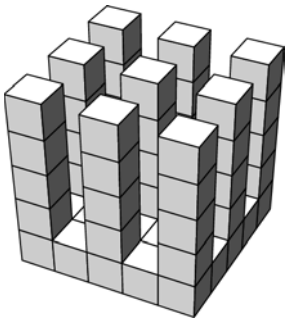
Mathematical Kangaroo 2014
Group Student (Grade 11 onwards)
Austria - 20.3.2014

- 3 point questions -

1. If one removes some 1×1×1 cubes from a 5×5×5 cube, you obtain the solid shown. It consists of several equally high pillars that are built upon a common base. How many little cubes have been removed?



(A) 56 (B) 60 (C) 64 (D) 68 (E) 80

2. Today is Carmen, Gerda and Sabine's birthday. The sum of their ages is now 44. How big will the sum of their ages be, the next time it is a two-digit number with two equal digits?

(A) 55 (B) 66 (C) 77 (D) 88 (E) 99

3. How big is the value of a^{-3k}, if $a^k = \frac{1}{2}$?

(A) $\frac{1}{8}$ (B) 8 (C) -8 (D) 6 (E) $\frac{1}{6}$

4. In three differently sized baskets there are 48 balls in total. Together the smallest and the biggest basket hold twice as many balls as the middle one. The smallest basket holds half as many balls as the middle one. How many balls are there in the biggest basket?

(A) 16 (B) 20 (C) 24 (D) 30 (E) 32

5. $\frac{2^{2014}-2^{2013}}{2^{2013}-2^{2012}} = ?$

(A) 2^{2011} (B) 2^{2012} (C) 2^{2013} (D) 1 (E) 2

6. For which of the following expressions is $b+1$ not a factor?

(A) $2b+2$ (B) b^2-1 (C) b^2+b (D) $-1-b$ (E) b^2+1

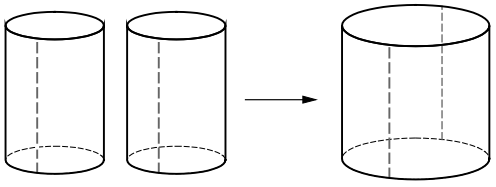
7. How many digits has the result of the calculation $(2^{22})^5 \times (5^{55})^2$?

(A) 22 (B) 55 (C) 77 (D) 110 (E) 111

8. Handsome Fritz has a secret e-mail-adress which is only known by four of his friends. Today he received eight e-mails at this address. Which of the following statements is definitely correct?

(A) Fritz has received two e-mails from each friend.
(B) Fritz cannot have received eight e-mails from one friend.
(C) Fritz has received at least one e-mail from each friend.
(D) Fritz has received at least two e-mails from one of his friends.
(E) Fritz has received at least two e-mails from at least two of his friends.

9. The curved surfaces of two identical cylinders are cut open along the vertical dotted line, as shown and then stuck together to create the curved surface of one big cylinder. What can be said about the volume of the resulting cylinder compared to the volume of one of the small cylinders?



(A) It is 2-times as big. (B) It is 3-times as big.
(C) It is π-times as big. (D) It is 4-times as big.
(E) It is 8-times as big.

10. In the year 2014 all digits are different and the last digit is bigger that the sum of the other three digits. How many years ago was this last the case?

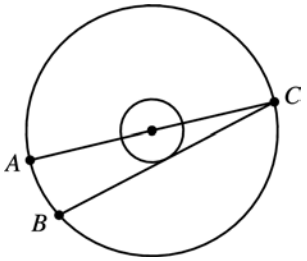
(A) 5 (B) 215 (C) 305 (D) 395 (E) 485

- 4 Point Questions -

11. A cuboid-shaped box has the measurements $a \times b \times c$ with $a < b < c$. If one increases a or b or c by 5 cm, the volume of the box increases as well. When is the increase biggest?
(A) If one increases a. (B) If one increases b.
(C) If one increases c. (D) The answer is depending on the values of a, b and c.
(E) The volume increases in the cases (A), (B) and (C) by an equal amount.

12. The winning team of a football match gets 3 points and the losing team 0 points. In the case of a draw both teams get one point each. Four teams A, B, C and D play a tournament. Each team plays each other team exactly once. At the end of the tournament Team A has 7 points, and Teams B and C have 4 points each. How many points has Team D got?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

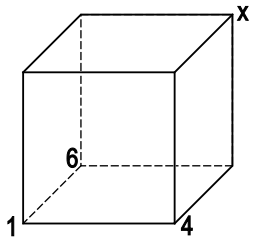
13. The ratio of the radii of two concentric circles is 1 : 3. The line AC a diameter of the biggest circle. A chord BC of the big circle touches the small circle (see diagram). The line AB has length 12. How big is the radius of the big circle?
(A) 13 (B) 18 (C) 21 (D) 24 (E) 26



14. How many whole number triples (a,b,c) with $a > b > c > 1$ fulfil the condition $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}>1$?
(A) none (B) 1 (C) 2 (D) 3 (E) infinitely many

15. Six weeks are $n!$ (= $n\cdot(n-1)\cdot\ldots\cdot2\cdot1$) seconds. n = ?
(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

16. The vertices of a die are numbered 1 to 8, so that the sum of the four numbers on the vertices of each face are the same. The numbers 1, 4 and 6 are already indicated in the picture. Which number is in position x?
(A) 2 (B) 3 (C) 5 (D) 7 (E) 8

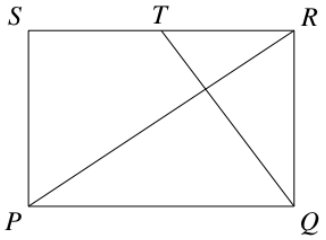


17. On the packaging of a soft cheese it says: total amount of fat 24%. On the same packaging it also says: 64% fat in the dry substance. How much water as a percentage is in the soft cheese?
(A) 88 % (B) 62·5 % (C) 49 % (D) 42 % (E) 37·5 %

18. The function $f(x) = ax + b$ fulfils the conditions $f(f(f(1))) = 29$ and $f(f(f(0))) = 2$. What is the value of a?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

19. Amongst 10 different positive whole numbers there are exactly 5 that are divisible by 5 and exactly 7 that are divisible by 7. Let M be the biggest amongst these numbers. What is the smallest possible value of M?
(A) 105 (B) 77 (C) 75 (D) 63 (E) another value

20. $PQRS$ is a rectangle. T is the midpoint of RS. QT is normal to the diagonal PR. What is the ratio of the lengths PQ : QR?
(A) 2 : 1 (B) $\sqrt{3}$: 1 (C) 3 : 2 (D) $\sqrt{2}$: 1 (E) 5 : 4

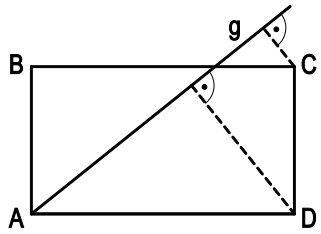


- 5 Point Questions -

21. Let *a,b,c* be different real numbers not equal to zero and *n* be a positive whole number. It is known that the numbers $(-2)^{2n+3} \times a^{2n+2} \times b^{2n-1} \times c^{3n+2}$ and $(-3)^{2n+2} \times a^{4n+1} \times b^{2n+5} \times c^{3n-4}$ have the same sign. Which of the following statements is definitely true?

(A) $a > 0$ (B) $b > 0$ (C) $c > 0$ (D) $a < 0$ (E) $b < 0$

22. The straight line *g* runs through the vertex *A* of the rectangle *ABCD shown*. The perpendicular distance from *C* to *g* is 2 and from *D* to *g* is 6. *AD* is twice as long as *AB*. Determine the length of *AD*.

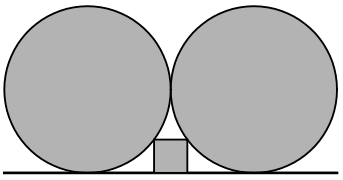


(A) 10 (B) 12 (C) 14 (D) 16 (E) $4\sqrt{3}$

23. There are 9 kangaroos that are called the Greatkangs. They are either coloured white or black. If three Greatkangs meet by chance, the probability that none of them is white is exactly two thirds. How many Greatkangs are black?

(A) 1 (B) 3 (C) 5 (D) 6 (E) 8

24. In the diagram on the right the following can be seen: a straight line, which is the common tangent of two touching circles with radius 1, and a square with one edge on the straight line and the other vertices one on each of the two circles. How big is the side length of the square?



(A) $\frac{2}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2}$

25. Thomas wants to write down pairwise, different positive whole numbers none of which should be bigger than 100. Their product should not be divisible by 54. How many numbers can he write down at the most?

(A) 8 (B) 17 (C) 68 (D) 69 (E) 90

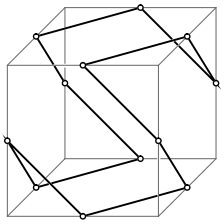
26. Two regular polygons with side length 1, lay on opposite sides of the common edge *AB*. One of them is the 15-sided polygon $ABC_1D_1E_1$... and the other one is the n-sided polygon $ABC_2D_2E_2$... . For which value of *n* is the distance between C_1 to C_2 exactly 1?

(A) 10 (B) 12 (C) 15 (D) 16 (E) 18

27. The chain of equations $k = (2014 + m)^{\frac{1}{n}} = 1024^{\frac{1}{n}} + 1$ should be valid for the positive whole numbers *k*, *m*, *n*. How many different values can *m* assume?

(A) none (B) 1 (C) 2 (D) 3 (E) infinitely many

28. In the diagram a closed polygon can be seen whose vertices are the midpoints of the edges of the die. The interior angles are as usual defined as the angle that two sides of the polygon describe in a common vertex. How big is the sum of all interior angles of the polygon?



(A) 720° (B) 1080° (C) 1200° (D) 1440° (E) 1800°

29. The mapping $f : Z \to Z$ fulfils the conditions $f(4) = 6$ and $xf(x) = (x-3)f(x+1)$. What is the value of the expression $f(4) \cdot f(7) \cdot f(10) \cdot \ldots \cdot f(2011) \cdot f(2014)$?

(A) 2013 (B) 2014 (C) 2013·2014 (D) 2013! (E) 2014!

30. In the forests of a magical island kingdom there are three kinds of animals: lions, wolves and goats. Wolves can eat goats and lions can eat wolves as well as goats. Since it is a magical island kingdom, the wolf that eats a goat changes into a lion. A lion that eats a goat changes into a wolf and a lion that eats a wolf changes into a goat. To begin with there were 17 goats, 55 wolves and 6 lions on the island. After some time no more eating is possible. How big is the maximum amount of animals that can still be on the island?

(A) 1 (B) 6 (C) 17 (D) 23 (E) 35