Känguru der Mathematik 2018
Level Junior (Grade 9 and 10)
Austria – 15.3.2018

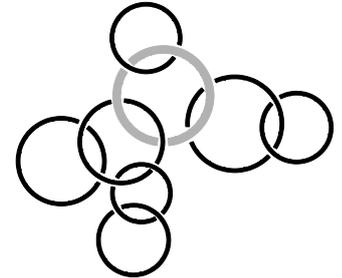**1.** Every child in my family has at least two brothers and at least one sister. What is the minimum number of children in my family?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

2. The rings shown are partially interlinked. How long is the longest chain built this way which also contains the thick light ring?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

3. In a triangle one side has length 5 and another side has length 2. The length of the third side is an odd whole number. Determine the length of the third side.

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

4. The distance between the top of the cat that is sitting on the table to the top of the cat that is sleeping on the floor is 150 cm. The distance from the top of the cat that is sleeping on the table to the top of the cat that is sitting on the floor is 110 cm. How high is the table?
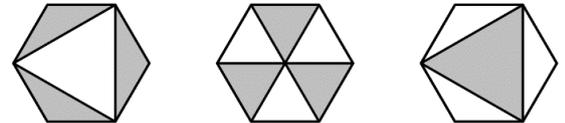
(A) 110 cm   (B) 120 cm   (C) 130 cm   (D) 140 cm   (E) 150 cm
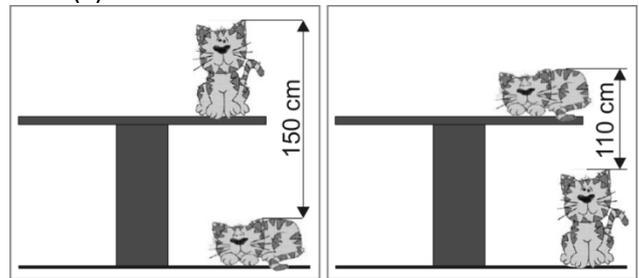
5. The sum of 5 consecutive whole numbers is 10^{2018}. What is the middle number of those numbers?

(A) 10^{2013} (B) 5^{2017} (C) 10^{2017} (D) 2^{2018} (E) $2 \cdot 10^{2017}$

6. In the three regular hexagons shown, X, Y and Z describe in this order the areas of the grey shaded parts. Which of the following statements is true?

(A) $X = Y = Z$ (B) $Y = Z \neq X$ (C) $Z = X \neq Y$ (D) $X = Y \neq Z$ (E) Each of the areas has a different value.

7. Maria wants to divide 42 apples, 60 peaches and 90 cherries fairly amongst her friends. In order to do so she divides the entire fruit into baskets, each with the same amount of apples, peaches and cherries, to then give each of her friends one such basket with fruit. At most, how many baskets of fruit can she fill this way?

(A) 3   (B) 6   (C) 10   (D) 14   (E) 42

8. In the (correct) calculation shown, some of the digits were replaced by the letters P, Q, R and S. What is the value of P + Q + R + S?

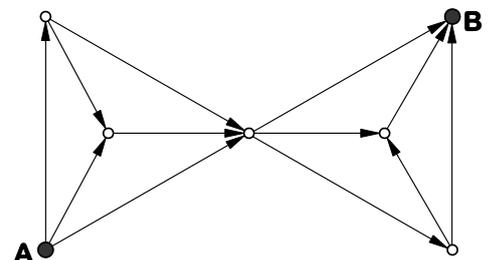(A) 14   (B) 15   (C) 16   (D) 17   (E) 24

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  P 4 5
+ Q R S
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  6 5 4
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9. How big is the sum of 25 % of 2018 and 2018 % of 25?

(A) 1009   (B) 2016   (C) 2018   (D) 3027   (E) 5045

10. In the diagram shown, you should follow the arrows to get from A to B. How many different ways are there that fulfill this condition?

(A) 20   (B) 16   (C) 12   (D) 9   (E) 6

11. The entrances of two student halls lie in a plain street 250 m apart from each other. There are 100 students in the first one and 150 students in the second one. Where should a bus stop be built if the total sum of the distances that each student of both halls has to cover to get to the bus stop should be a minimum?

(A) directly in front of the first hall   (B) 100 m away from the entrance of the first hall
(C) 100 m away from the entrance of the second hall   (D) directly in front of the second hall
(E) in any place between the two hall entrances

12. 105 numbers are written in a row: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5,…Where each number n is written exactly n-times. How many of those numbers are divisible by 3?

(A) 4   (B) 12   (C) 21   (D) 30   (E) 45

13. Eight congruent semi-circles are drawn inside a square with side length 4. How big is the area of the white part?

(A) 2π (B) 8   (C) $6 + \pi$ (D) $3\pi - 2$ (E) 3π
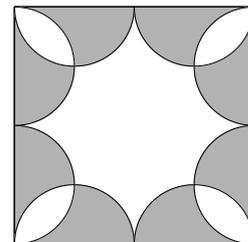
14. On one particular day there are a total of 40 trains from one of the towns M, N, O, P and Q to exactly one other of those towns. There are 10 trains either from or to M. There are 10 trains either from or to N. There are 10 trains either from or to O. There are 10 trains either from or to P. How many trains are there either from or to Q?

(A) 0   (B) 10   (C) 20   (D) 30   (E) 40

15. At a humanistic university you can study languages, history and philosophy. Some of the students there study exactly one language. (Nobody studies several languages at the same time.) Amongst those, 35 % study English. Amongst all students of the university 13 % study a language other than English. Which percentage of the students studies a language?

(A) 13 %   (B) 20 %   (C) 22 %   (D) 48 %   (E) 65 %

16. Peter wants to buy a book but has no money. He can only buy this book with his father's and his two brother's help. His father gives him half as much money as his brothers give him jointly. His older brother gives him a third of the sum that the two others give him. The youngest brother gives him 10 €. How expensive is the book?

(A) 24 €   (B) 26 €   (C) 28 €   (D) 30 €   (E) 32 €

17. How many three-digit numbers are there with the property that the two-digit number obtained by deleting the middle number is exactly a ninth of the original number?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

18. How often does the summand 2018² appear under the root, if the following statement is correct?

$$\sqrt{2018^2 + 2018^2 + \ldots + 2018^2} = 2018^{10}$$

(A) 5   (B) 8   (C) 18   (D) 2018^8 (E) 2018^{18}

19. How many digits has the final result of the calculation $\frac{1}{9} \cdot 10^{2018} \cdot (10^{2018} - 1)$?

(A) 2017   (B) 2018   (C) 4035   (D) 4036   (E) 4037

20. In a regular 2018-sided shape the vertices are numbered 1 to 2018 in order. Two diagonals of the polygon are drawn in, where one of them connects the vertices 18 and 1018 and the other one the vertices 1018 and 2000. How many vertices do the three resulting polygons have?

(A) 38, 983, 1001   (B) 37, 983, 1001   (C) 38, 982, 1001   (D) 37, 982, 1000   (E) 37, 983, 1002

21. Some whole numbers are written on a board, amongst them the number 2018. The sum of all these number is 2018. The product of all these number is also 2018. Which of the following numbers could be the amount of numbers on the board?

(A) 2016   (B) 2017   (C) 2018   (D) 2019   (E) 2020

22. Given are four positive numbers. Take three of them, work out their mean and then add the fourth number. This can be done in four different ways. The results obtained this way are 17, 21, 23 and 29. Which number is the biggest of the four numbers?

(A) 12   (B) 15   (C) 21   (D) 24   (E) 29

23. The points $A_0, A_1, A_2, ...$ all lie on a straight line. It is true that $\overline{A_0 A_1} = 1$ and A_n is the midpoint of every line segment $A_{n+1}A_{n+2}$, for every non-negative index n. How long is the line segment A_0A_{11}?

(A) 171   (B) 341   (C) 512   (D) 587   (E) 683

24. Two concentric circles with radii 1 and 9 form an annulus. n circles without overlap are drawn inside this annulus, where every circle touches both circles of the annulus. (The diagram shows an example for n=1 and the other radii as given.) What is the biggest possible value of n?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5
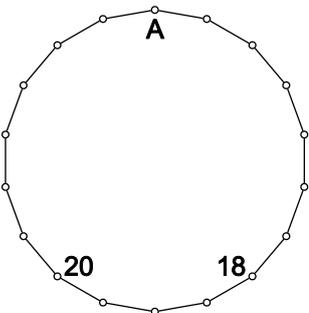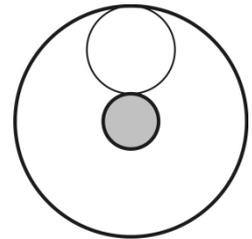
25. A number is to be written into every vertex of the 18-sided shape so that it is equal to the sum of the two numbers from the adjacent vertices. Two of these numbers are given. Which number is written in vertex A?

(A) 2018   (B) −20   (C) 18   (D) 38   (E) −38

26. Diana draws a rectangle made up of twelve squares onto a piece of squared paper. Some of the squares are coloured in black. She writes the number of adjacent black squares into every white square. The diagram shows an example of such a rectangle. Now she does the same with a rectangle made up of 2018 squares. What is the biggest number that she can obtain as the sum of all numbers in the white squares?

(A) 1262   (B) 2016   (C) 2018   (D) 3025   (E) 3027
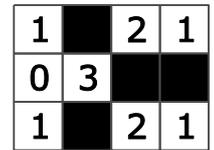
27. Seven little dice were removed from a 3 x 3 x 3 die, as can be seen in the diagram. The remaining (completely symmetrical) figure is cut along a plane through the centre and perpendicular to one of the four space diagonals. What does the cross-section look like?
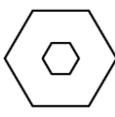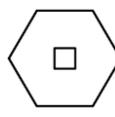
(A)   (B)   (C)   (D)   (E)
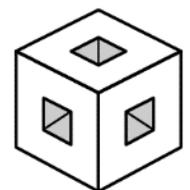
28. Every number of the set {1, 2, 3, 4, 5, 6} is written into exactly one cell of a 2 x 3 table. In how many ways can this be done so that the sum of the numbers in every column and every row is divisible by 3?

(A) 36   (B) 42   (C) 45   (D) 48   (E) another number

29. Ed forms a big die using several identical small white dice and colours some of the faces of the big die, red. His sister Nicole drops the die and it again breaks into the original small dice. 45 of which do not have a red face. How many faces of the big die did Ed colour in red?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

30. Two chords AB and AC are drawn into a circle with diameter AD. $\angle BAC = 60°$, $\overline{AB} = 24$ cm, E lies on AC so that $\overline{EC} = 3$ cm, and BE is perpendicular to AC. How long is the chord BD?

(A) $\sqrt{3}$ cm   (B) 2 cm   (C) 3 cm   (D) $2\sqrt{3}$ cm   (E) $3\sqrt{2}$ cm