Ratio and Proportion

Ratio, Proportion and unitary method

Ratio

↳ • Compare two quantities

• ↳ Comparison is done by division

eg. Kabir's wt. $\Rightarrow$ 40 kg ✓

Kishore's wt. $\Rightarrow$ 50 kg. ✓

Comparison by division

we have, $\dfrac{wt. \text{ of } kabir}{wt. \text{ of } Kishore} = \dfrac{40 \text{ kg}}{50 \text{ kg}} = \dfrac{4}{5}$

$A \to 4$

$B \to 8$

$A:B = \dfrac{A}{B} = \dfrac{4}{8} = \dfrac{1}{2}$

$A:B = 1:2$

$$\frac{\text{wt. of kabir}}{\text{wt. of kishore}} = \frac{4}{5}$$

We can say that, wt. of kabir is $\frac{4}{5}$ times wt. of Kishore. ✓

Check

wt. of kishore = 50 kg.

wt. of kabir $= \frac{4}{5} \times (\text{wt. of kishore})$

$= \frac{4}{5} \times 50$ kg

$= 40$ kg.

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eg.

Kabir's height = 150 cm

Kishore's height = 125 cm

Comparison by division method:

$$\frac{Kabir's\ height}{Kishore's\ height} = \frac{150\ cm}{125\ cm} = \frac{6}{5}$$

$$= \frac{6}{5}$$

Kabir's ht. is $\frac{6}{5}$ times of Kishore's ht.

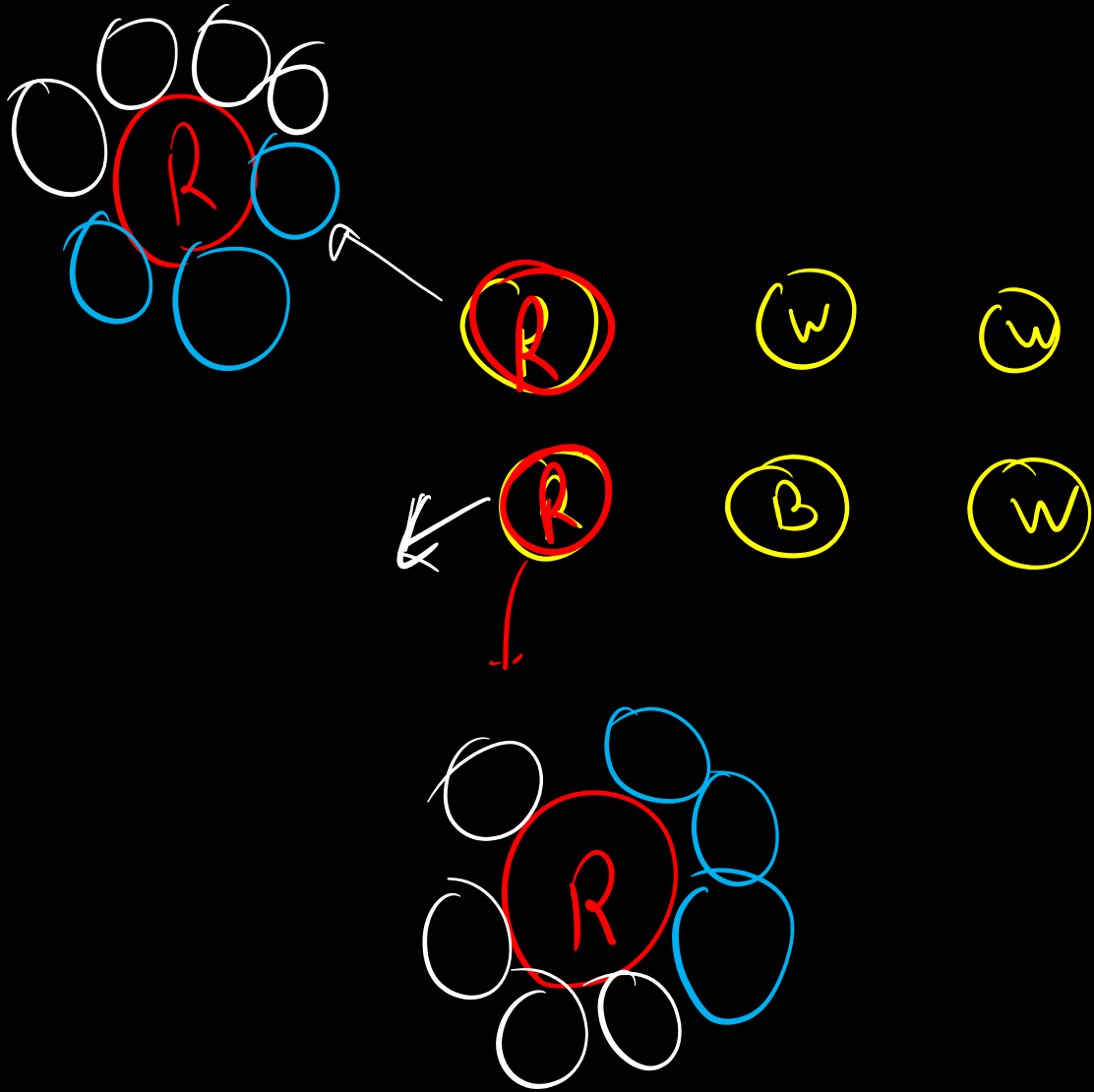$$3 \times \left(\quad 1 \quad \text{Banana} \qquad\qquad 3 \quad \text{appl...} \qquad\qquad \right)$$

5

$\underline{\underline{3 \ B}}$ $\qquad\qquad\qquad \underline{9 \ Apple}$

4 Y	12 R

1 Y	3 R

R : B : W

1 : 3 : 4

Ratio

Comparing two quantity or number by division.

a is to b = a
: :

a : b
1 : 2

4 : 8
is to

⇒
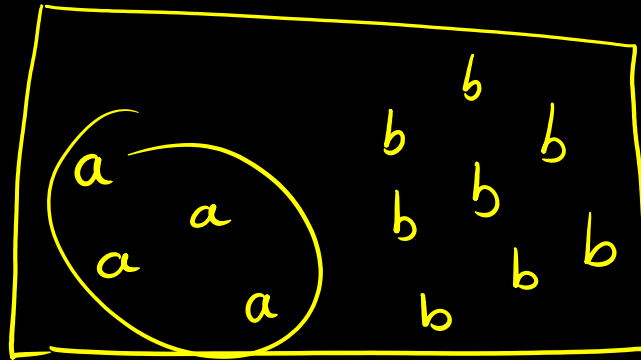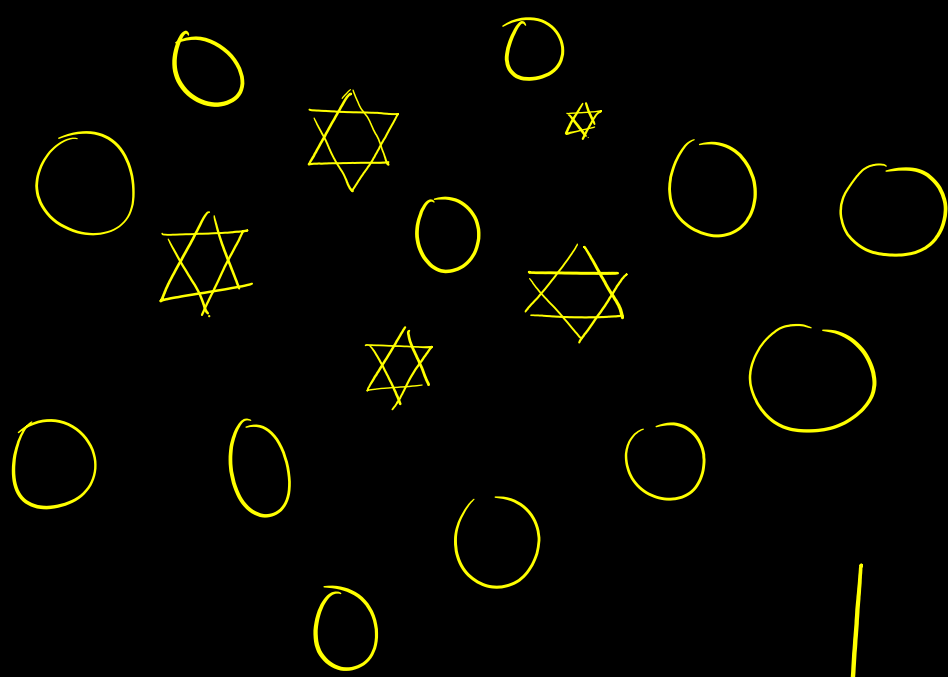
Star __to__ circle

star is to circle

5 is to 12

5 : 12

circle is to star

$\boxed{12 : 5}$

<u>Class</u>

<u>25 girls</u> and <u>25 boys.</u>

Ratio of girl is to boy.

25 : 25

girls : boy.

5 : 5

1 : 1

$\Rightarrow$ Ratio can be represented in the form of fraction

$$a : b = \frac{a}{b}$$

$\Rightarrow$ Just like equivalent fraction we can have equivalent ratios.

for eg: ① $\boxed{3 : 1}$ has equivalent ratios

6 : 2

9 : 3

12 : 4

$$\frac{3^{\times 2}}{1^{\times 2}} = \frac{6}{2}$$

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3 l water for 1 kg mutton.

Water : mutton
3 : 1

$$\frac{5 \text{ kg mutton}}{}$$

↓

15 water.

15 : 5

3 : 1

$$\boxed{a \; : \; b}$$

antecedent   consequent

$$\boxed{2 \; : \; 5} \quad \times \; 3$$

$$\boxed{6 \; : \; 15}$$

antecedent   consequent

3 apples   is to   9 oranges.

$$3 \; : \; 9$$

$$\boxed{or}$$
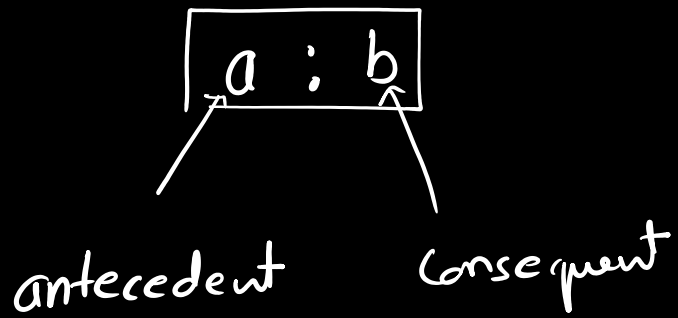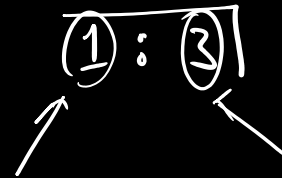
$$\boxed{① \; : \; ③}$$
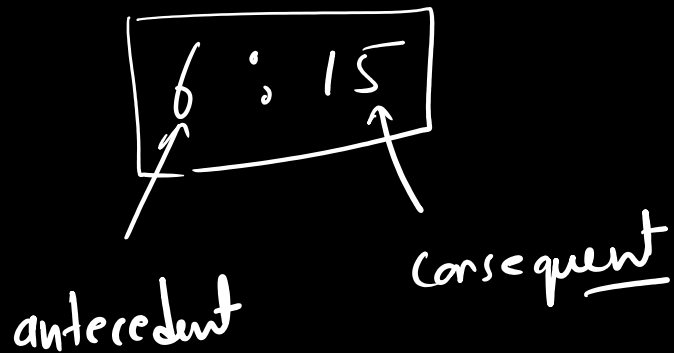
Q. Express the given ratio in simplest form.

(i) 150 : 400

$\Rightarrow$ HCF of 150, 400 is :$\rightarrow$ 50

The ratio 150 : 400 can be written in the form of fraction as $\dfrac{150}{400}$

To simplify.

$$\dfrac{150 \div 50}{400 \div 50} = \boxed{\dfrac{3}{8}}$$

$\dfrac{3}{8} = \boxed{3 : 8}$

(II) $\left(27 : 57 \right) \div 3$

$\Rightarrow$ 9 : 19

(III) a dozen to a score
(12) (20)

12 : 20 $\Rightarrow$ 4 is HCF

3 : 5 Simplest form.

Q. Find the ratio of 200 grams to 4 kg.

Imp: Quantities should be of same kind.

200 grams to 4000 grams.

Given ratio, 200 : 4000

HCF = 200

$$\boxed{1 : 20}$$

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Q. A girl earned ₹ 40,000 and Paid Rs. 5000 as income tax.
Find the ratio of:

(i) income tax to income

(ii) income to income tax.

$\Rightarrow$ 5000 : 40,000

$\Rightarrow$ HCF = 5000

$\Rightarrow$ 1 : 8

$\Rightarrow$

$5000 \overline{)40,000}$

$\begin{array}{r} 8 \\ 5\overline{)40} \\ .40 \\ \hline 0 \end{array}$

Q. The no. of boys and girls in a school are 480 and 384 respectively. Express the ratio of the number of boys to that of girls in the simplest form.

$\frac{480, 384}{}$

Boys : Girls

480 : 384

HCF is 96

∴ 5 : 4

$$13 \overline{) 5493} ($$

Comparison of Ratios

① Compare the ratios   5 : 12 and 3 : 8

HCF $(12, 8) = 24$

$$\frac{5 \times 2}{12 \times 2} \qquad \boxed{\frac{10}{24}}$$

$$\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$$\frac{10}{24} > \frac{9}{24}$$

$$\frac{5}{12} > \frac{3}{8}$$

$$5 : 12 > 3 : 8$$

Equivalent ratio ✓

Ratio : $\boxed{6 : 4}$

Equivalent ratio of $6:4$

$$\frac{6 \times 2}{4 \times 2} = \frac{12}{8} \qquad = \quad 12:8$$

$$\frac{6 \times 3}{4 \times 3} = \frac{18}{12} = \quad \underline{18:12}$$

$$\frac{6 \div 2}{4 \div 2} \quad = \quad \frac{3}{2} = \quad \underline{3:2}$$

which one is larger ratio.

(i) 9:20 or 8:13

∴ 9:20 < 8:13

$$\boxed{6:4}$$ 3 2 $$\boxed{\$24:16}$$

$$\boxed{6:4 = 24:16}$$

↳ Proportion

Con two ratio be equal. ?

Proportion

$\hookrightarrow$ An equality of two ratios is called proportion.

$$\boxed{6:18 \;=\; 1:3}$$

$\downarrow$

$\underline{6:18}$ and $\underline{1:3}$ are in proportion.

$6:18$ and $1:3$ are proportional.

$\Rightarrow$ "Four numbers $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ are said to be in proportion, if the ratio of the first two numbers is equal to the ratio of the last two numbers".

$$a:b \;\bigcirc\!\!=\; c:d$$

$$a:b \;::\; c:d$$

is proportional to

$$1\;,\;2\;,\;4\;,\;8$$

$$1:2 \;\bigcirc\!\!=\; 1:2$$

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① 40, 30, 60, 45   are in proportion or not?

40 : 30
——————
↓
4 : 3

60 : 45
——————
↓
4 : 3

a, b, c and d are in proportion

if $a:b = c:d$
——————

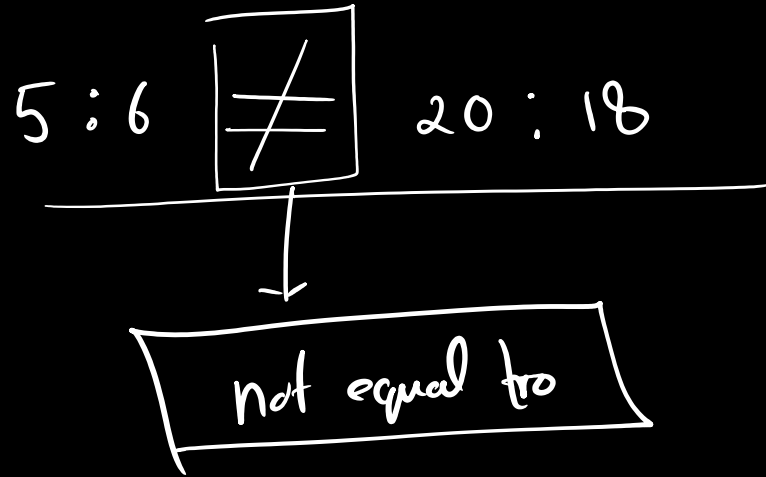or $\boxed{\dfrac{a}{b} = \dfrac{c}{d}}$

$\dfrac{40:30}{} = \dfrac{60:45}{}$

∴ 40, 30, 60, 45 are in proportion

40 : 30 :: 60 : 45

$a:b :: c:d$

$\dfrac{\cancel{60}^{12}}{\cancel{45}_{9}} = \dfrac{\cancel{12}^{4}}{\cancel{9}_{3}}$, 4:3

②   5 , 6 , 20 , 18   are in proportion or not?

$$5 : 6 \boxed{\neq} 20 : 18$$

not equal to

Q. The first, third and fourth terms of a proportion are 12, 8 and 14 respectively. Find the second term.

$$12, \; x, \; 8, \; 14$$

$$\Rightarrow \quad 12 : x = 8 : 14 \quad \checkmark$$

$$\Rightarrow \quad \frac{12}{x} = \frac{8}{14}$$

If,
$$a : b = c : d$$

then,

$$\boxed{a \times d = b \times c}$$

$$\Rightarrow \quad 12 \times 14 = 8 \times x$$

$$\Rightarrow \quad \boxed{8} \times x = 12 \times 14^{\,3}$$

$$x = \frac{\cancel{12} \times \cancel{14}^{\,7}}{\cancel{8}} = 3 \times 7 = \underline{21}$$

Q. The first, second and fourth terms of a proportion are 6, 18 and 25 respectively. Find its third term.

$6, 18, x, 25$

$6 : 18 = x : 25$

$$\frac{6}{18} = \frac{x}{25}$$

$$\boxed{\frac{1}{3} = \frac{x}{25}}$$ ✓

$$\frac{x}{\textcircled{25}} = \frac{1}{3}$$

$$x = \frac{1}{3} \times 25 = \boxed{\frac{25}{3}}$$

$$\boxed{6, 18, \frac{25}{3}, 25}$$
↓
are in proportion.

$2\textcircled{x} = 6$

$\frac{2 \times \textcircled{x}}{2} = \frac{6}{2}$

$$\boxed{x = 3}$$

$3x = 6$

$x = \frac{6}{3} = 2$

$\frac{x}{\textcircled{2}} = 4$

$x = 4 \times 2 = 8$

$$\boxed{\frac{x}{3} = 7}$$

$\frac{x}{7} = 319$

$x = 319 \times 7$
$=$

Q. Find the value of x, if $x : 6 :: 5 : 3$

Q. The ratio of the length of a school ground to its width is $5 : 2$. Find the length, if the width of the ground is 50 m.

Sol. $\Rightarrow$ length is x metres.

$$\Rightarrow x : 50 = 5 : 2$$

$$\text{or,} \quad \frac{x}{50} = \frac{5}{2}$$

$$\text{or,} \quad x = \frac{5}{2} \times 50$$

$$x = \frac{250}{2} = \underline{\underline{125\,m}}$$

Find x,

$$18 : x = 27 : 3$$

$$\frac{18}{x} = \frac{27}{3}$$

$$18 \times 3 = x \times 27$$

$$x = \frac{\overset{2}{\cancel{18} \times \cancel{3}}}{\underset{9}{\cancel{27}}} = 2$$

$$\boxed{x = 2}$$

$$\boxed{7 : 14 = 15 : x}$$

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Q. Solve: $2\frac{1}{3} - 1\frac{2}{3} + 4\frac{1}{3}$

$\Rightarrow \quad \dfrac{7}{3} - \dfrac{5}{3} + \dfrac{13}{3}$

$\Rightarrow \quad \boxed{5}$

Q. Value of $(-2) \times (-3) \times 6 \times (-1)$

$= \underline{\underline{-36}}$

If $a \Delta b = -a + b - (-2)$, then find:

(i) $4 \Delta 3$

(ii) $(-2) \Delta (-3)$

Find the value of: $373 + (-245) + (-373) + 145 + 3000$.

Find the value of: $-26 - 20 + 33 - (-33) + 21 + 24 - (-25) - 26 - 14 - 34$

Q. The sum of two integers is -26 and one of them is 14. Find the other integer.

Q. If $x = (-23) + 22 + (-23) + 22 + \ldots (40 \text{ terms})$ and $y = 11 + (-10) + 11 + (-10) + \ldots$

(20 terms). Find $\underline{y - x}$.

$x \Rightarrow$ $\underbrace{(-23) + 22}_{2 \text{ term}} + \underbrace{(-23) + 22}_{} + \ldots (40 \text{ term})$

$(-23) + (-23) +$

Evaluate $7 \times |-15| - |-9| \times 8$

$= \boxed{33}$

$|-x| = \underline{\underline{x}}$

$\underbrace{\text{mod}}\ x$
$\downarrow$
$\underline{\underline{\text{modulus}}}$

Comparison of of Ratio

eg① Compare the ratio 5:12 and 3:8

$$\frac{5}{12} \qquad \frac{3}{8}$$

$$\boxed{\frac{10}{24}} \;>\; \frac{9}{24}$$

$$\boxed{5:12 \;>\; 3:8}$$

Equivalent Ratios

eg.   6 : 4

$$\frac{6}{4} \quad = \quad \frac{12}{8} \quad , \quad \frac{18}{12} \quad , \quad \frac{24}{16}$$

equivalent Ratio

$$\text{ey.} \quad \frac{14}{21} = \frac{\boxed{2}}{3} = \frac{6}{\boxed{9}}$$

$$A \quad = \quad B \quad = \quad C$$

$$\text{ey.} \quad \frac{12}{20} = \frac{\boxed{3}}{5} = \frac{9}{\boxed{15}}$$

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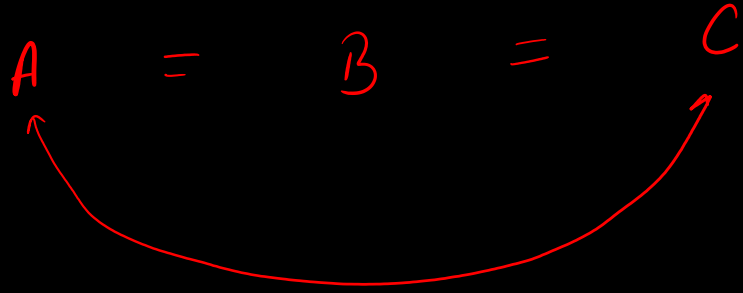
Proportion

$$a \;,\; b \;,\; c \text{ and } d$$

$$a:b \;=\; c:d$$

"a, b, c and d are in proportion"

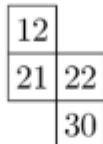Four numbers a, b, c and d are in proportion, if

$$a:b = c:d.$$
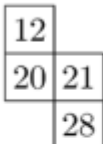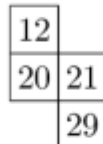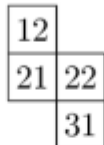
Are the numbers 5, 6, 20 and 18 are in proportion.

$$5:6 \neq 20:18$$

Therefore, 5, 6, 20, and 18 are not in proportional.

1	2	3	4	5	6	7	8
9	10	11	12				

1. Holger writes the numbers up to 40 in the table in the same way as shown.

Which of the pieces A to E can he then cut from the table?

(A)
```
12
22 23
   33
```

(B)
```
12
20 21
   28
```

(C)
```
12
20 21
   29
```

(D)
```
12
21 22
   30
```

(E)
```
12
21 22
   31
```

**2.** Matchsticks are arranged to form numbers as shown. To form the number 15 one needs 7 matchsticks.

To form the number 8 one needs the same amount.

What is the biggest number that one can build using 7 matchsticks?

0 1 2 3 4 5 6 7 8 9

(A) 31   (B) 51   (C) 74   (D) 711   (E) 800

1. What is (20+22) ÷ (20−22) = ?

(A) -42   (B) -21   (C) -2   (D) 22   (E) 42

2. Meike paddles around five buoys with her boat (see diagram).
Which of the buoys does she paddle around in a clockwise direction?

(A) 2, 3 and 4   (B) 1, 2 and 3   (C) 1, 3 and 5   (D) 2, 4 and 5   (E) 2, 3 and 5
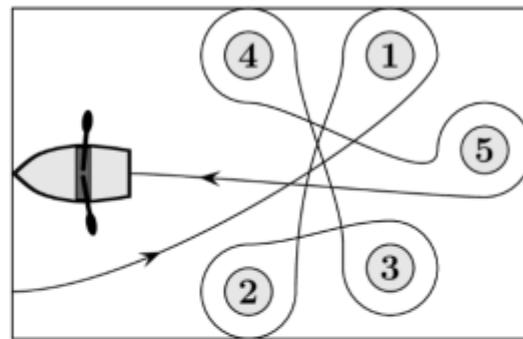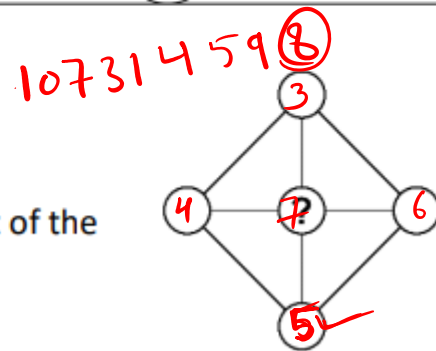
3. Beate arranges the five cards so that the smallest nine-digit number is
created. Which card is furthest on the right?

(A) $\boxed{4}$ (B) $\boxed{8}$ (C) $\boxed{31}$ (D) $\boxed{59}$ (E) $\boxed{107}$

4. The numbers 3, 4, 5, 6, 7 are written inside the five circles of the shape. The product of the
numbers in the four outer circles is 360. Which number is in the inner circle?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

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$(20+22) \div (20-22) = ?$

$42 \div (-2) \Rightarrow -(\ )$

$-(42 \div 2)$

$(-42 \div 2)$

$(-42) \div (-2)$

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28. Mowgli asks a bear and a panther which day of the week it is. The bear always lies on Monday, Tuesday and Wednesday. The panther always lies on Thursday, Friday and Saturday. On all other days they both always speak the truth. The bear says: „Yesterday was one of my lying days." The panther says: „Yesterday was also one of my lying days." On which day of the week did this conversation take place?

(A) Thursday   (B) Friday   (C) Saturday   (D) Sunday   (E) Monday

29. Some points are marked on a straight line. Renate marks another point between every pair of adjacent points. She repeats this process three more times.
Now 225 points are marked on the straight line. How many points were there to begin with?

(A) 10   (B) 12   (C) 15   (D) 16   (E) 25

30. In total there are 2022 kangaroos and some koalas living within seven parks. As many kangaroos live in each park as there are koalas in all other parks together. How many koalas in total live in the seven parks?

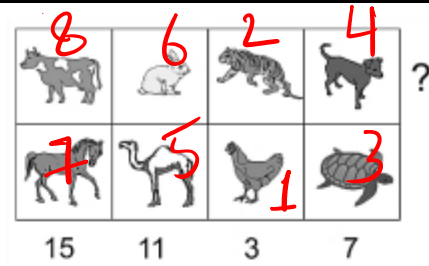(A) 288   (B) 337   (C) 576   (D) 674   (E) 2022
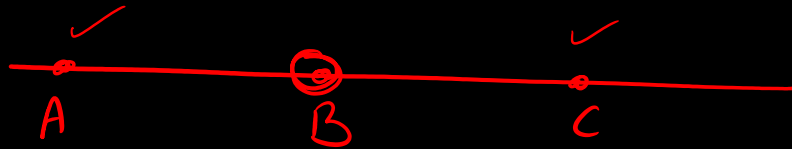
22. Each animal in the picture on the right represents a natural number greater than zero.
Different animals represent a different numbers.
The sum of the two numbers of each column is written underneath each column.
What is the maximum value the sum of the four numbers in the upper row can have?
(A) 18   (B) 19   (C) 20   (D) 21   (E) 22

15   11   3   7

23. 30 people are sitting around a round table. Some of them are wearing a hat.
Those who do not wear a hat, have to speak the truth.
Those who wear a hat can either speak the truth or lie.
They all claim: „At least one of my two neighbouring people wears a hat."
What is the biggest number of people that do not wear a hat?
(A) 5   (B) 10   (C) 15   (D) 20   (E) 25

24. Kai has to insert the numbers 3, 4, 5, 6 and 7 into the five circles of the diagram on the
right in the following way: The product of the three numbers in the vertices of each triangle
has to be equal to the number stated within the triangle.
How big is the sum of the numbers in the vertices of the triangle with the number 168?
(A) 12   (B) 14   (C) 15   (D) 17   (E) 18

105   84
210   168
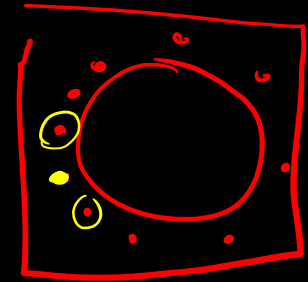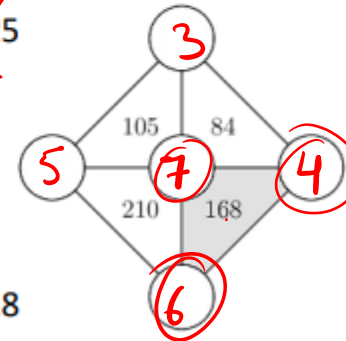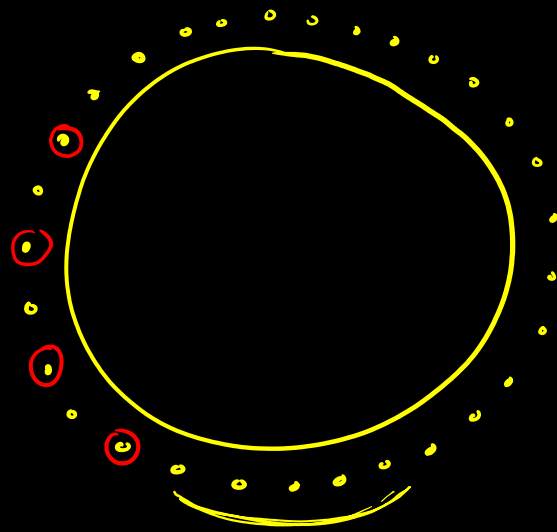
Q1. Amit has 16p picture cards. If each picture card costs Rs. 3pq, determine the cost of ~~picture~~ all picture cards.

Q2. Rohit covers 9y cm in one step, what is the distance moved by him in 7x steps?

Q3. The volume of a cuboid is given by the product of its length, breadth and height. The length of a cuboid is 3 times its breadth and the height is one-half of the length. Find its volume if breadth is y cm.

④

length $= 2x^2y$ cm

breadth $= 3xy^2$ cm.

Area :-

$$\frac{x^{\textcircled{3}} \times x^3}{}$$

$(x^{3 \times 2})$

⑤ The cost of painting a rectangular metal sheet is square of its area. If the length and breadth of the rectangle are $2xy$ and $3x^2y$, find the cost.

⑥ In a room there are x^2 rows of chairs and each row contains $2x^2$ chairs. Find the total number of chairs in the room.

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