}\, a^{3}$

$\sqrt{2}$

$\sqrt{1} = 1$

$\sqrt{4} = \sqrt{2 \times 2} = 2$

$\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

$\sqrt{9} = 3$

$\sqrt{3 \times 3} = 3$

$\sqrt{16} = \sqrt{4 \times 4} = 4$

$= \sqrt{2 \times 2 \times 2 \times 2}$ (2, 2) $= 2 \times 2 = 4$

$x^{15} = x \cdot x \cdot x \cdot x \cdot x \ldots$ 15 times.

$x^{999} =$ ~~xxxxxx~~ $x \cdot x \cdot x \cdot \ldots$ repeated multiplication 999 times

$\underbrace{7 \times a \times a \times a \times a \times b \times b}_{\text{product form}} = \underbrace{7a^4b^2}_{\text{Exponential form.}}$

Q. Write $10x^3y^3z^4$ in product form.

$a^4 b^2 \times a^7 b^3$

$\Rightarrow$ $a \times a \times a \times a \times b \times b \times a \times a \times a \times a \times a \times a \times a \times b \times b \times b$

$\Rightarrow$ $a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times b \times b \times b \times b \times b$

$= a^{11} b^5$

$a^{(4+7)} b^{(2+3)}$

$a^{11} b^5$

2

x

(ii) $3a^2b^3 \times 2a^1b^4$

$= 3 \times 2 \times a^{(2+1)} \times b^{(3+4)}$

$= 6a^3b^7$

(iii) $4x^2y^3 \times 3x^1y^2 \times 5x^3y$

$= 4 \times 3 \times 5 \times x^{(2+1+3)} y^{(3+2+1)}$

$= 4 \times x^2 \times y^3 \times 3 \times x^1 \times y^2 \times 5 \times x^3 \times y^1$

$= 4 \times 3 \times 5 \times x^2 \times x^1 \times x^3 \times y^3 \times y^2 \times y^1$

$= 60 x^6 y^6$

Laws of exponent

(i) $x^m \times x^n = x^{(m+n)}$

$x^3 \times x^2 = x^5$

$x^2 \times x^1 \times x^3 = x^{(2+1+3)}$

$x^2 \times x^1 \times x^3 =$

Q. $a^{4} \div a^{2} = \frac{a^{4}}{a^{2}} = \frac{a \times a \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a}} = a \times a = a^{2}$

$= a^{(4-2)} = a^{2}$

eg 2 $b^{6} \div b^{2} = b^{(6-2)} = b^{4}$

(I) Add $\frac{2}{5} + \frac{4}{3}$

(II) Subtract $\frac{21}{25}$ from $\frac{18}{20}$

(III) Simplify: $\frac{2}{3} + \frac{3}{4} + \frac{1}{2}$

(IV) Simplify: $4\frac{2}{3} - 3\frac{1}{4} + 2\frac{1}{6}$

(v) Replace □ by correct number

(a) $\square - \frac{5}{8} = \frac{1}{4}$ → $\frac{7}{8}$

(b) $\frac{1}{2} - \square = \frac{1}{6}$ → $\frac{2}{6}$

$(-) \times (-) = +$

$(+) \times (-) = -$

$(+) \times (+) = +$

$-(-2) = +2$

(vi) Evaluate:

$-26 - 20 + 30 - (-33) + 21 + 24 - (-25) - 26 - 14 - 34$ ✗ $-2 - 3 - 9 - 7$

(vii) If $\frac{45}{60}$ is equivalent to $\frac{3}{x}$, then $x = ?$ ✓

(viii) Subtract the sum of -5020 and 2320 from -709.

$\Rightarrow -709 - (-2700) \Rightarrow -709 + 2700 \Rightarrow 1991$

(1) Add: $3x + 2y + 6x + y + x$ ✓

[Hint: Same type of literals can be added or subtracted.]

[eg. $2x + 1x = 3x$

$2x - x = x$]

$3x + 6x + x + 2y + y$

$3 + 6 + 1 + 1$

$10x + 3y$ ⇒ final result

(11) Simplify. $7x - 3x + 2x$

$\Rightarrow 7x + 2x - 3x$

$\Rightarrow 9x - 3x$

$\Rightarrow 6x$

$\Rightarrow 7x - x$

$\Rightarrow 6x$

$7x - 3x + 2x$

$4x + 2x$

$\Rightarrow 6x$

$7y^5 = 7y \times 7y \times 7y + 7y + 7y$ ✗

$= 7 \times y \times y \times y \times y \times y$ ✓

$(7y)^5 = 7y \times 7y \times 7y \times 7y \times 7y$

$2x = 2 \times x$

Value of:

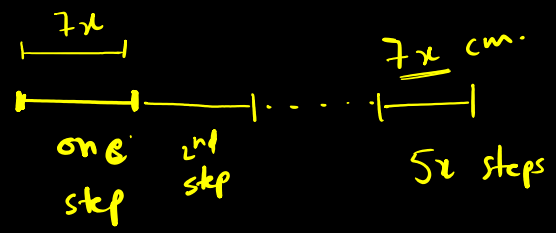
$(2x)^3 = 2x \times 2x \times 2x$

$= 2 \times 2 \times 2 \times x \times x \times x$

$= 8 \times 2^3 \times x^3$

$= 8x^3$

Q. Rohit covers $7x$ cm in one step. What is the distance moved by him in $5x$ steps!



distance covered in one step = $7x$ cm.

$\therefore$ distance covered in $5x$ steps = $5x \times 7x$ cm.

$= 5 \times x \times 7 \times x$

$= 5 \times 7 \times x \times x$

$= 35x^2$ cm

Q. Melin has $14a$ picture cards. If each picture card costs $\$ 3ab$, determine the cost of picture cards possessed by Melin.

Sol:

Total no. of picture cards = $14a$

Cost of 1 card = $\$ 3ab$

Total cost = $14a \times 3ab$

$= 14 \times a \times 3 \times a \times b$

$= 14 \times 3 \times a \times a \times b$

$= \$ 42a^2b$

Q. In a classroom there are $2x$ rows of benches. If each row has $3xy$ benches and each bench can accomodate x students, determine the number of students in the room if it is full up to its capacity.

Sol:

Total rows $= 2x$

Benches in each raw $= 3xy$.

Total Benches in the room $= 2x \times 3xy$.

$= 2 \times x \times 3 \times x \times y$

$= 6x^2y$

$x^2 \times x = x^3$

$x \times x \times x = x^3$

Each bench can accomodate x students

$\therefore$ Total no. of students $= 6x^2y \times x = 6 \times x \times x \times xy \times x$

$= 6x^3y$.

$7 \times a \times b \times 7 \times a \times a \times b \times b$ $\Rightarrow$ Product form

Exponential Representation : $7^2 a^3 b^3$

↑ Exponential form

a^{20} in product form

↓

$a \times a \times a \times \ldots$ 20 times

1. The number of bacteria in a culture is x. It becames Square of itself after one week. What will be its number after 2 weeks.

Sol:

Now $\longrightarrow$ x

after 1 week $\longrightarrow$ $\boxed{x^2}$

after 2 weeks $\longrightarrow$ $(x^2)^2$

$\Rightarrow$ $x^{2\times 2} = x^4$

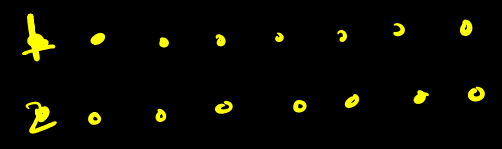
$$(2^3)^4 = 2^{3\times 4} = 2^{12}$$

Laws of exponent:

$$(x^m)^n = x^{m\times n}$$

If there are x^2 rows of chairs and each row contain $\frac{x^3}{2}$ chairs.
Determine the total number of chair

$\Rightarrow$



$\rightarrow$ 27 chair
$\rightarrow$ 27 chair
$\rightarrow$ 27 chair

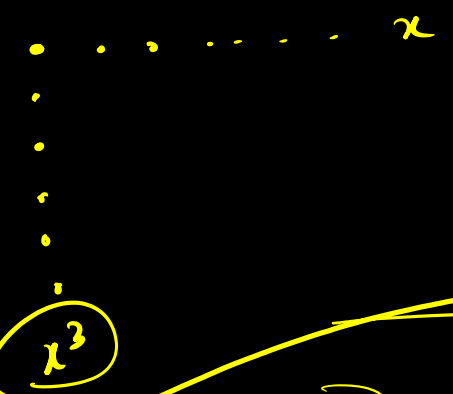
$27 \times 3 = 81$

$\rightarrow$ 8 chair
$\rightarrow$ 8 chair

$8 \times 2 = 16$

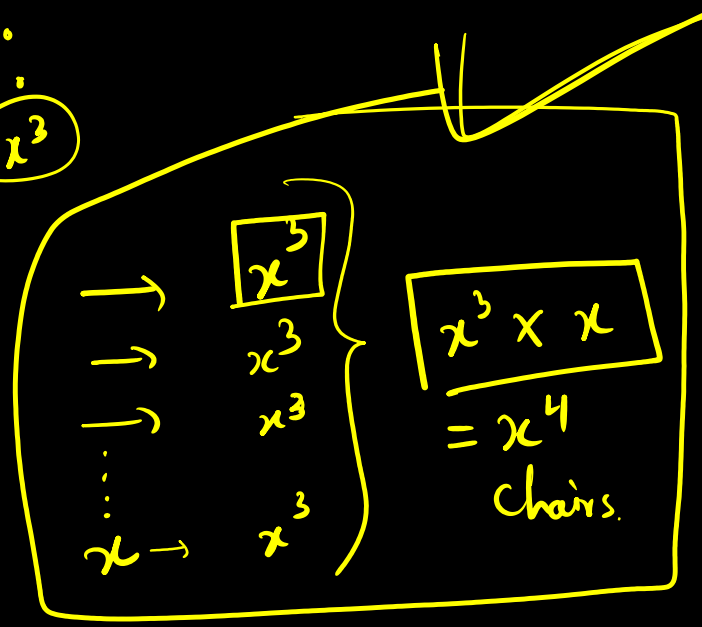
3^3
3^3
3^3

$\rightarrow 6^3$
$\rightarrow 6^3$



x^3

4^3
4^3

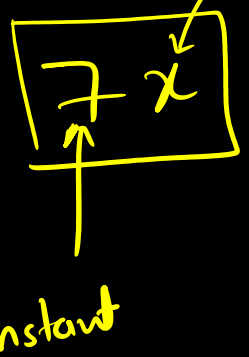


$\rightarrow x^3$
$\rightarrow x^3$
$x \rightarrow x^3$

$x^3 \times x = x^4$ chairs.

$A = l \times b$

Variables and Constants



Constant: Symbols having fixed value

Variable: Symbol that can take various numerical values.

$$\text{Circumference} = \underbrace{2\pi}_{\text{Constants}} \overbrace{r}^{\text{variable}}$$

End of the chapter

Introduction to Algebra