1. Integers

① Number system (Natural & whole nos.) [abhinav@ekademy.in]

- Integers
- Rational numbers (Fractions / Decimal numbers)
- Operation on rational nos.

② Algebra

- Algebraic expression / notations
- Exponents
- Equations (Linear equation)
- Ratios and Proportions.
- Sequences

1 Integers

Natural number : Counting numbers.

$\{1, 2, \ldots, \infty\}$

whole numbers : 0 + Natural numbers

$\{0, 1, 2, \ldots \infty\}$

Negative numbers: $-1, -2, -3, \ldots \underline{-\infty}$

(Negative integers) negative side of infinity.

$$1024 \pm (\pm 1024) + 1024 = (=1024) + (-1024)$$

$$\Rightarrow 1024 + 1024 + 1024 + 1024 - 1024$$

$$\Rightarrow 3 \times (1024)$$

$$= 3072$$

$$9 + (-4)$$

$$9 - 4$$

Integers (I or Z) : Collection of positive numbers, negative number and zero.

$\{-\infty \ldots, -3, -2, -1, \boxed{0}, 1, 2, 3, \ldots \infty\}$

Zero (0) is neither positive nor negative

All positive integers : $1, 2, 3, 4, \ldots \infty$.

All negative integers : $-1, -2, -3, \ldots -\infty$.

All non-positive integers : $0, -1, -2, -3, \ldots -\infty$

All non-negative integers : $0, 1, 2, 3, \ldots \infty$

Multiplication of integers

→ $\{-\infty \ldots, -2, -1, 0, 1, 2, \ldots \infty\}$

for eg: $6 \times 2 = 12$

$(-3) \times (7) = -21$

$(3) \times (-7) = -21$

① Closure Property : Product of two or more integers is always an integer.

Property ② Commutative property :

If a and b are two integers, then,

$$a \times b = b \times a$$

Multiplication is commutative.

eg: $(-3) \times (7) = -7 \times 3$ ✗

$a \quad b$

$= 7 \times (-3)$ ✓

⊕	×	⊕	=	⊕
⊕	×	⊖	=	⊖
⊖	×	⊕	=	⊖
⊖	×	⊖	=	⊕

$(-2) \times (-2) \times (-2) = -8$

(3) Associative Property

If a, b and c are integers.

$$a \times (b \times c) = (a \times b) \times c$$

eg. $a = (-3)$

$b = (4)$

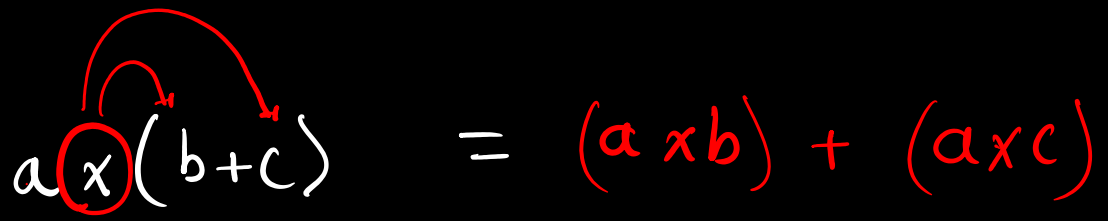
$c = (-7)$

(4) Distributive property.

[or]

– Distributivity of multiplication over addition/subtraction.

If a, b and C are three integers.



$$a \times (b+c) = (a \times b) + (a \times c)$$

$$a(p+q) = ap + aq$$

$$a(p+\overline{q \times h}) = a(p+qh) = a \times p + a \times qh$$

$$\underbrace{(-3)}_{a} \times \left[\underbrace{(-5)}_{b} + \underbrace{2}_{c}\right] = \underbrace{(-3) \times (-5)} + \underbrace{(-3)(2)}$$

$$= (15) + (-6)$$

$$= 15 - 6$$

$$= \underline{9}$$

(5) Multiplicative Identity

$a \times 1 = a$ "Integer 1 is multiplicative identity."

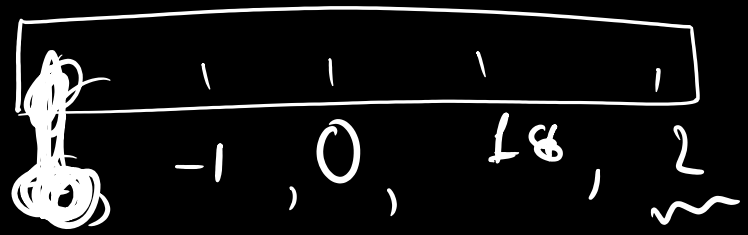
$a \times 0 / 7 / 1$ + Zero is additive identity

(1) Additive inverse

For any integer 'a', $-a$ is additive inverse of 'a'.

eg: 6, $\Rightarrow$ '-6'

-3, $\Rightarrow$ $-(-3) = 3$



$-2, -\frac{1}{2}, 0, \frac{1}{2}, 2$

7) $\underbrace{(-a_1) \times (-a_2) \times (-a_3) \times \cdots \times (-a_n)}_{n \text{ integers}}$

↳ if n is even, $(a_1) \times (a_2) \times (a_3) \times (a_4) \times \cdots \times (a_n)$

↳ If n is odd,

$$= -(a_1 \times a_2 \times a_3 \times \cdots \times a_n)$$

eg: $\underbrace{(-1) \times (-1) \times (-1) \times \cdots \times (-1)}_{98 \text{ times}} = 1$

$\underbrace{(-1)\,(-1)\,(-1) \times \cdots \times (-1)}_{151 \text{ times}} = -1$

Problems:

(I) $9 \times (-3) \times (-6)$

$= 162$

(II) $(-12) \times (-13) \times (-5)$

$= -780$

(III) $15625 \times (-2) + (-15625) \times 98$

$(-15625) \times 2 + (-15625) \times 98$

(a) (b) (a) (c)

(IV) $7298 \times 2 + 7298 \times 98$

$= 7298(2 + 98)$

$= 7298 \times (100)$

$= 729800$

$= -15625(2 + 98)$

$= -15625(100)$

$= -1562500$

$$a \times (b + c) = a \times b + a \times c$$

$$a \times b + a \times c = a(b + c)$$

$$2 \times (-7) = -14$$

$$2 \times (-7) = (-2) \times (7)$$

$$(-2) \times (7) = -14$$

(V)

$1569 \times 887 - 569 \times 887$

$\Rightarrow 887(1569 - 569)$

$\Rightarrow 887(1000)$

$\Rightarrow 887000$

(iv) $28945 \times 99 - (-28945)$

$\Rightarrow \underbrace{\overbrace{28945}^{a} \times \overbrace{99}^{b}}_{} + \overbrace{28945}^{a} \times \overbrace{1}^{c}$

$\Rightarrow 28945(99+1)$

$\Rightarrow 28945(100)$

$\Rightarrow 2894500$

(vii) $18946 \times 99 - (-18946) \times 1$

1894600

Find the value of:

(i) $19 \times \{7 + (-3)\}$

$= 19 \times \{7 - 3\}$

$= 19 \times 4$

$= 19 \times 4$

$= 76$

(ii) $(-23)\{(-5) + (+19)\}$

$\Rightarrow -322$

Division of Integers

$100 \div 5 = \frac{100}{5}$

(i) $50 \div 5 = 10$

(ii) $-50 \div 5 = -(50 \div 5) = -10$

(iii) $50 \div (-5) = -(50 \div 5) = -10$

(iv) $(-50) \div (-5) = (-)\times(-)(50 \div 5) = 10$

eg: $(-35) \div (-7) = \frac{(+35)}{(+7)} = \frac{35}{7} = 5$

(iii) $(-91) \div 13 = \frac{-91}{13} = \frac{(-1) \times 91}{13} = (-1)(7)$

$= -7$

(iv) $324 \div (-27) = \frac{324}{-27} = \frac{324}{(-1)(27)} = \frac{1}{(-1)} \times \left(\frac{324}{27}\right)$

$= -\left(\frac{324}{27}\right)$

$= -12$

$= (-1)\left(\frac{324}{27}\right)$

$= -12$

① $[32 + 2 \times 17 + (-6)] \div 15$

$= [32 + 34 - 6] \div 15$

$= (66 - 6) \div 15$

$= 60 \div 15$

$= 4$

⑪ $\{36 \div (-9)\} \div \{(-24) \div 6\}$

$= \frac{(-4)}{} \div \frac{(-4)}{}$

$= \frac{+4}{+4} = \boxed{1}$

(iii) $(-5) - (-48) \div (-16) + (-2) \times 6$

$= \boxed{-5 \ -3 \ -12}$

$= -8 \ -12$

$= -20$

$-5 - 3$

$-1 - 2$

(iv) $(-20) + (-8) \div (-2) \times 3$

$= (-20) + 4 \times 3$

$= (-20) + 12$

$= -8$

Practice Problems (Integers)

(I) Simplify: $48 - [18 - \{16 - (5 - \overline{4-1})\}]$

(II) Simplify: $[29 - (-2)\{6 - (7-3)\}] \div [3\{5 + (-3) \times (-2)\}]$

(III) Simplify: $63 - [(-3)\{-2 - \overline{8-3}\}] \div [3\{5 + (-2)(-1)\}]$

(IV) Simplify: $222 - [\frac{1}{3}\{42 + (56 - \overline{8+9})\} + 108]$

$63 - [21] \div [21]$

$63 \div [21) - [21]$

$21 - 3$

$3 - 21$

$63 - 1$

62

End of the chapter