

KÄNGURU DER MATHEMATIK 2022

17. 3. 2022



Level: Student, Grades 11–13

Full name:	
School:	
Class:	

Time: 75 min.

30 starting points

each correct answer to questions 1. – 10.: 3 points

each correct answer to questions 11. – 20.: 4 points

each correct answer to questions 21. – 30.: 5 points

each questions left unanswered: 0 points

each incorrect answer: minus $\frac{1}{4}$ of the points for the question

Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 bis 30). Write clearly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Zustimmungserklärung zur Datenverarbeitung für den österreichischen Wettbewerb „Känguru der Mathematik“

Mit meiner Unterschrift gebe ich das Einverständnis, dass meine angeführten personenbezogenen Daten (Vor- und Zuname, Klasse, Schulstufe, Schulstandort und Schularart) zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von schulweiten Reihungen, sowie zur Erstellung und Veröffentlichung der Siegerlisten auf unserer Vereinshomepage (sofern mindestens 50 % der zu erreichenden Punktezahl erlangt werden bzw. ich unter den besten 10 einer Kategorie liege) verwendet werden dürfen.

Betroffenenrechte

Die Verwendung dieser Daten ist bis 31. Dezember des 2. Folgejahres gestattet. Nach diesem 31. Dezember werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei dieser durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art anonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage der DSGVO erlaubt.

Ich habe ein Recht auf Auskunft über meine gespeicherten personenbezogenen Daten, sowie das Recht auf Berichtigung, Datenübertragung, Widerspruch, Einschränkung der Bearbeitung sowie Sperrung oder Löschung unrichtig verarbeiteter Daten.

Ich kann die erteilte Einwilligung jederzeit auf der Homepage des Vereines Känguru der Mathematik unter www.kaenguru.at mittels des dafür bereitgestellten Formulars mit Wirkung für die Zukunft widerrufen (Art. 21 Abs. 1 DSGVO).

Ein Widerruf hat zur Folge, dass die personenbezogenen Daten nach gegenseitiger Rücksprache innerhalb von 31 Tagen gelöscht werden.

Durch den Widerruf wird die Rechtmäßigkeit der aufgrund der Einwilligung bis zum Widerruf erfolgten Verarbeitung nicht berührt. (Art. 7 Abs. 2 DSGVO)

Ort, Datum

Unterschrift



Information über den Känguruwettbewerb: www.kaenguru.at
Wenn du mehr in dieser Richtung machen möchtest,
gibt es die Österreichische Mathematikolympiade.
Infos unter: www.oemo.at

Känguru der Mathematik 2022

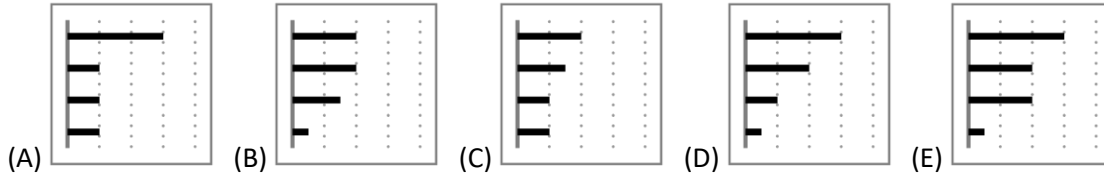
Level Student (Schulstufe 11, 12 and 13)

Austria – 18. 3. 2021



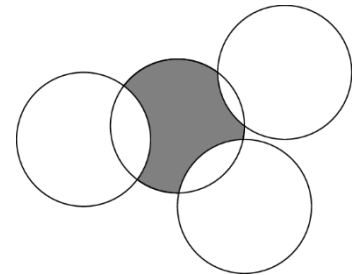
- 3 Point Examples -

1. Martin's smartphone displays the diagram on the right. It shows how long he has worked with four different apps in the previous week. The apps are sorted from top to bottom according to the amount of time they have been used. This week he has spent only half the amount of time using two of the apps and the same amount of time as last week using the other two apps. Which of the following pictures **cannot** be the diagram for the current week?



2. How many positive three-digit numbers are divisible by 13?
 (A) 68 (B) 69 (C) 70 (D) 76 (E) 77
3. Bella is older than Charly and younger than Lily. Which two can be the same age if Teddy is older than Bella?
 (A) Charly and Teddy (B) Teddy and Lily (C) Lily and Charly
 (D) Bella and Lily (E) Teddy and Bella
4. Which one of the following numbers is **not** divisible by its own digit sum?
 (A) 2022 (B) 2023 (C) 2024 (D) 2025 (E) 2027
5. 5^8 pencils are distributed evenly among 25 empty boxes. How many pencils are in each box?
 (A) 25^3 (B) 25^2 (C) 5^3 (D) 5^2 (E) 5
6. The product of the digits of a ten-digit number is 15. How big is the sum of the digits of this number?
 (A) 8 (B) 12 (C) 15 (D) 16 (E) 20

7. Four circles with radius 1 intersect each other as seen in the diagram. What is the perimeter of the grey area?

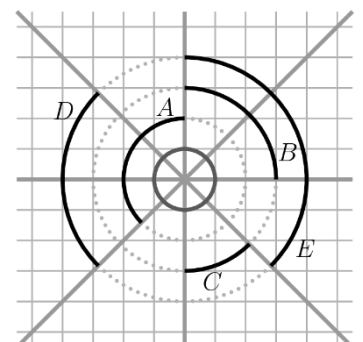


- (A) π (B) $\frac{3\pi}{2}$ (C) a number between $\frac{3\pi}{2}$ and 2π (D) 2π (E) π^2
8. All integers from 2 to 2022 which can be written using only the digits 0 and 2 are written in ascending order in a list.
 Which number is the middle number on that list?
 (A) 200 (B) 220 (C) 222 (D) 2000 (E) 2002

9. How many real solutions does the equation $(x - 2)^2 + (x + 2)^2 = 0$ have?
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4



10. The points A , B , C and D are marked on a straight line in this order as shown in the diagram. We know that A is 12 cm from C and that B is 18 cm from D . How far apart from each other are the midpoints of the line segments AB and CD ?
 (A) 6 cm (B) 9 cm (C) 12 cm (D) 13 cm (E) 15 cm



4 Point Examples -

11. Four straight lines that intersect in one single point form eight equal angles (see diagram). Which one of the black arcs has the same length as the circumference of the little (grey) circle?
 (A) A (B) B (C) C (D) D (E) E

12. a , b and c are real numbers not equal to zero. It is known that the numbers $-2a^4b^3c^2$ and $3a^3b^5c^{-4}$ have the same sign. Which of the following statements is definitely correct?

- (A) $ab > 0$ (B) $b < 0$ (C) $c > 0$ (D) $bc > 0$ (E) $a < 0$

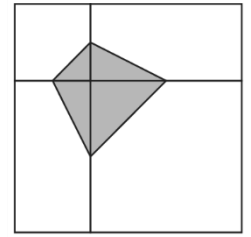
13. We check the water meter and see that all digits on the display are different. What is the minimum amount of water that has to be used before this happens again?

- (A) 0.006 m^3 (B) 0.034 m^3 (C) 0.086 m^3 (D) 0.137 m^3 (E) 1.048 m^3



14. The square pictured, is split into two squares and two rectangles. The vertices of the shaded quadrilateral with area 3 are the midpoints of the sides of the smaller squares. What is the area of the non-shaded part of the big square?

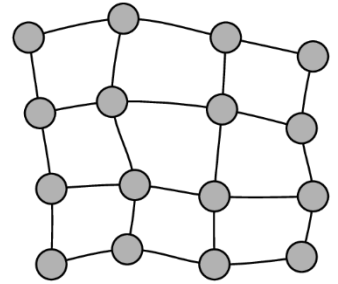
- (A) 12 (B) 15 (C) 18 (D) 21 (E) 24



15. What is the largest common divisor of $2^{2021} + 2^{2022}$ and $3^{2021} + 3^{2022}$?

- (A) 2^{2021} (B) 1 (C) 2 (D) 6 (E) 12

16. The diagram shows a map with 16 towns which are connected via roads. The government is planning to build power plants in some towns. Each power plant can generate enough electricity for the town in which it stands as well as for its immediate neighbouring towns (i.e. towns that can be reached via a direct connecting road).

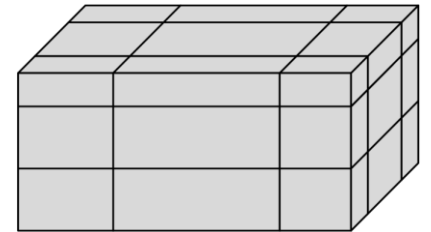


What is the minimum number of power plants that have to be built?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

17. In a tournament with 8 participants the players are randomly paired up in four teams for the first round and the winner of each encounter then proceeds to the second round. There are two games in the second round and the two winners then play the final. Anita and Martina are the two best players and will win against all others; in case they have to play against each other, Anita will win. How big is the chance that Martina will get to the final?

- (A) 1 (B) $\frac{1}{2}$ (C) $\frac{2}{7}$ (D) $\frac{3}{7}$ (E) $\frac{4}{7}$

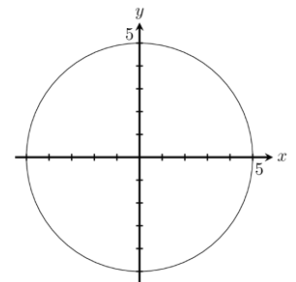


18. A cuboid with surface area X is cut up along six planes parallel to the sides (see diagram). What is the total surface area of all 27 thus created solids?

- (A) X (B) $2X$ (C) $3X$ (D) $4X$ (E) It depends on the position of the planes.

19. The arithmetic mean of five numbers is 24. The mean of the three smallest numbers is 19 and that of the three biggest is 28. What is the median of the five numbers?

- (A) 20 (B) 21 (C) 22 (D) 23 (E) 24



20. A circle with midpoint $(0|0)$ has a radius of 5. How many points are there on the circumference where both co-ordinates are integers?

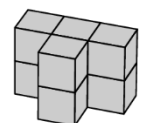
- (A) 5 (B) 8 (C) 12 (D) 16 (E) 20

- 5 Point Examples -

21. The vertices of a 20-gon are labelled using the numbers 1 to 20 so that adjacent vertices always differ by 1 or 2. The sides of the 20-gon whose vertices are labelled with numbers that only differ by 1 are drawn in red. How many red sides does the 20-gon have?

- (A) 1 (B) 2 (C) 4 (D) 5 (E) 10

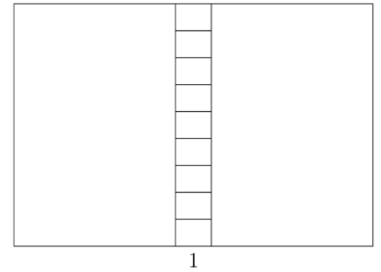
22. Which two building blocks can be joined together so that the object shown is created?



- (A) (B) (C) (D) (E)

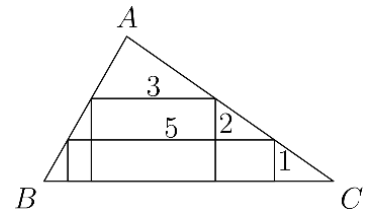
23. A rectangle is split into 11 smaller rectangles as shown.
 All 11 small rectangles are similar to the initial rectangle.
 The smallest rectangles are aligned like the original rectangle (see diagram).
 The lower sides of the smallest rectangles have length 1.
 How big is the perimeter of the big rectangle?

(A) 20 (B) 24 (C) 27 (D) 30 (E) 36



24. Two rectangles are inscribed into a triangle as shown in the diagram.
 The dimensions of the rectangles are 1×5 and 2×3 respectively.
 How big is the height of the triangle in A?

(A) 3 (B) $\frac{7}{2}$ (C) $\frac{8}{3}$ (D) $\frac{6}{5}$ (E) another number

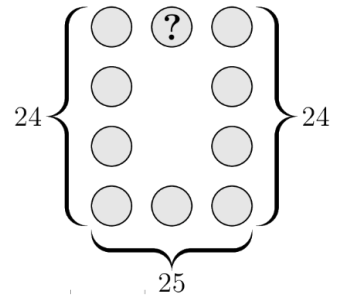


25. How many three-digit numbers are there that are equal to five times the product of their digits?

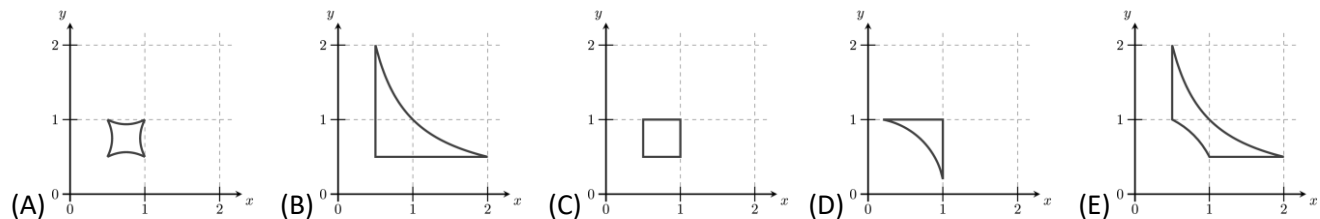
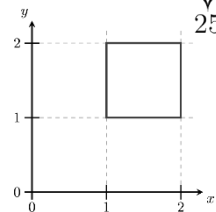
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

26. The numbers 1 to 10 were written into the ten circles in the pattern shown in the picture.
 The sum of the four numbers in the left and the right column is 24 each and the sum of the three numbers in the bottom row is 25. Which number is in the circle with the question mark?

(A) 2 (B) 4 (C) 5 (D) 6 (E) another number



27. A square is placed in a co-ordinate system as shown. Each point $(x|y)$ of the square is deleted and replaced by the point $(\frac{1}{x} | \frac{1}{y})$.
 Which diagram shows the resulting shape?



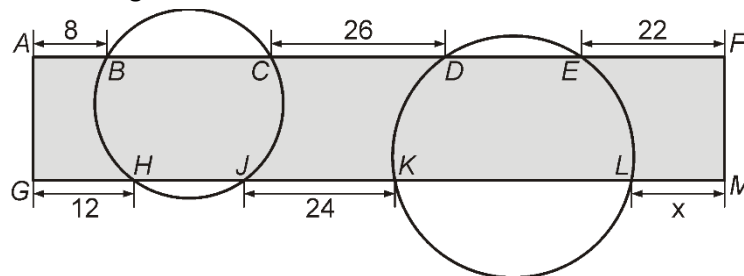
28. Let N be a positive integer. How many integers are between $\sqrt{N^2 + N + 1}$ and $\sqrt{9N^2 + N + 1}$?

(A) $N + 1$ (B) $2N - 1$ (C) $2N$ (D) $2N + 1$ (E) $3N$

29. A sequence $\langle a_n \rangle$ has $0 < a_1 < 1$. For all $n \geq 1$ we know that $a_{2n} = a_2 \cdot a_n + 1$ and $a_{2n+1} = a_2 \cdot a_n - 2$.
 We know that $a_7 = 2$. What is the value of a_2 ?

(A) a_1 (B) 2 (C) 3 (D) 4 (E) 5

30. Two circles intersect a rectangle $AFMG$ as shown in the diagram. The line segments along the long side of the rectangle that are outside the circles have length $AB = 8$, $CD = 26$, $EF = 22$, $GH = 12$ and $JK = 24$.
 How long is the length x of the line segment LM ?



(A) 14 (B) 15 (C) 16 (D) 17 (E) 18