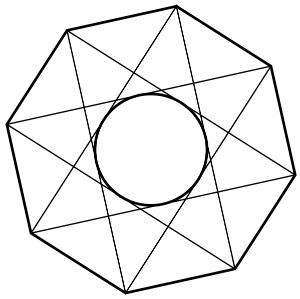
Mathmatical Kangaroo 2013
Group Student (Grade 11. and above)
Austria - 21.3.2013

- 3 Point Questions -

1. Which of the following numbers is biggest?
(A) 2013 (B) 2^{0+13} (C) 20^{13} (D) 201^3 (E) $20 \cdot 13$



2. The regular eight-sided shape on the right has sides of length 10. A circle touches all inscribed diagonals of this eight-sided shape. What is the radius of this circle?
(A) 10 (B) 7,5 (C) 5 (D) 2,5 (E) 2

3. The surface of a prism is made of 2013 faces. How many edges does the prism have?
(A) 2011 (B) 2013 (C) 4022 (D) 4024 (E) 6033

4. The third root of 3^{3^3} takes which value? (Note: $a^{b^c} = a^{(b^c)}$.)
(A) 3^3 (B) 3^{3^3-1} (C) 3^{2^3} (D) 3^{3^2} (E) $(\sqrt{3})^3$

5. The date 2013 is made up of four consecutive digits 0, 1, 2, 3. How many years before the year 2013 was the date last made up of four consecutive digits?
(A) 467 (B) 527 (C) 581 (D) 693 (E) 990

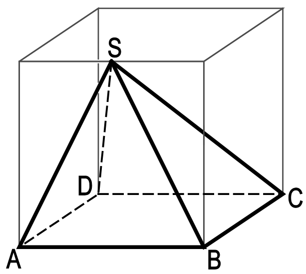
6. Let f be a linear function for which $f(2013) - f(2001) = 100$ gilt. holds true. What is the value of $f(2031) - f(2013)$?
(A) 75 (B) 100 (C) 120 (D) 150 (E) 180

7. We know that the relationship $2 < x < 3$ is valid for a number x**.** How many of the following statements are true in this case?

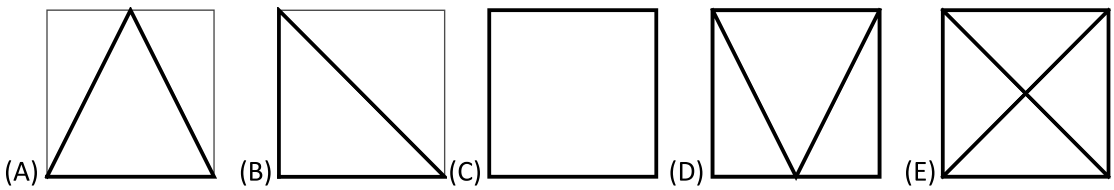
$4 < x^2 < 9$ $\quad 4 < 2x < 9$ $\quad 6 < 3x < 9$ $\quad 0 < x^2 - 2x < 3$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. Each of six lone heros has captured wanted people. In total they have captured 20 wanted people: the first hero one wanted person, the second hero two wanted people, the third hero three wanted people. The fourth hero has captured more wanted people than any other hero. Determine the smallest number of wanted people that the fourth hero could have captured, so that this statement could be true.
(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

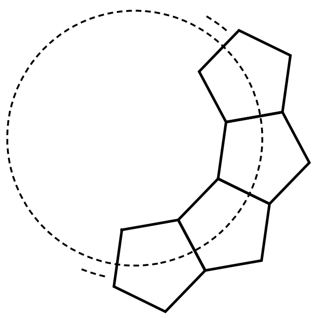


9. Inside the cube lattice pictured on the side one can see a solid, non-seethrough pyramid $ABCDS$ with square base $ABCD$, whose top S is exactly in the middle of one edge of the cube. If you look at the pyramid from above, from below, from the front, from the back, from the right and from the left – which of the following views cannot be possible?



10. If a certain substance melts the volume increases by $\frac{1}{12}$.
By how much does the volume decrease if the substance solidifies again?
(A) $\frac{1}{10}$ (B) $\frac{1}{11}$ (C) $\frac{1}{12}$ (D) $\frac{1}{13}$ (E) $\frac{1}{14}$

- 4 Point Questions -



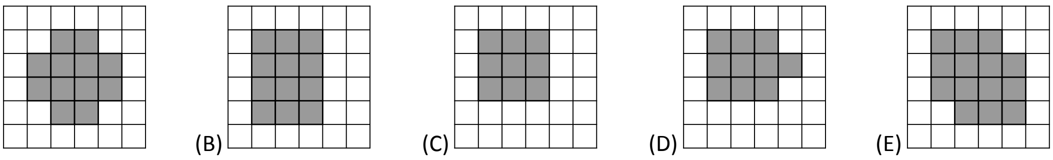
11. Ralf has a number of equally big plastic plates each in the form of a regular five sided shape. He glues them together along the sides to form a complete ring (see picture). Out of how many of these plates is the ring made up?

(A) 8 (B) 9 (C) 10 (D) 12 (E) 15

12. How many positive integers n are there with the property that $\frac{n}{3}$ as well as $3n$ are three-digit numbers?

(A) 12 (B) 33 (C) 34 (D) 100 (E) 300

13. . A circular carpet is placed on a floor which is covered by equally big, square tiles. All tiles that have at least one point in common with the carpet are coloured in grey. Which of the following cannot be a result of this?



(A) (B) (C) (D) (E)

14. We are looking at the following statement about a function defined for all integers x $f: Z \to Z$: "For each even x $f(x)$ is even." What would be the negation of this statement?

(A) For each even x $f(x)$ is odd.

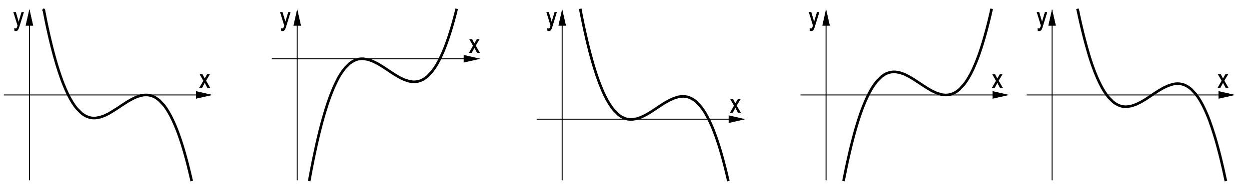
(B) For each odd x $f(x)$ is even.

(C) For each odd x $f(x)$ is odd.

(D) There is a number x, for which $f(x)$ is odd.

(E There is an odd number x, for which $f(x)$ is odd.

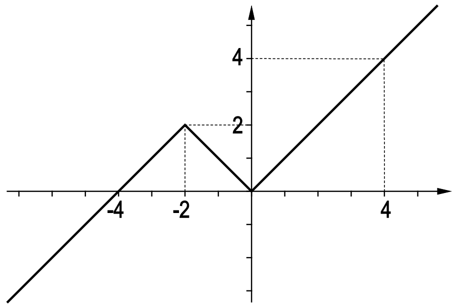
15. Amongst the graphs shown below there is the graph of the function $f(x) = (a-x)(b-x)^2$ with $a < b$. Which is it?



(A) (B) (C) (D) (E)

16. We are considering rectangles which have one side of length of 5.0 cm. Amongst these there are some that can be cut to make a square and a rectangle, one of which having an area of 4.0 cm^2. How many such rectangles are there?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5



17. Peter has drawn the graph of a function $f: R \to R$ which consists of two rays and a line segment as indicated on the right. How many solutions has the equation $f(f(f(x))) = 0$?

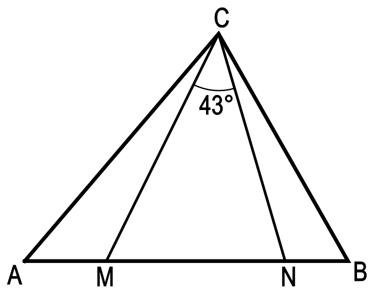
(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

18. How many pairs of positive integers (x, y) solve the equation $x^2 \times y^3 = 6^{12}$

(A) 6 (B) 8 (C) 10 (D) 12 (E) A different number.

19. In a box there are 900 cards that are numbered from 100 to 999. On any two different cards there are always different numbers. Franz picks a few cards and works out the sum of the digits on each card. What is the minimum number of cards he has to pick to have at least three with the same sum?

(A) 51 (B) 52 (C) 53 (D) 54 (E) 55



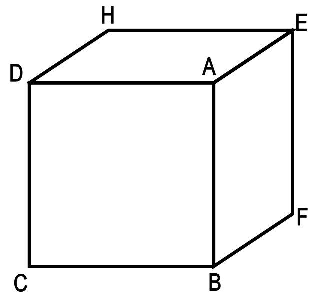
20. In a triangle ABC the points M and N are placed on side AB so that AN = AC and BM = BC. Determine $\angle$ACB if $\angle$MCN = 43°.

(A) 86° (B) 89° (C) 90° (D) 92° (E) 94°

- 5 Point Questions -

21. How many pairs of integers (x, y) with $x \le y$ are there such that their product is exactly five times their sum?

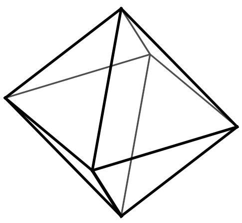
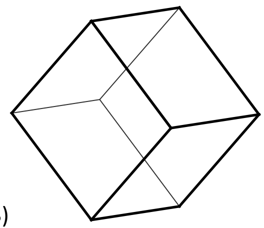
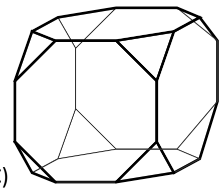
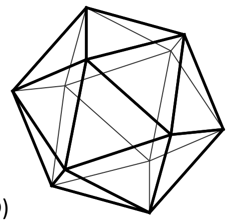
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



22. The function $f: R \to R$ is defined by the following properties:
f is periodic with period 5, and for $-3 \le x < 2$ $f(x) = x^2$ holds true. How big is $f(2013)$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

23. The cube pictured on the side is intersected by a plane that passes through the three points adjacent to A, that is *D*, *E* and *B*. In a similar way the cube is also intersected by those planes that go through the three points adjacent to each of the other seven vertices. These planes dissect the cube into several pieces. What does the piece that contains the centre of the cube look like?

(A)  (B)  (C)  (D) 

(E) The centre of the cube belongs to several pieces.

24. How many solutions (x, y) with a real *x* and *y* has the equation $x^2 + y^2 = |x| + |y|$?
(A) 1 (B) 5 (C) 8 (D) 9 (E) unendlich viele

25. Let $f: N \to N$ be the function that is defined by $f(n) = \frac{n}{2}$ for even *n*, and by $f(n) = \frac{n-1}{2}$ for odd *n*. If *k* is a positive integer then let $f^k(n)$ describe the expression $f(f(...f(n)...))$, in which *f* appears *k*-times. The number of solutions to the equation $f^{2013}(n) = 1$ is
(A) 0 (B) 4026 (C) 2^{2012} (D) 2^{2013} (E) unendlich

26. On the co-ordinate plane a number of straight lines have been drawn. Line *a* intersects exactly three other lines and line *b* intersects exactly four other lines. Line *c* intersects exactly *n* other lines with $n \neq 3, 4$. How many lines were drawn on the plane?
(A) 4 (B) 5 (C) 6 (D) 7 (E) eine andere Anzahl

27. If you add the first *n* positive integers, you obtain a three-digit number with all digits being the same. How big is the sum of the digits of *n*?
(A) 6 (B) 9 (C) 12 (D) 15 (E) 18

28. On an island live only Truthtellers (who always speak the truth) and Liars (who never speak the truth). I met two inhabitants and asked the taller one whether they are both Truthtellers. He replied but from his answer I couldn't decide which group they belonged to. So I asked the smaller one whether the taller one is a Truthteller. He answered and then I knew which type they both were. Which statement is correct?
(A) Both were Truthtellers.
(B) Both were Liars.
(C) The taller one was a Truthteller and the smaller one a Liar.
(D) The taller one was a Liar and the smaller one a Truthteller.
(E) Not enough information is given to be able to decide for certain.

29. Julian wrote an algorithm to form a sequence of numbers. $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$ holds true for all positive integers *m* and *n*. Determine the value of a_{100}.
(A) 100 (B) 1000 (C) 2012 (D) 4950 (E) 5050

30. Five cars enter a roundabout at the same time (see picture). Each car leaves the roundabout having completed less than a whole round and exaclty one car leaves at each exit. How many different combinations are there of how the cars can leave the roundabout?

(A) 24 (B) 44 (C) 60 (D) 81 (E) 120