Linear Equations with One Variable

one unknown
(x)

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① $-9-1(9x-6) = 3(4x+6)$

-1

$-9 \quad -9x \quad +6 = 12x + 18$

$-3 \quad -18 = 12x + 9x$

$\frac{-21}{21} = \frac{21x}{21}$

$-1 = x$ $\quad$ $x = -1$

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(11) $2 - 16t = 6(-3t + 2)$

$2 - 16t = -18t + 12$

$-16t + 18t = 12 - 2$

$2t = 10$

$t = 5$

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(111) $10(0.4 + 0.\overline{5}g) = 4g$

$4 + 5g = 4g$

$5g - 4g = -4$

$\boxed{g = -4}$

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Number of Solutions to equations

- One Solution
  - $x = 7$
- No Solution
  - $2 = 7$
- Infinitely many Solutions
  - $9 = 9$

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Determine the number of solutions for each of these equations:

$-7x + 2 = 2x + 2 - 9x$

$-2 \quad\quad -2$

$-7x = 2x - 9x$

$\frac{-7x}{-7x} = \frac{-7x}{-7x}$

$1 = 1$ ✓

$0 = 0$ ✓

Infinitely many solution

$-7x + 3 = 2x + 2 - 9x$

$-7x + 3 = -7x + 2$

$+7x \quad\quad +7x$

$3 = 2$

No solution

$-7x + 3 = 2x + 2$

$-7x - 2x = 2 - 3$

$-9x = -1$

$x = \frac{1}{9}$

One solution

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Solve for x:

$$8(3x+10) = 28x - 14 - 4x$$

$$24x + 80 = 24x - 14$$

$$24x - 24x = -14 - 80$$

$$0 = -94$$

No solution

Value of x is not defined

$$0 \cdot x = -94$$

$$x = \frac{-90}{0}$$

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How many solutions does the following equation have?

$$-6(x+7) = -4x - 2$$

$$-6x - 42 = -4x - 2$$

$$-6x + 4x = -2 + 42$$

$$-2x = 40$$

$$x = \frac{40}{-2} = -20$$

One solution

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How many solutions does the following equation have?

$$3(y+41) = 3y+123$$

$$3y+123 = 3y+123$$

$$3y-3y = 123-123$$

$$0 = 0$$

(Many Solution)

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How many solutions does the following equation have?

$$14(z+3) = 14z + 21$$

$$14z + 42 = 14z + 21$$

$$42 = 21$$

No solution

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Creating an equation with no solution.

- Fill in the box to form a linear equation with no solutions:

$$-11x + 4 = \boxed{-11}x + \boxed{5}$$

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Creating a linear equation with infinitely many solutions.

Fill in the box to form a linear equation with infinitely many solutions.

$$4(x-2)+x = 5x+\boxed{-8}$$

$4x - 8 + x$

$5x - 8$

- LHS = RHS ⇒ Many solutions
- x = constant ⇒ one solution
- LHS ≠ RHS ⇒ No solution

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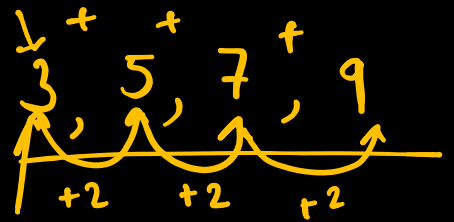
Word Problems :

Q. The sum of 4 consecutive odd integers is 136. What are the 4 integers?



4 Consecutive odd integers:

- x ⟶ Smallest odd integer (31)
- $x+2$ ⇒ (33)
- $x+4$ ⇒ (35)
- $x+6$ ⇒ (37)

$$x + x+2 + x+4 + x+6 = 136$$

$$4x + 12 = 136$$

$$4x = 136 - 12$$

$$4x = 124$$

$$x = \frac{124}{4}$$

$$x = 31$$

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Q. The sum of three consecutive odd integers is 231. What is the largest integer?

x is smallest odd integer

$x + (x+2) + (x+4) = 231$

$3x + 6 = 231$

$3x = 231 - 6$

$3x = 225$

$x = \frac{225}{3} = 75$

Largest integer $= 75 + 4 = 79$.

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Linear Equations and Functions

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Graph of Proportional Relationships.

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Proportional Relationship :→

A relationship between two variables where the ratio between them is always the same.
(constant)

⇒ When one variable changes, the othe changes with a constant rate

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Constant of Proportionality (k)

∴ The ratio of the y-value to the x-value is k/a constant of proportionality (k)

[Proportionality Constant (k)]

x	y	$\frac{y}{x}$
1	6	$\frac{y}{x} = \frac{6}{1}$
2	12	$\frac{y}{x} = \frac{6}{1}$
3	18	$\frac{y}{x} = \frac{6}{1}$

$$k = \frac{y}{x}$$

$$\frac{y}{x} = k$$

$$y = k \cdot x$$

$y = k \cdot 0$

$y = 0$

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Graphing proportional relationship using unit rate.

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Example 1: Graph the line that represents a proportional relationship between y and x with unit rate 0.4; i.e., a change of 1 unit in x corresponds to a change in 0.4 units in y. What would be the equation of this line?

x	y
0	0
1	0.4
2	0.8
3	1.2
4	1.6
5	2

$(0,0)$ $(1, 0.4)$ $(5,2)$

$$y = kx$$

$$k = \frac{y}{x}$$

$k = 0.4$

$k = \frac{2}{5}$

$$y = kx$$

$$y = \frac{2}{5}x \Rightarrow 5$$

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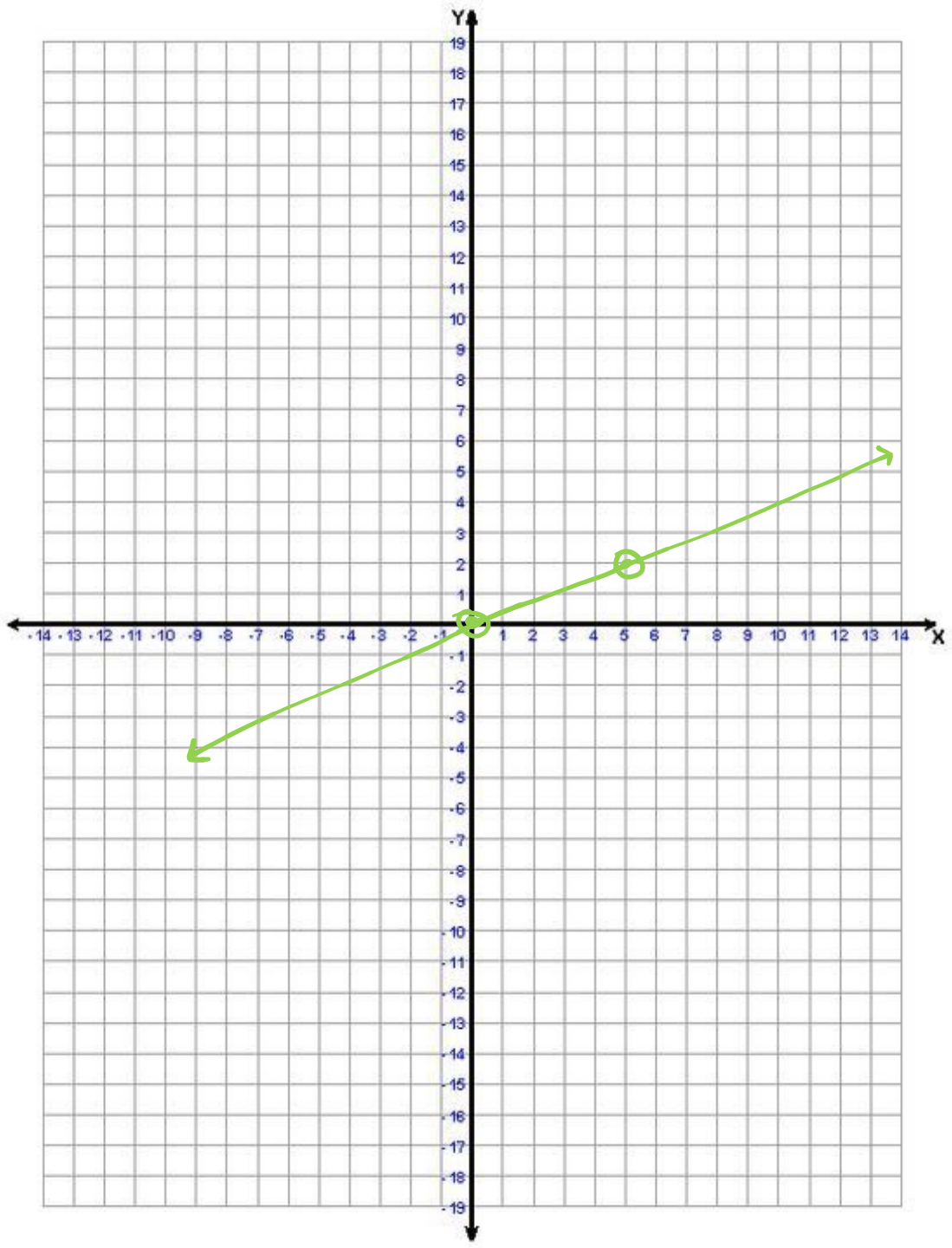
$y = kx$

$k = \frac{2}{5}$

$y = \frac{2}{5}x$

$5y = 2x$

$5y - 2x = 0$



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Graphing proportional relationship from a table

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Ex: Graph the proportional relationship shown in the table below:

x	0	3	6	9	12
y	0	0.5	1.0	1.5	2.0

What is the slope of the line that represents this relationship?

(0, 0)

(6, 1)

x: 0 → 3

y: 0 → 0.5

$\frac{0.5}{3}$

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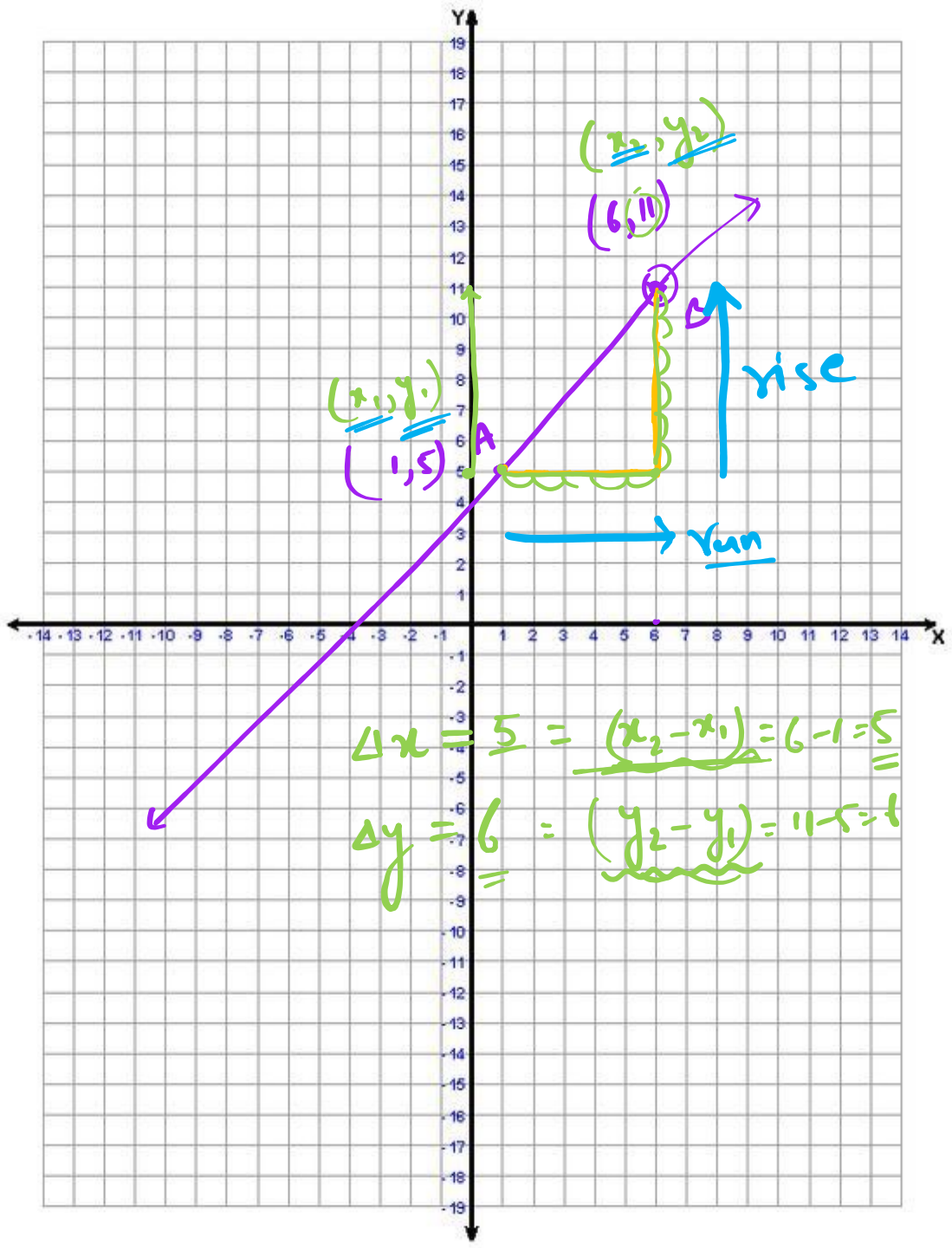
$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

Δy = change in y $= 1$

Δx = change in x $= 6$

$$\text{Slope} = \frac{1}{6}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



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Graphing Proportional Relationship from an equation.

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Ex: Graph $y = 2.5x$. Find the slope of the line represented by this equation.

$y = 2.5x$

x	y	
0	0	←
1	2.5	
2	5	←

(0, 0)

(2, 5)

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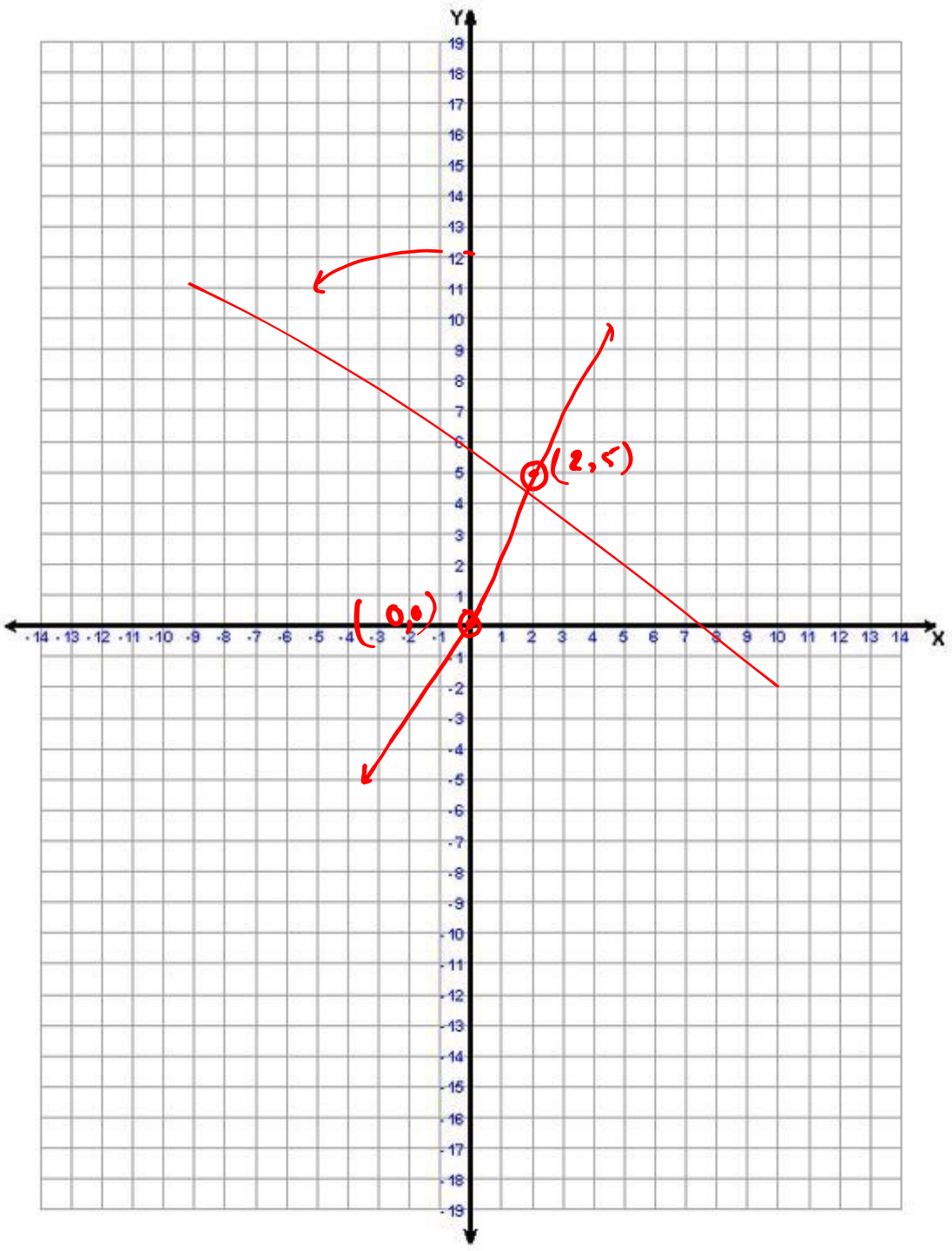
$y = mx + c$

$y = 2.5x$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5}{2}$$

$$= 2.5$$



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Practice: A proportional relationship is shown in the table below:

x	0	3	6	9	12	15
y	0	0.6	1.2	1.8	2.4	3

what is the slope of the line that represents this relationship? Graph the line that represents this relationship.

$$\frac{\Delta y}{\Delta x} = \frac{2.4 - 1.2}{12 - 6} = \frac{1.2}{6} = \frac{12}{60} = \frac{1}{5}$$

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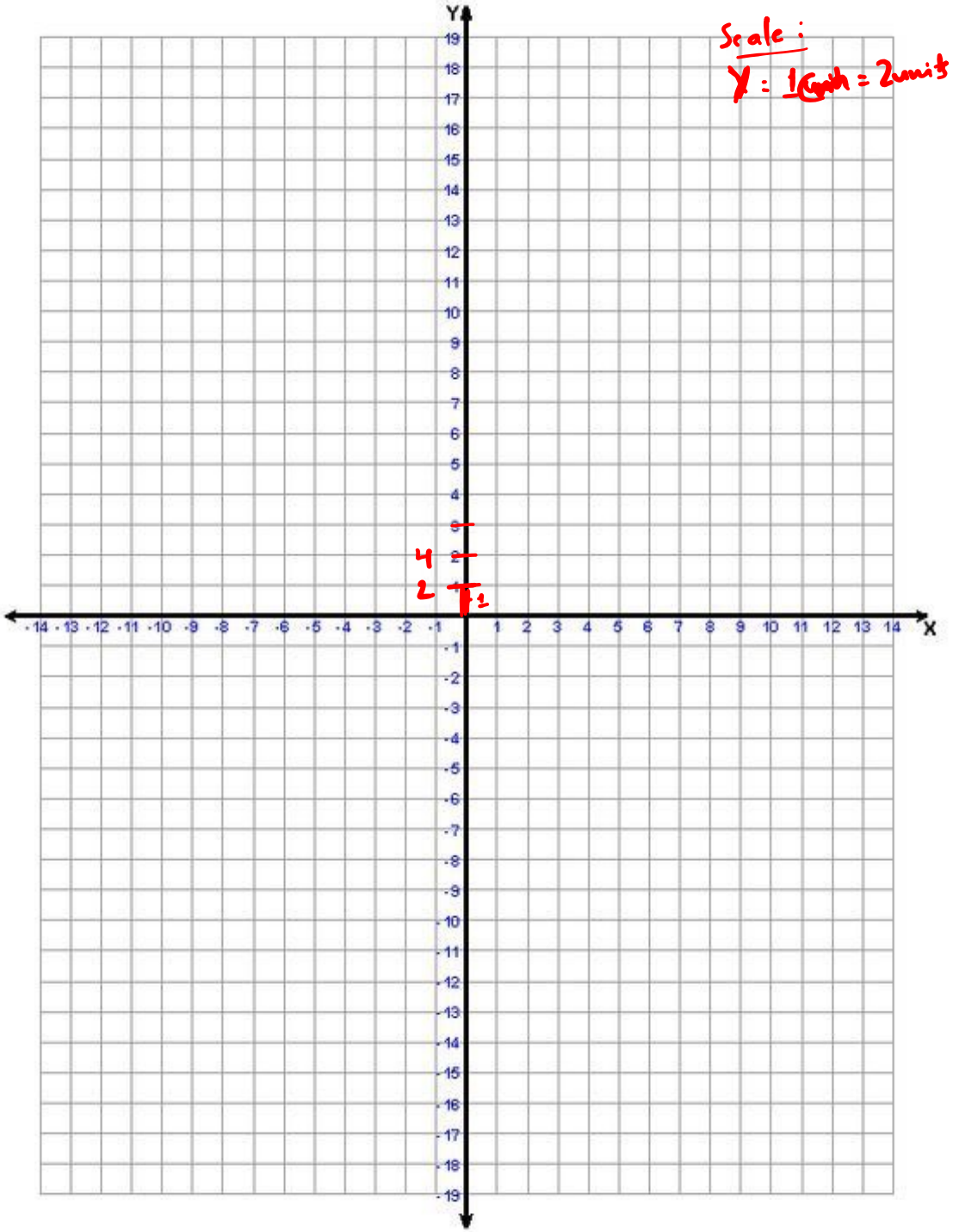

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Solutions to Linear Equations

$ax + by = c$ ⇒ Linear equation in two variables.

Unknowns: x and y
(Variables)

a = Coefficient of x

b = Coefficient of y

c = constant value. ∈ Real number

On solving this kind of equation we get a line on XY-plane.

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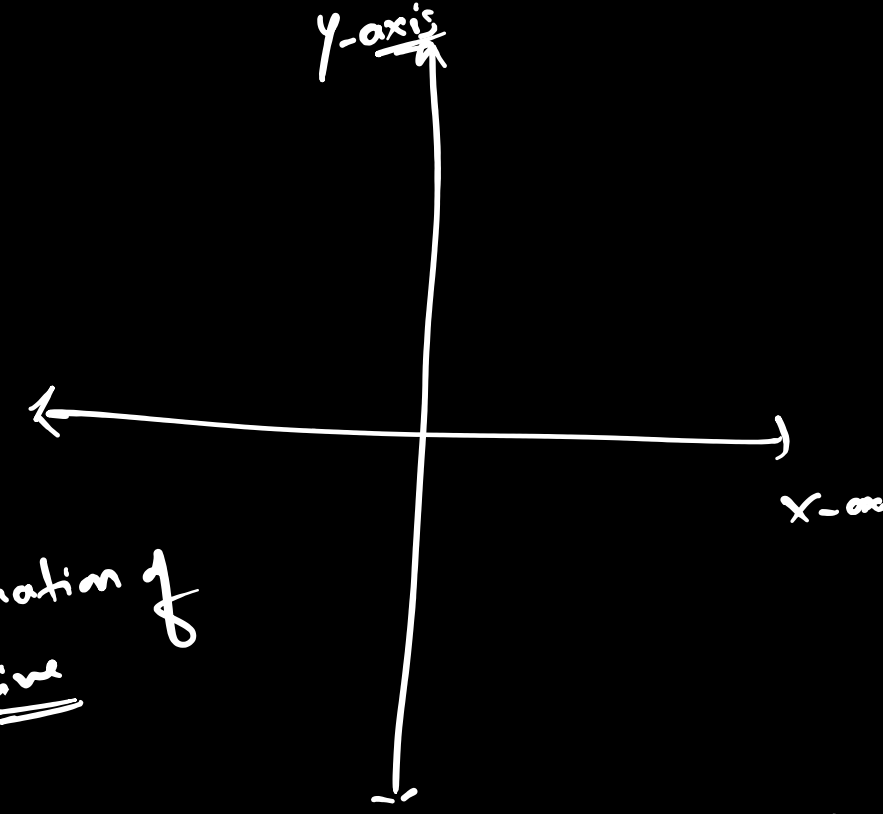
Rene Descartes

De-cart

(15th to 16--)

Geometry



$y = 2x + 1$ ⇒ Equation of Line

x	y
0	1
1	3
2	5

(Cartesian coordinate System)

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$-3x - y = 6$ ⟹ a line

$(-4, 4)$

$x \quad y$

$(-3, 3)$ Solution

$(-4, 4)$

LHS: $-3(-4) - 4$

$= 12 - 4$

$= 8 \neq$ RHS

$(-3, 3)$

LHS: $-3(-3) - 3$

$= 9 - 3$

$= 6 =$ RHS

$-3x - y = 6$

x	y
0	-6
1	-9
-1	-3
-3	3

$(-3, 3)$

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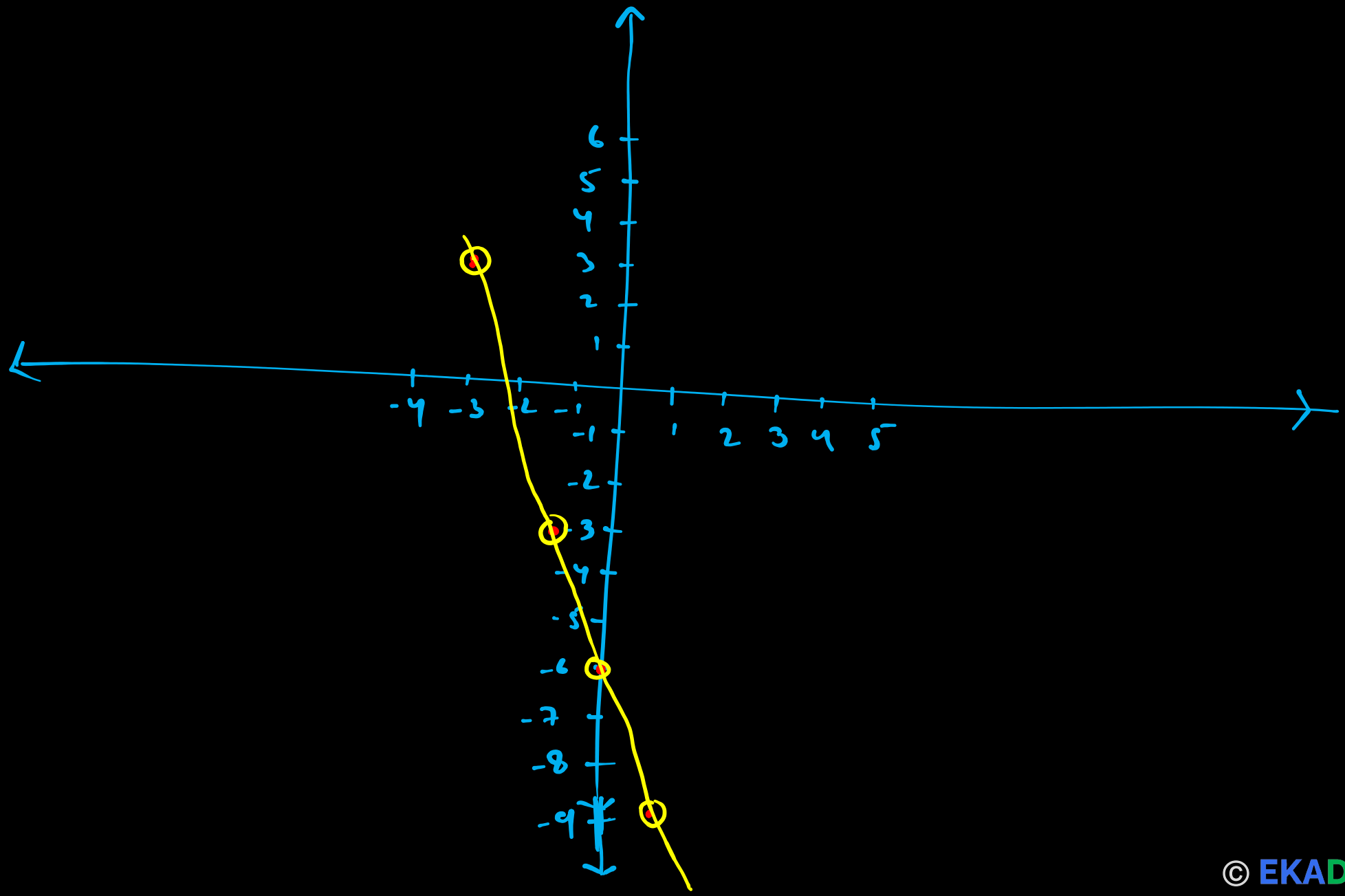

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$4x - 1 = 3y + 5$

Solve for x and y and draw the line that you obtained.

$y = 4x - 2$

$4x - 1 = 3y + 5$ $\Rightarrow$ $-1 = 3y + 5$

$-3y = 6$

$y = \frac{6}{-3} = -2$

$\Rightarrow \quad -3y = 5 + 1 - 4x$

$\Rightarrow \quad -3y = 6 - 4x$

$\Rightarrow \quad y = \frac{6 - 4x}{-3}$ $\quad \times(-1)$ $\quad \times(-1)$

$= \frac{-(6 - 4x)}{3}$

$y = \frac{4x - 6}{3}$

x	y
0	-2
2	$\frac{2}{3}$
3	2

$(3, 2)$

$(2, \frac{2}{3})$

Equation of line

$y = 3x + 1$

$y = 3x$

$y = 2 - 4x$

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$2x + 3y = 5x - y$

$(0, 0)$

$2x + 3y = 5x - y$

$y = 0$

$2x + 3(0) = 5x - 0$

$2x = 5x$

$2x - 5x = 0$

$-3x = 0$

$x = 0$

$-3x + 7y = 5x + 2y$

$(-5, -8)$ Find the missing value.

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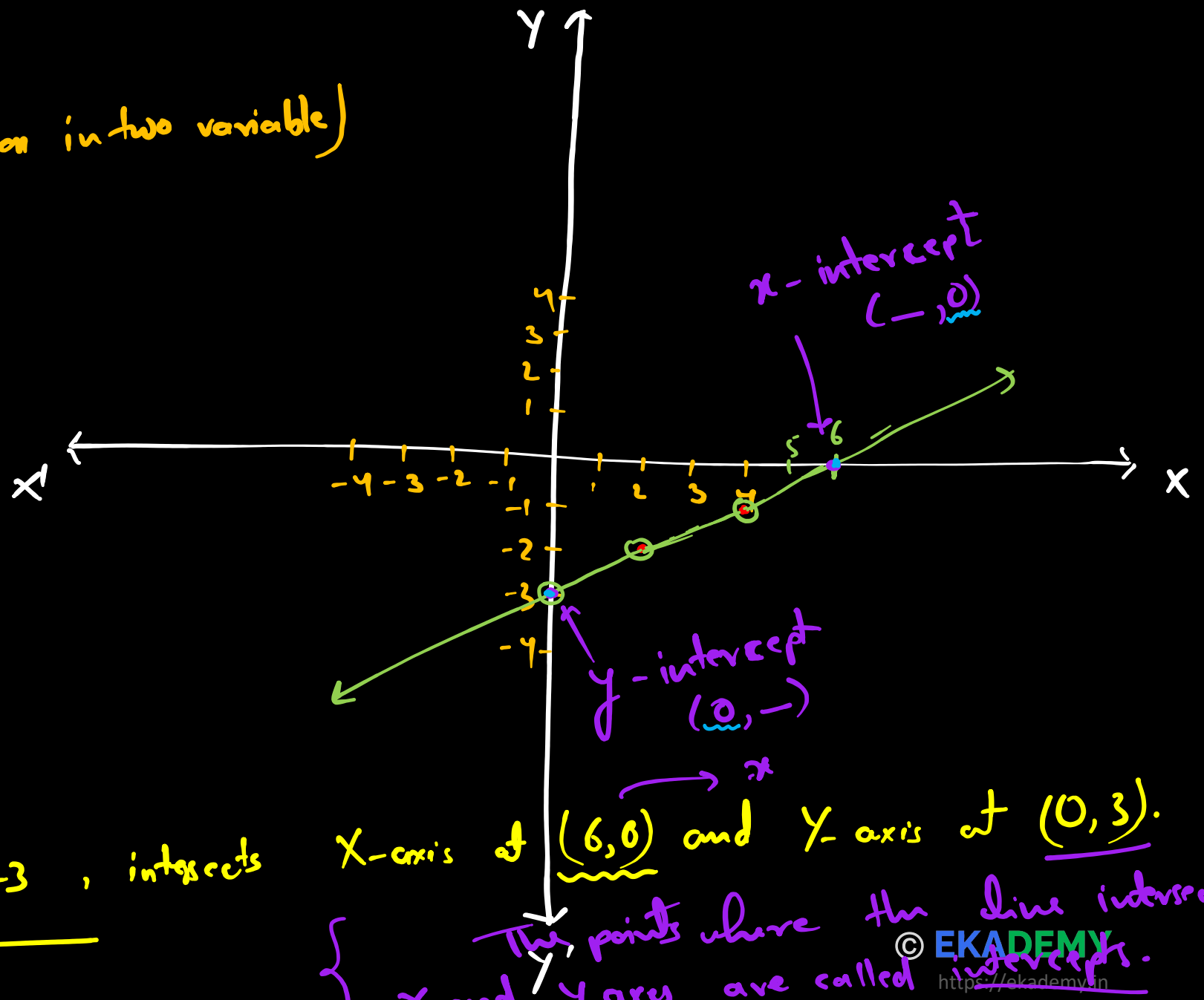
Intercepts

Equation of a line (linear equation in two variable)

$y = \frac{1}{2}x - 3$

Lets draw this line on graph paper.

x	y
0	-3
2	-2
4	-1
6	0



The Line represented by $y = \frac{1}{2}x - 3$, intersects X-axis at (6,0) and Y-axis at (0,3).

These points where the line intersects x and y axes are called intercepts.

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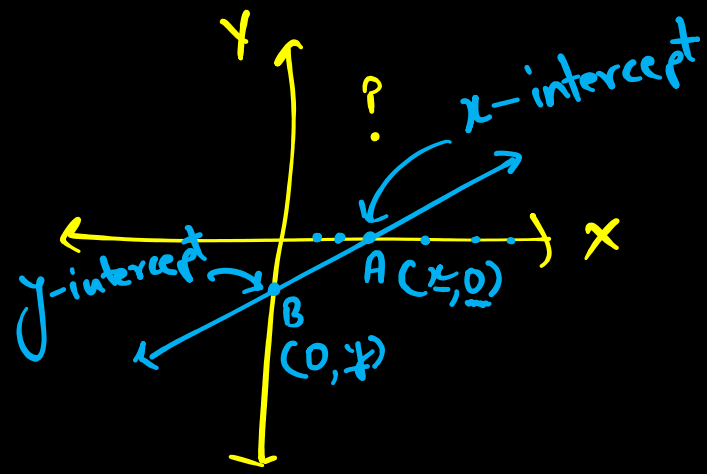
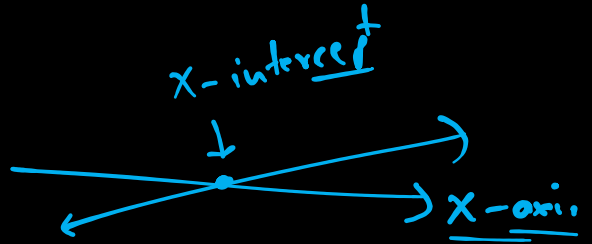
Intercept "Defined w.r.t. a Line"



X-intercept :⇒

It is the point where a line crosses the X-axis.
(intersects)

* For x intercept, $y = 0$.



Y-intercept

It is the point where a line crosses the Y-axis.

For Y-intercept, $x = 0$

We can find x and Y intercept without plotting graph.

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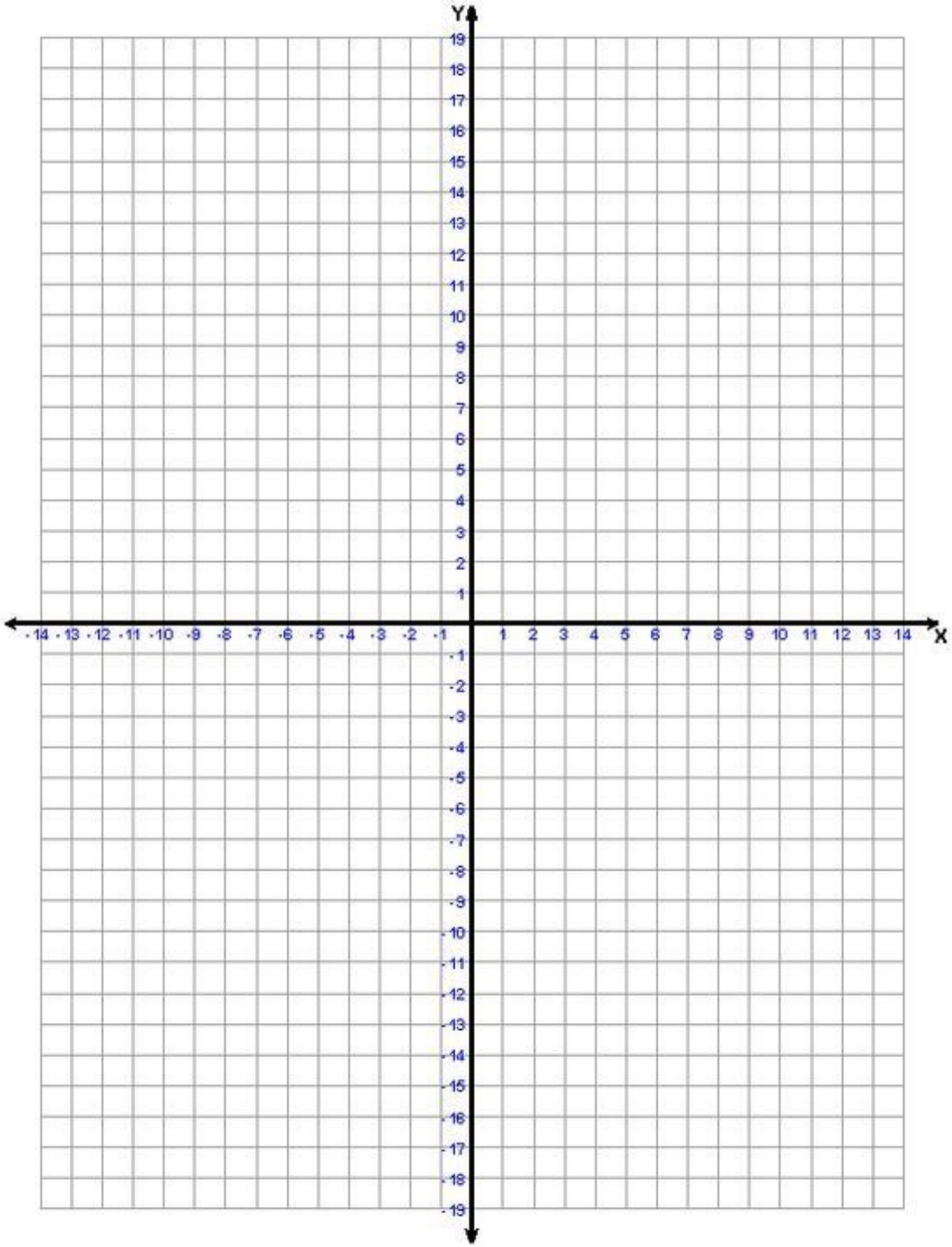

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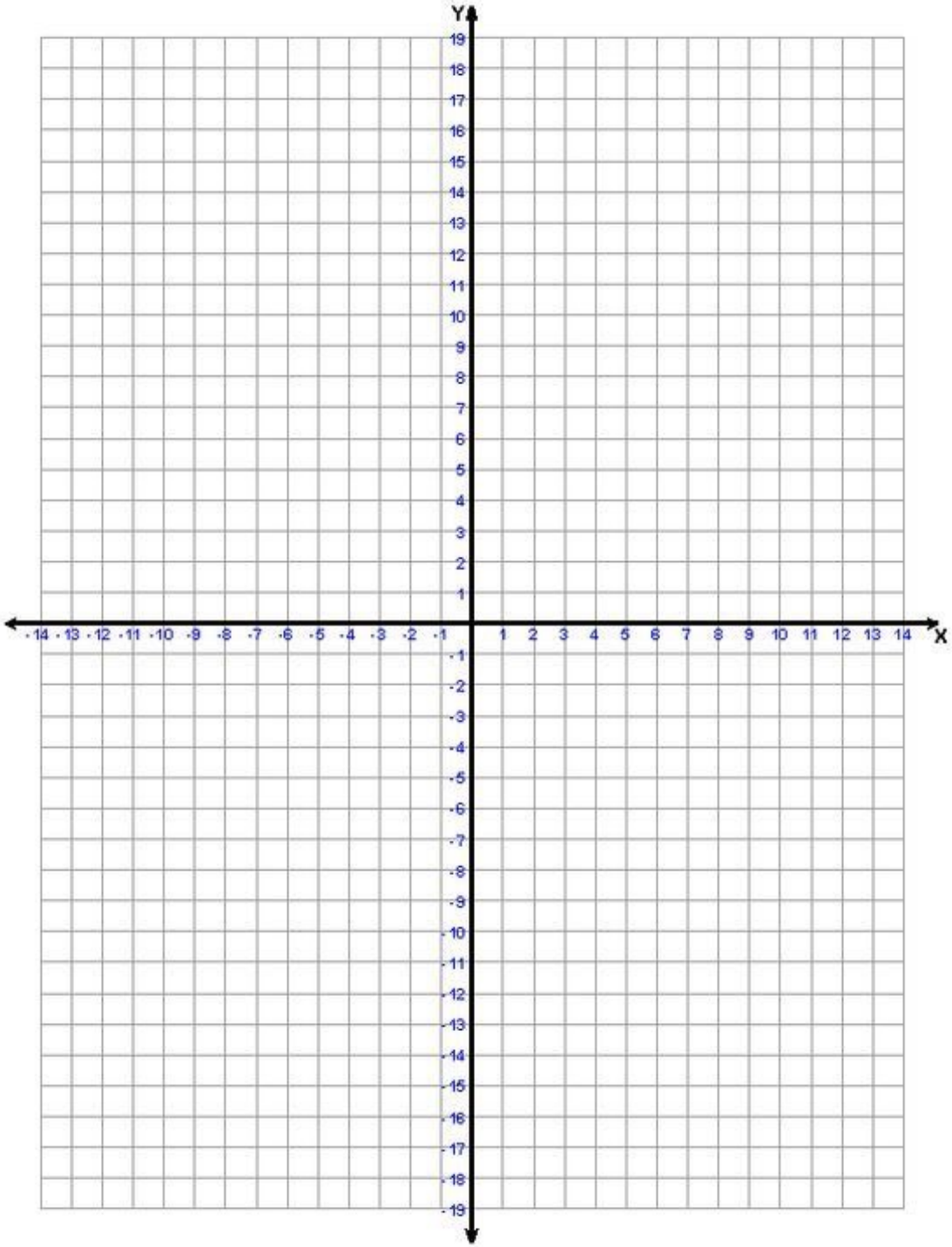

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Ex. $5x + 6y = 30$, find intercepts without plotting the graph.

Sol: For x intercept $y = 0$.

$$5x + 6(0) = 30$$

$$5x = 30$$

$$\boxed{x = 6}$$

x-intercept : $(6, 0)$

For y-intercept, $x = 0$

$$5(0) + 6y = 30$$

$$6y = 30$$

$$\boxed{y = 5}$$

y-intercept : $(0, 5)$

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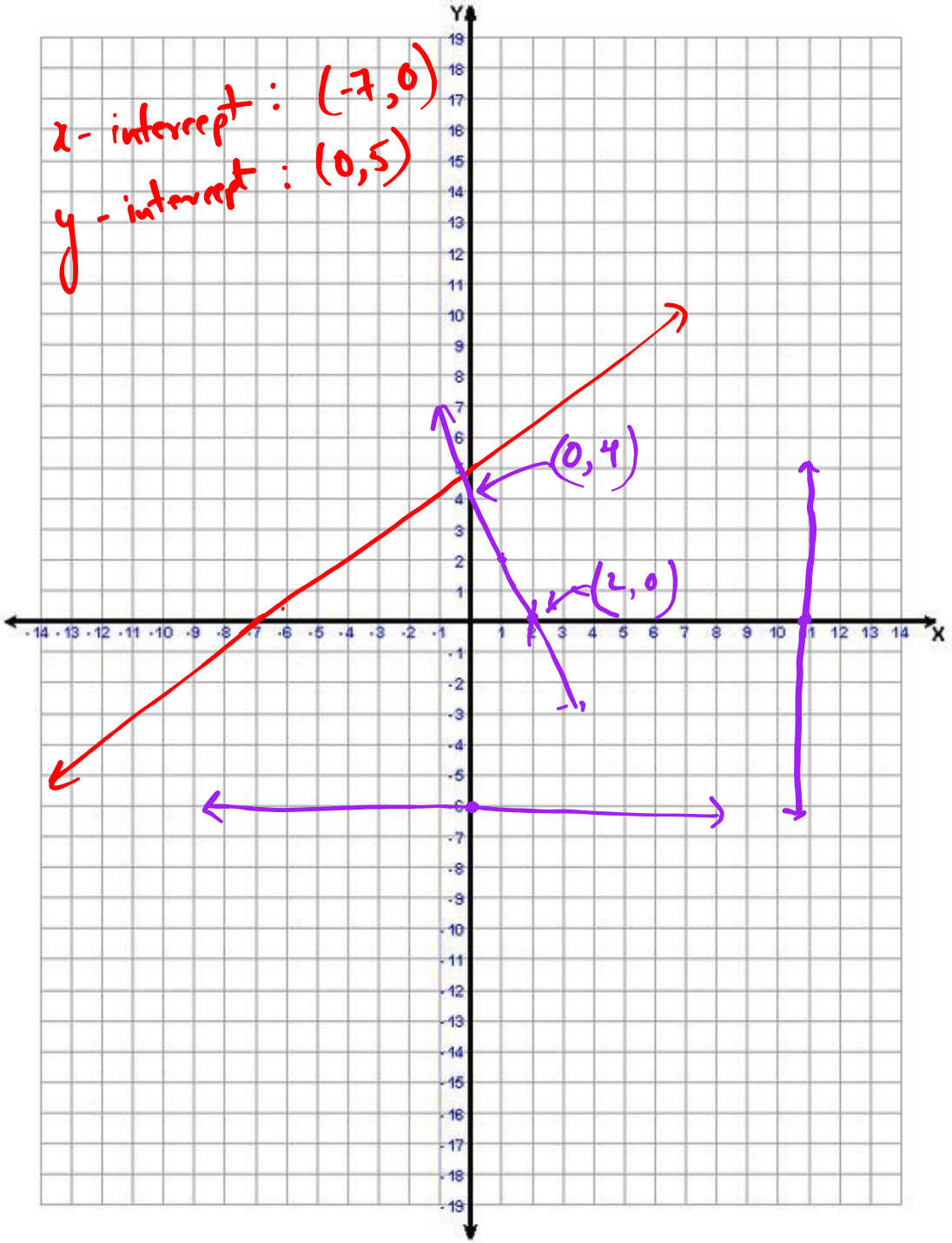

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$-5x + 4y = 20$ ✓ Find the intercepts and plot the graph.

$4x - 3y = 17$

Equation of a line. or a linear equation in two variable.

$y = mx + c$

$-3y = -4x + 17$

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Intercepts from a table

x	y
-2	8
1	2
2	0
4	-4

x-intercept

Use the values given in the table to determine x- & y- intercepts.

x	y
-2	8
-1	6
0	4
1	2
2	0
4	-4

+1, -2; +1, -2; +1, -2; +3, -6

at $x = 0$

$y = 4$

$(0, 4)$ y-intercept

x- intercept $(2, 0)$.

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x	y
1	-9
3	-6
5	-3
7	0

Determine x and y intercepts.

y-intercept: (0, -10.5) ✓

x-intercept: (7, 0) ✓

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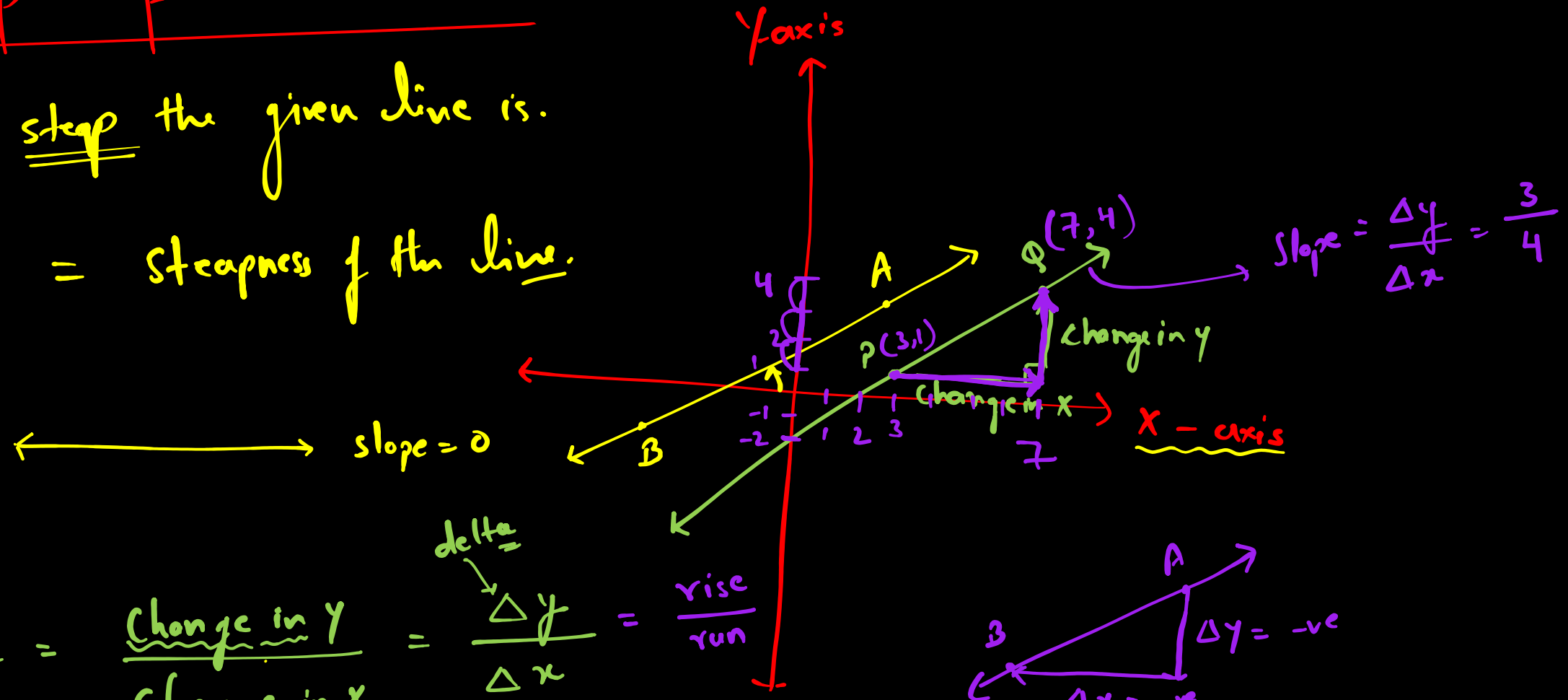
Slope of a line

How steep the given line is.

Slope = Steepness of the line.



$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{3}{4}$

slope = 0

$\text{Slope} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$

(delta)

$\text{Slope} = \frac{-ve}{-ve} = +ve$

=

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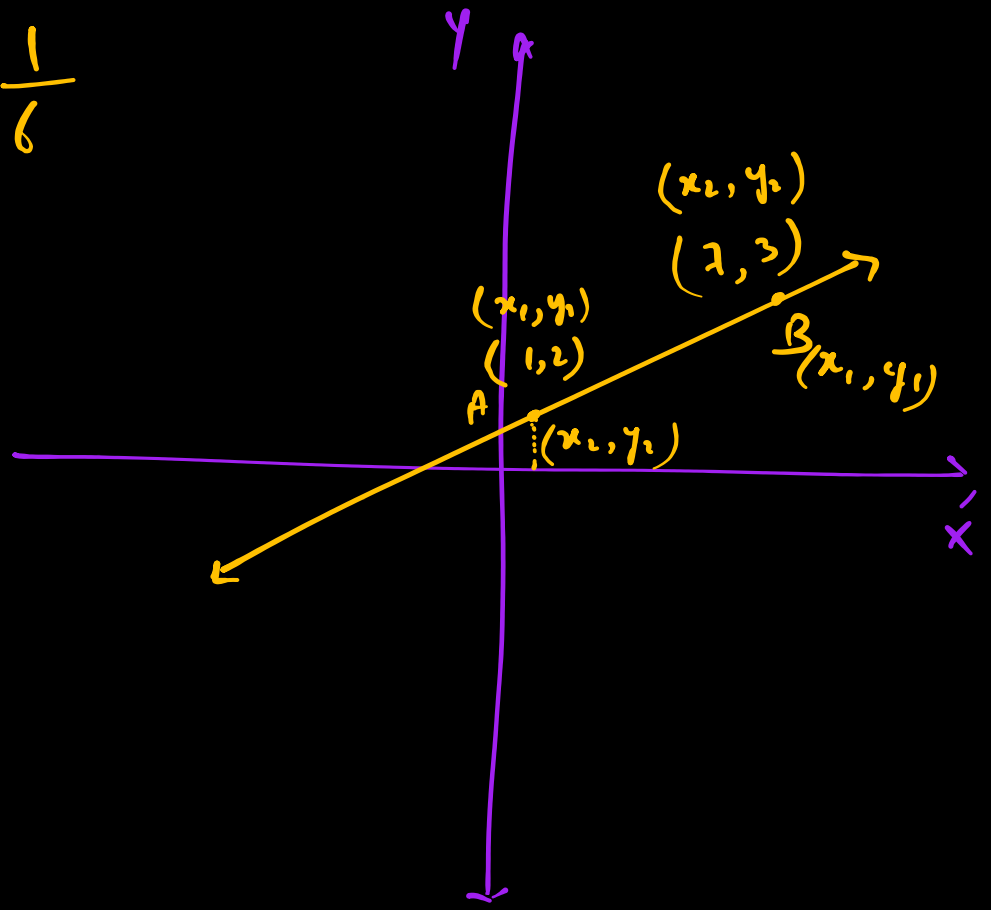
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{7-1} = \frac{1}{6}$$

$$= \frac{2-3}{1-7} = \frac{-1}{-6} = \frac{1}{6}$$



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Positive and Negative Slope

Δx = +ve
&
Δy = +ve

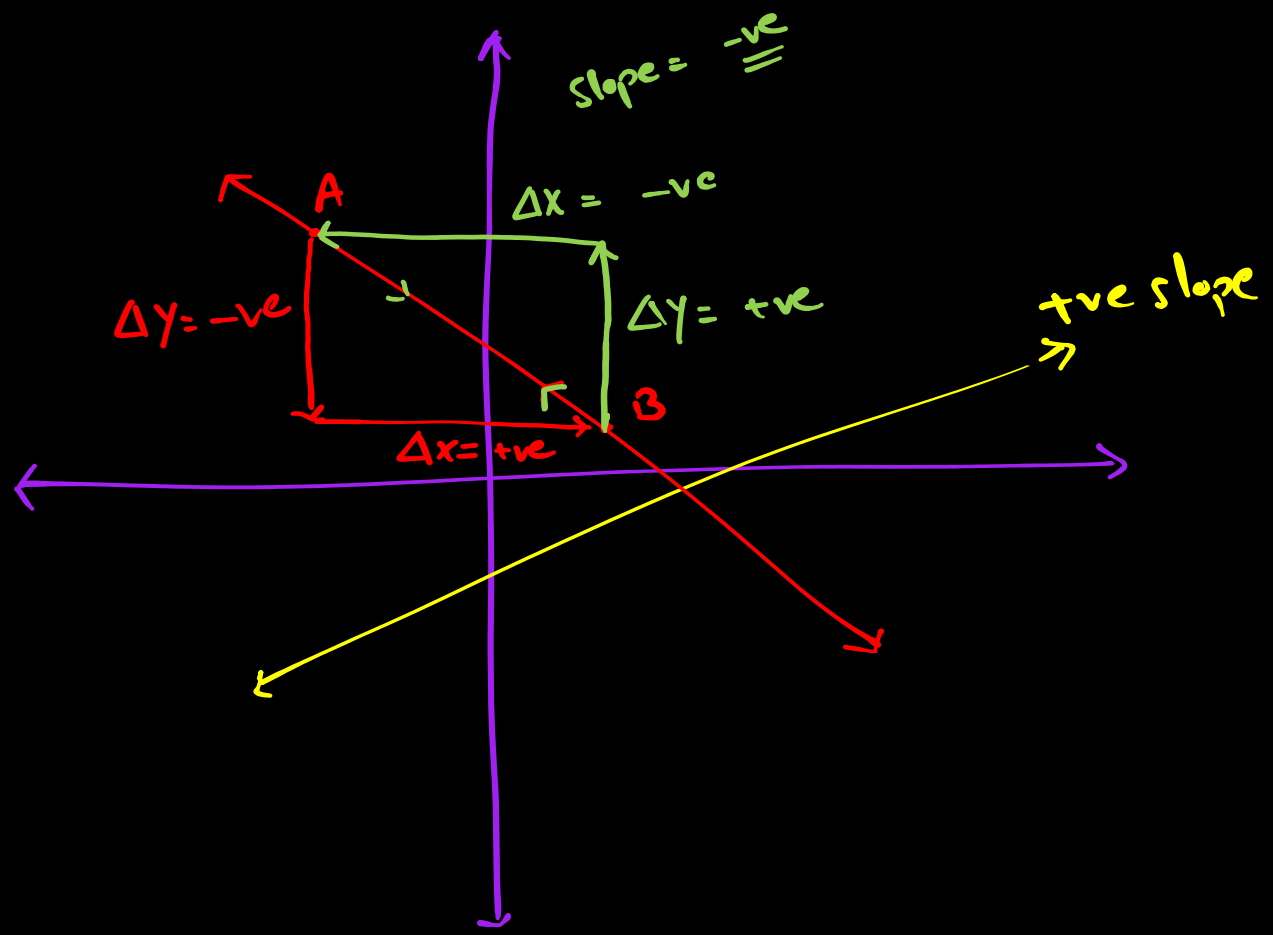
or

Δx = -ve
and
Δy = -ve

Δx = +ve
and
Δy = -ve

or

Δx = -ve
and
Δy = +ve



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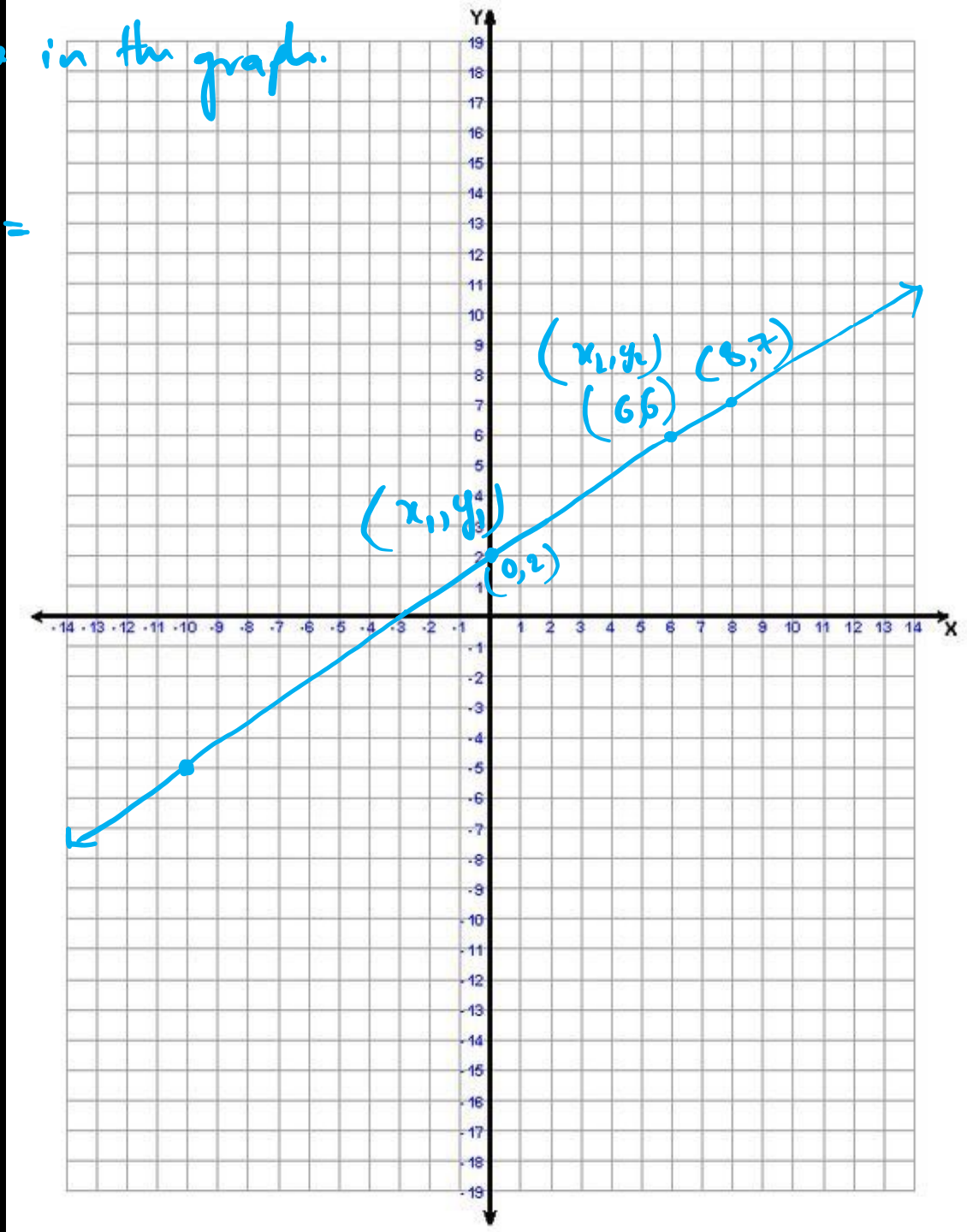
Find the slope of the line in the graph.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

$$= \frac{6-2}{6-0}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$



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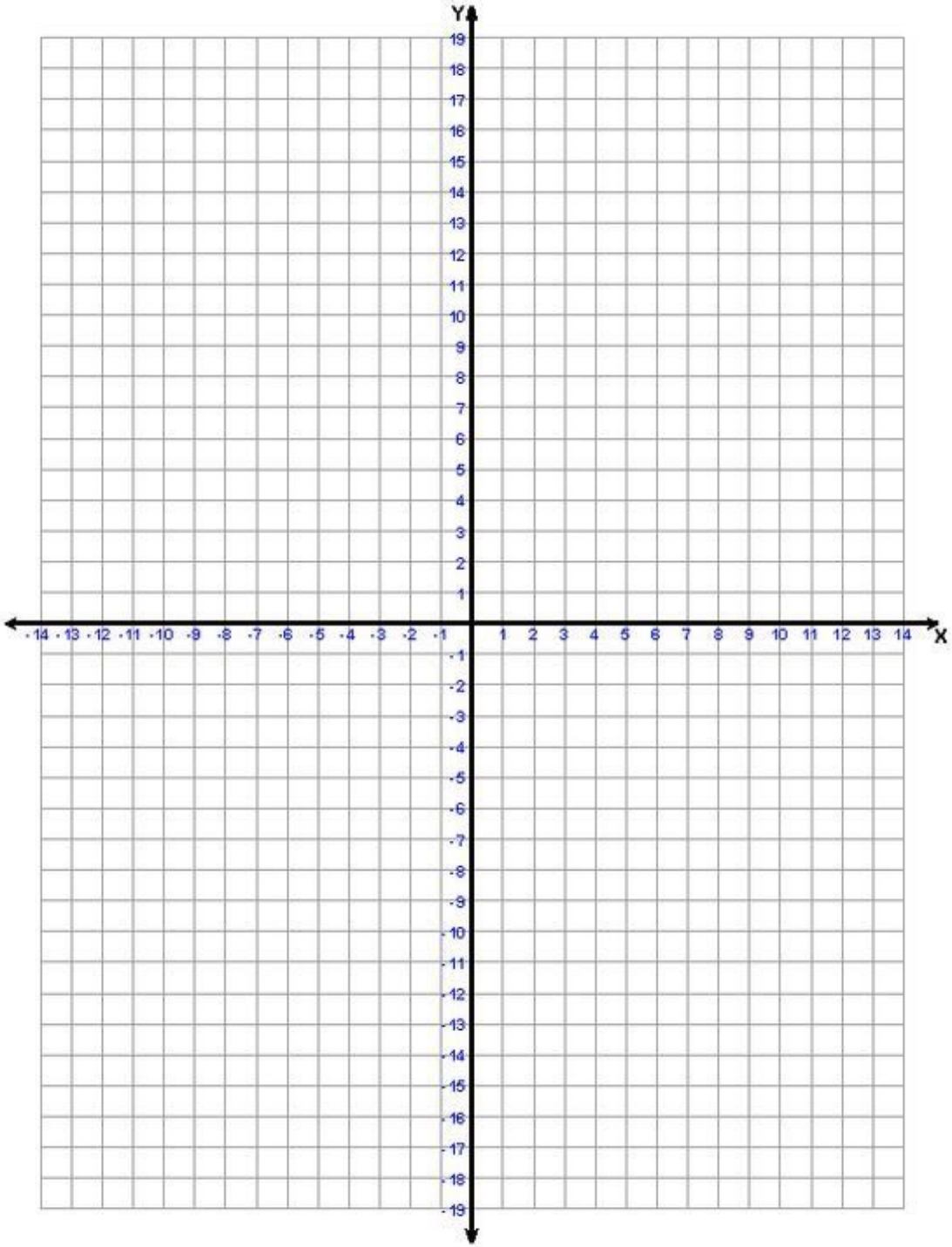

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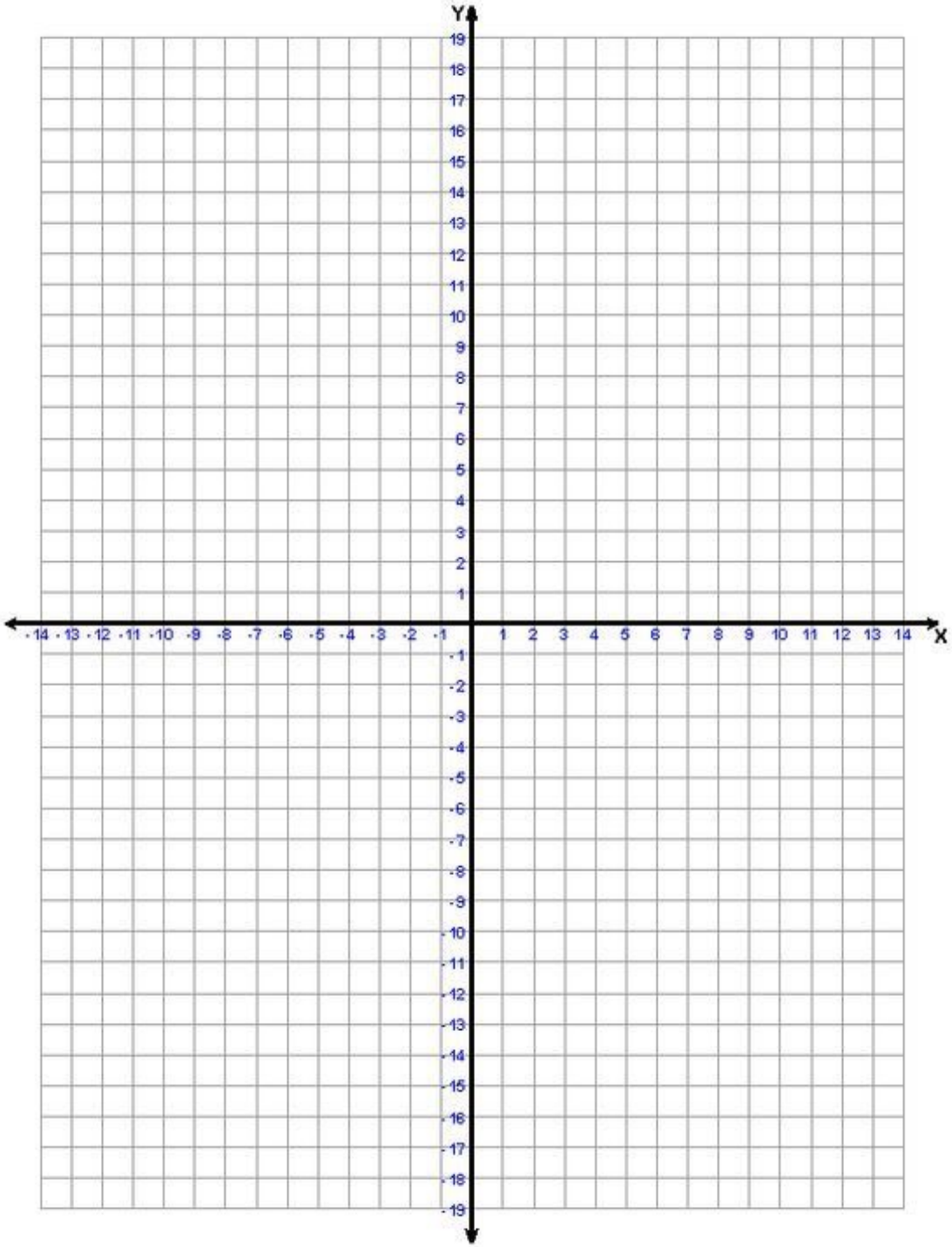

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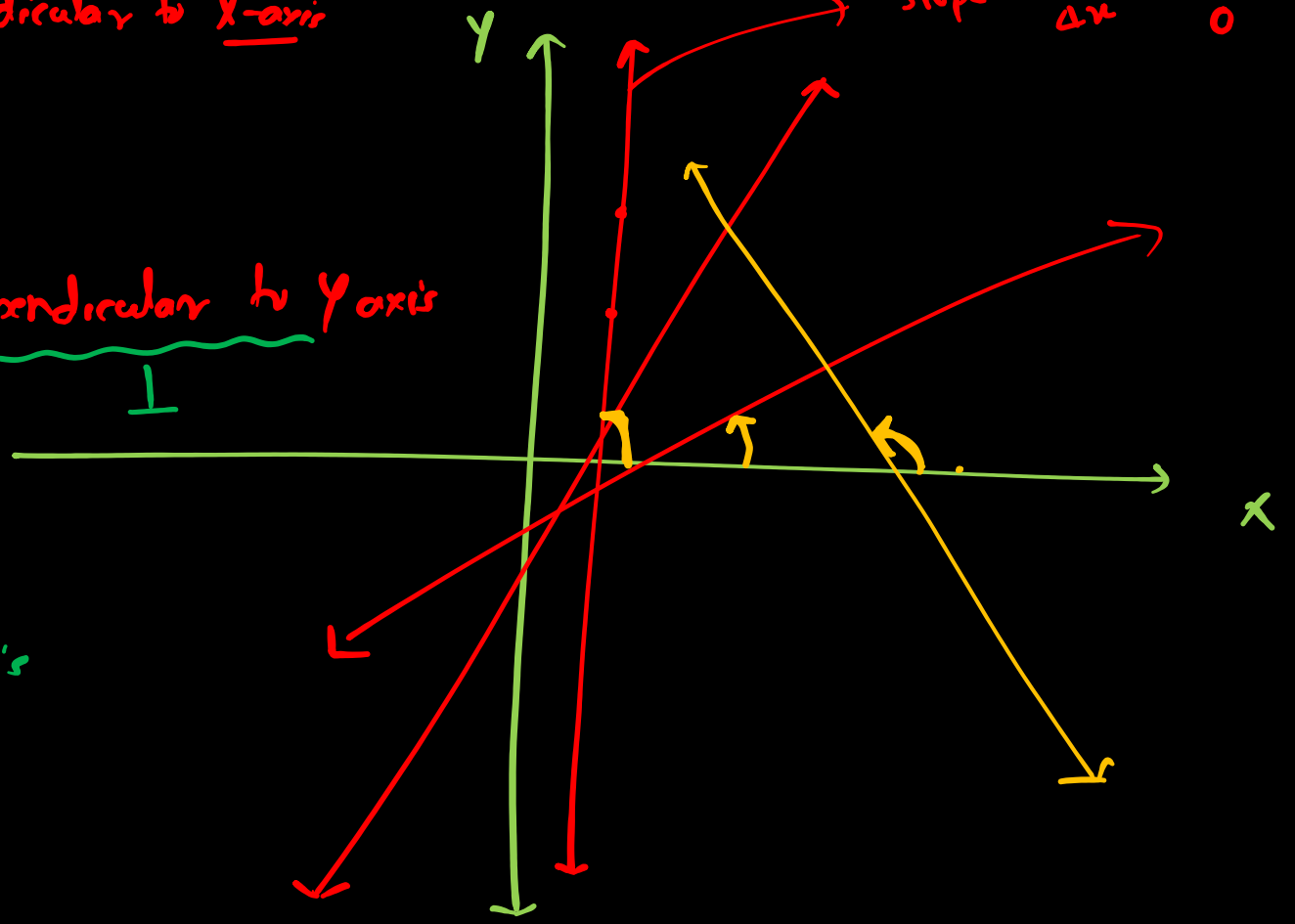
- A line parallel to Y-axis or perpendicular to X-axis has undefined slope.

- A line parallel to X axis or perpendicular to Y axis has zero slope.

- A line ∥ to X axis and ⊥ to Y axis



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Linear equation in Slope intercept form

$5x + 2y = 3$

Can be written in various ways

Different forms

Different ways in which the given linear eq. can be written:

(i) $5x = 3 - 2y$

or,

(ii) $2y = 3 - 5x$

or,

(iii) $2y = -5x + 3$

or,

(iv) $\boxed{y = \frac{-5}{2}x + \frac{3}{2}}$

or,

(v) $5x + 2y - 3 = 0$

$y = \left(\frac{-5}{2}x\right) + \frac{3}{2}$

$y = \frac{-5x + 3}{2}$

$y = \frac{-5x}{2} + \frac{3}{2}$

y-intercept

slope

y-intercept

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$$y = \frac{-5}{2}x + \frac{3}{2}$$

Slope-intercept form

in general

$$y = mx + c$$

m: slope

c: y-intercept.

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$$y = mx + c$$

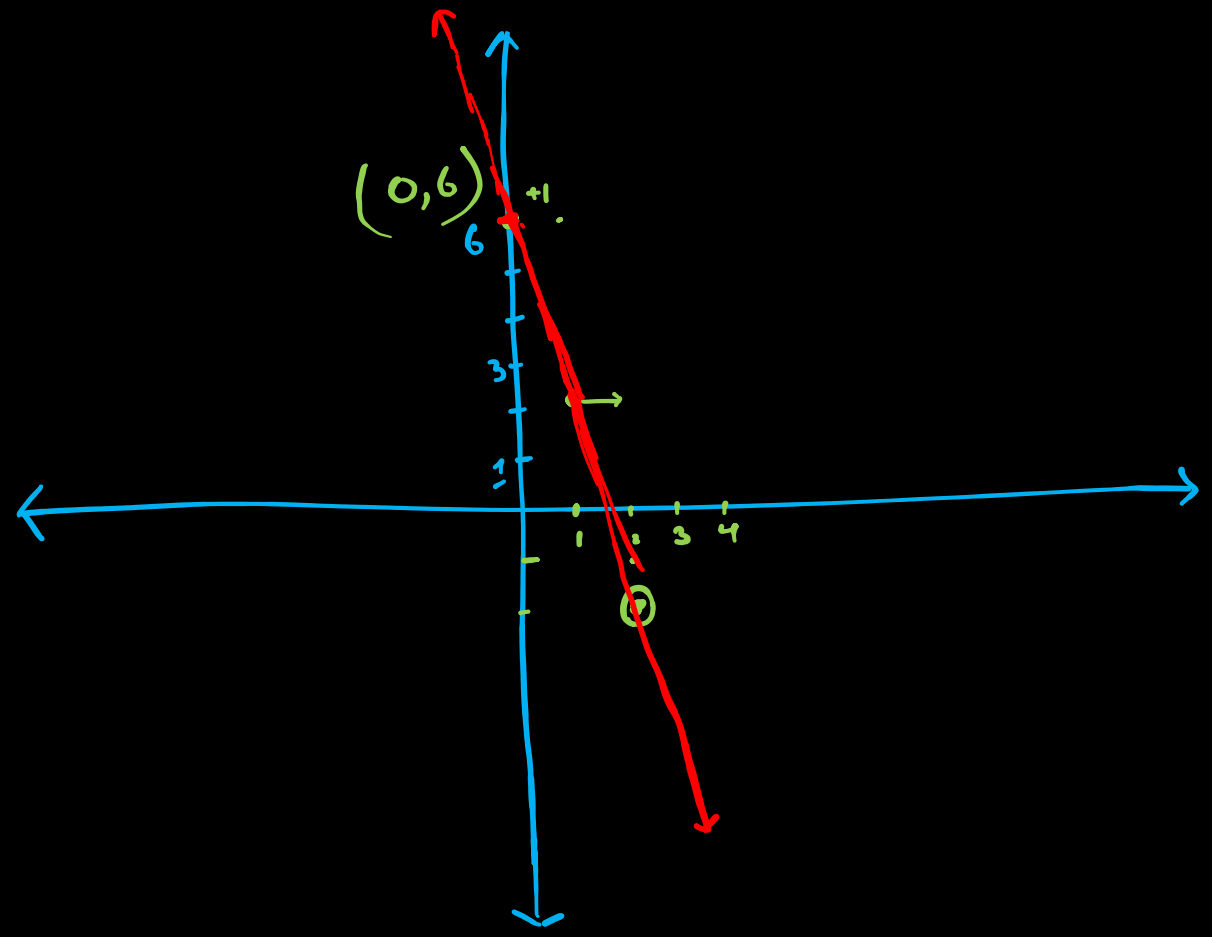
$$4x + 2y = y + 6$$

$$4x + 2y = y + 6$$

$$2y - y = -4x + 6$$

$$y = -4x + 6$$

$$y = mx + c \Rightarrow$$



$$\frac{\Delta y}{\Delta x} = \frac{-4}{1}$$

$$m = -4$$

$$c = +6$$

$\Rightarrow$ Y-intercept

& one of the point = $(0, c)$

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$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{-4}{1}$$

Steepness

[inclination of a line w.r.t. X-axis]

$$\frac{\Delta y}{\Delta x} = \frac{-4}{1}$$

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$y = 5x - 7$

$y = mx + b$

$m = 5$

$5x - 2y + 6 = 3x + y$ | slope! | y-intercept!

$5x - 3x + 6 = 3y$

$3y = 2x + 6$

$y = \frac{2x+7}{3} \Rightarrow y = \frac{2}{3}x + \frac{7}{3}$

$y = mx + b$

$m = \frac{2}{3}$

$b = \frac{7}{3}$

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$y = -3x - 2$

graph it using slope and y-intercept.

$m = -3$

$b = -2$

Original coordinates

$(0, -2)$

Next coordinates

$(0+1, -2+(-3))$

$(1, -5)$

$\frac{\Delta y}{\Delta x} = \frac{-3}{1}$

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$y = 3x - 1$

$\boxed{m = 3}$

$\frac{\Delta y}{\Delta x} = \frac{3}{1}$

A $(0, -1)$

B $(0+1, -1+3)$

$(1, 2)$

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$y = \frac{2}{3}x + 1$ plot.

$m = \frac{2}{3} = \frac{\Delta y}{\Delta x}$

y intercept $(0, 1)$ A

$(0+3, 1+2)$ B

$(3, 3)$ B

$(6, 5)$ C

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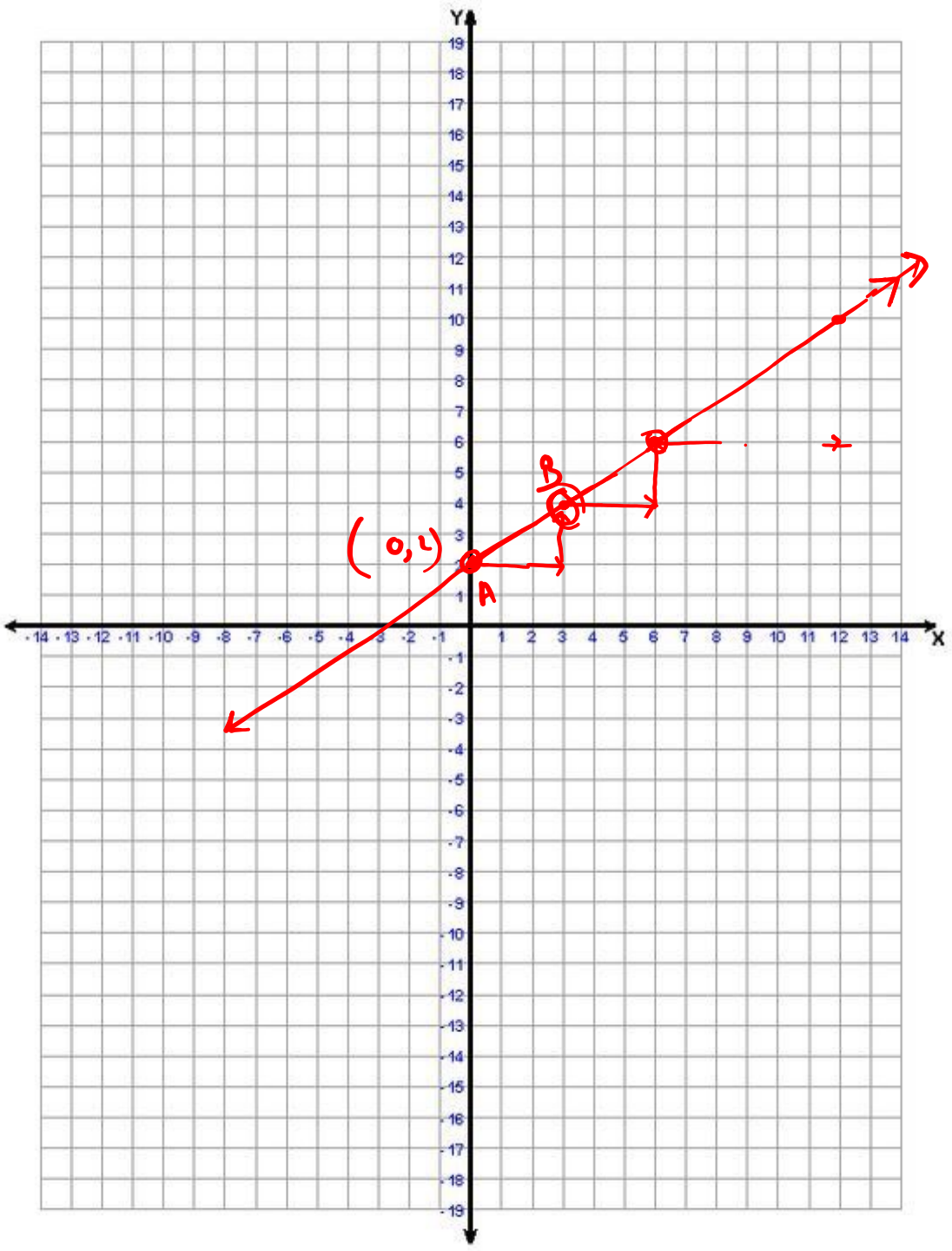
y-intercept = 2

$$m = \frac{2}{3} = \frac{\Delta y}{\Delta x}$$



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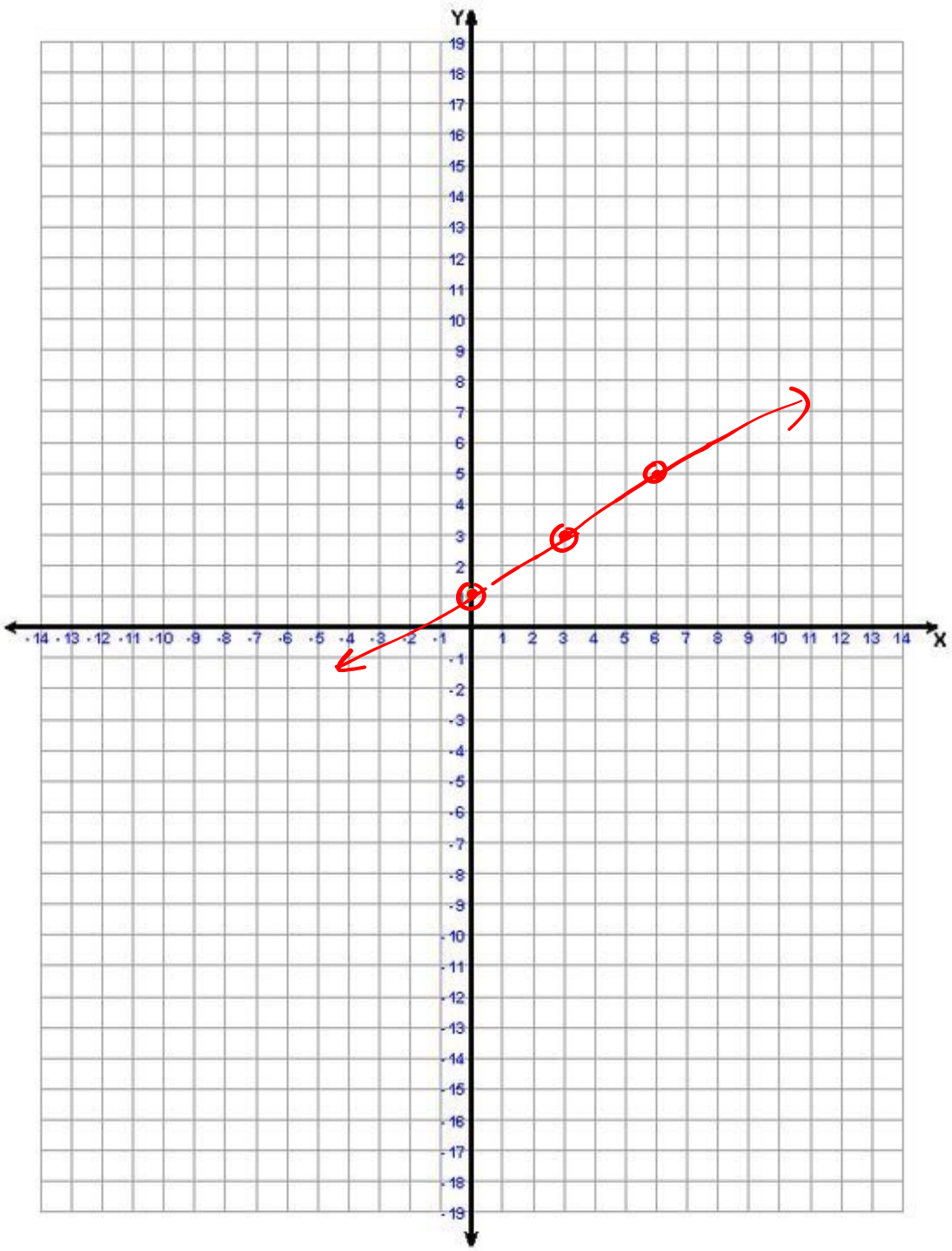
(0,1)

(3,3)

(6,5)



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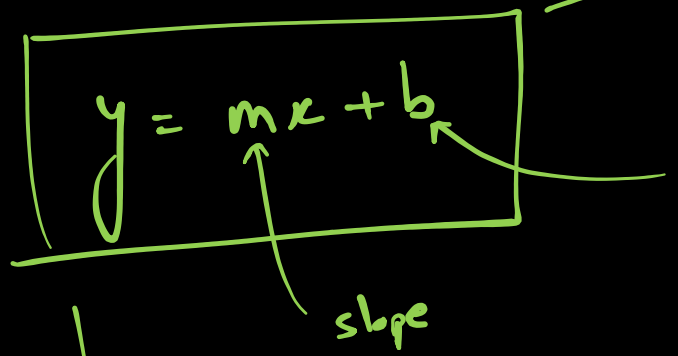
Writing Slope-intercept equations



→ This specific form of lin. eq. is called slope-intercept equations.

→ Linear equation

→ Equation of a line

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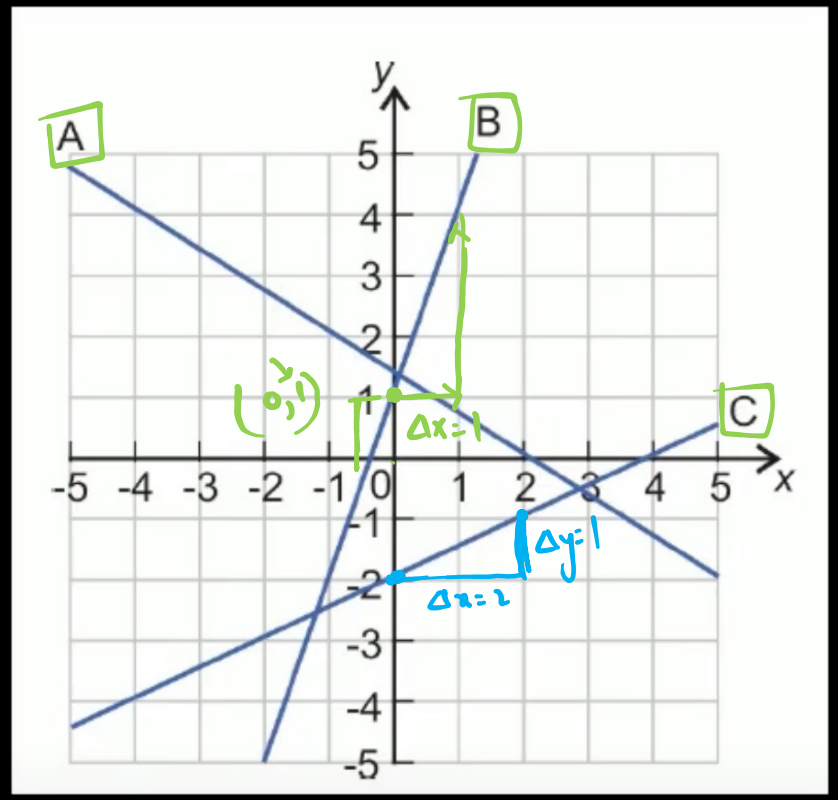
Slope - intercept equation from graph



Line A:

$$y = mx + b$$

If $x = 0$, $y = b$

$$m = \frac{\Delta y}{\Delta x} = -\frac{2}{3}$$

Δx	Δy
3	−2
1	$-\left(\frac{2}{3}\right)$

$$m = -\frac{2}{3}$$

$$y = -\frac{2}{3}x + b$$

$$b = 1\frac{1}{3}$$

$$y = -\frac{2}{3}x + \frac{4}{3}$$

$$b = \frac{4}{3}$$

Line B: $m = 1$ $\quad b = ?$

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$$

$x = 0$, $y = 1$ $\quad b = 1$

$$y = 3x + 1$$

Line C:

$$m = \frac{\Delta y}{\Delta x} = \frac{1}{2}$$

$b = -2$

$$y = \frac{1}{2}x + (-2) \quad \Big| \quad y = \frac{1}{2}x - 2$$

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① $y = 2x + 5$

② $y = -0.2x + 7$

③ $y = -x$

④ $y = 3.75$

$y = -0.2x + 7$

$b = 7$

$m = -0.2$

$= -\frac{2}{10}$

$\frac{\Delta y}{\Delta x} = -\frac{1}{5}$

③ $y = -x$

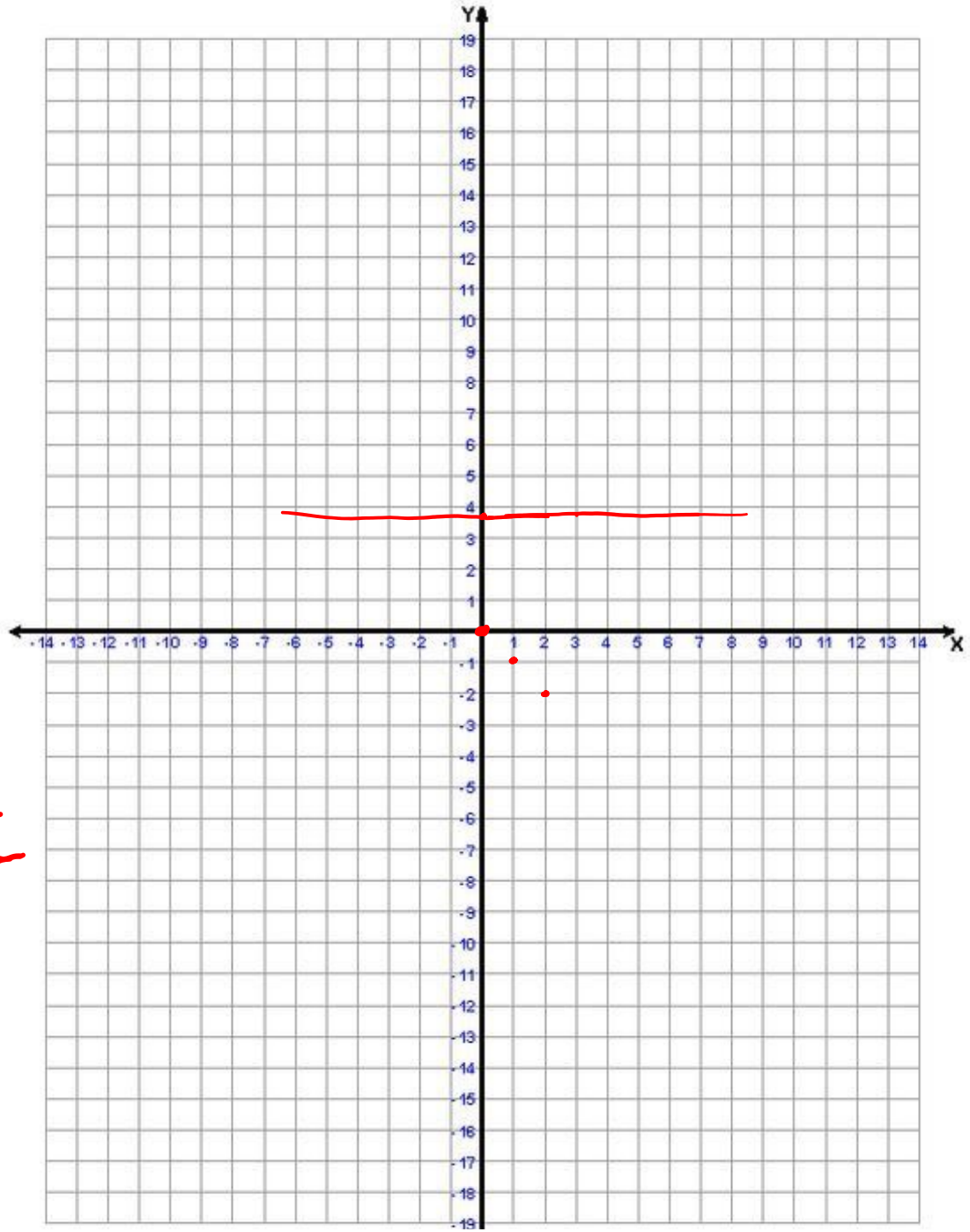
$y = -1x + 0$

$m = -1$

$\frac{\Delta y}{\Delta x} = \frac{-1}{1}$

④ $y = 3.75$

$y = 0.x + 3.75$



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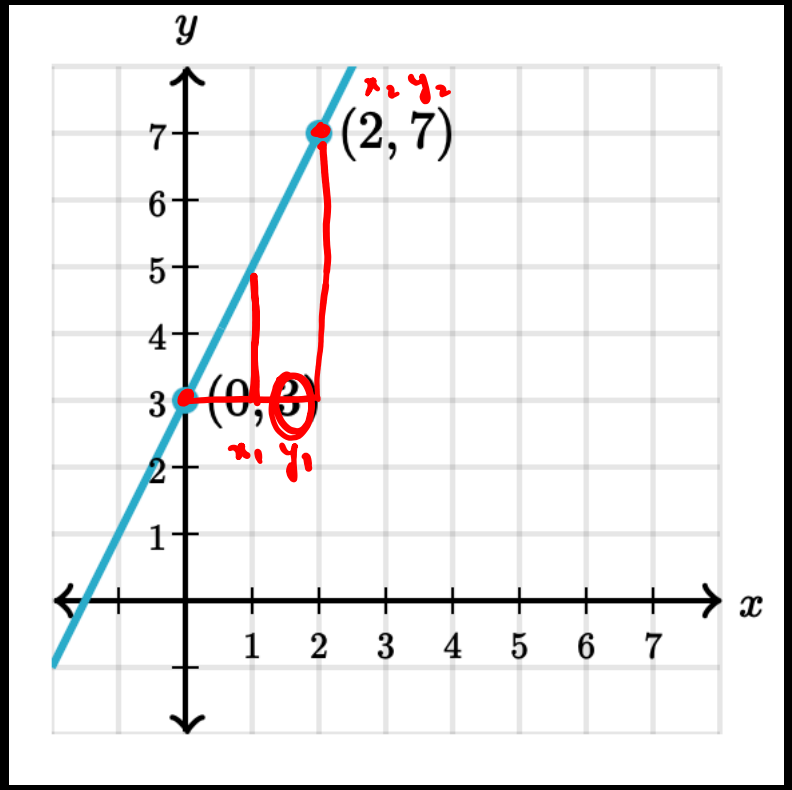
Problem: Write slope intercept equation for the given graph.



$$m = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{7-3}{2-0} = \frac{4}{2} = 2$$

$b = \{ y \text{ at } x = 0 \}$

$$\boxed{b = 3}$$

$$\boxed{y = 2x + 3}$$

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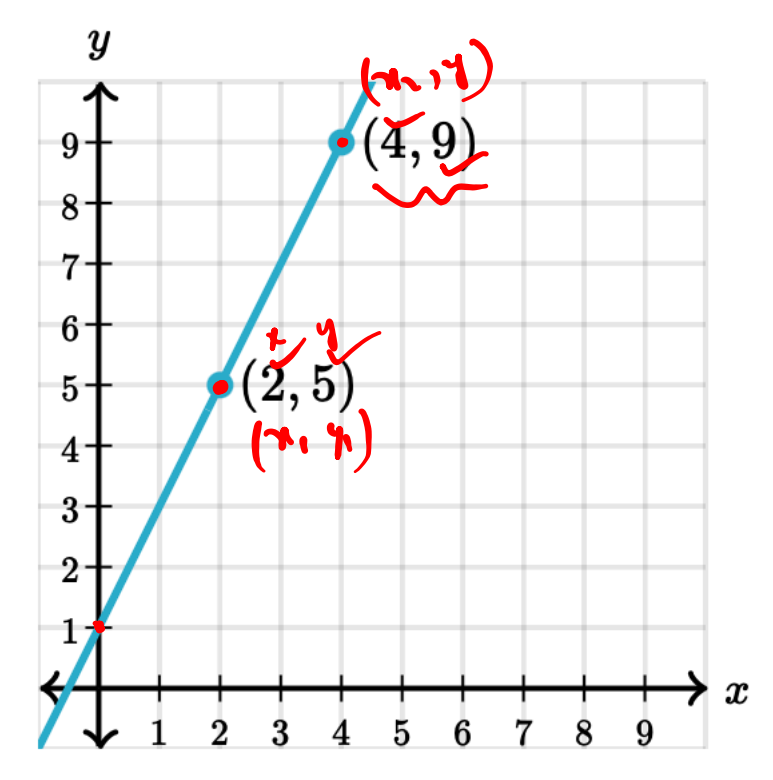
Writing equations from any two points.



$m = ?$ $\quad b = ?$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{4-2} = \frac{4}{2} = 2$$

$$y = 2x + b$$

(using (2,5)

$$5 = 2(2) + b$$

$$b = 5 - 4$$

$$b = 1$$

$$y = 2x + 1$$

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Slope-intercept equation from slope and point.

Ex:→ A line has a slope of $-3/4$ and goes through the point $(0, 8)$. What is the equation of this line, in slope-intercept form?

$m = \frac{-3}{4}$, $(0, 8)$ (0 → x, 8 → b)

$b = ?$

$y = mx + b$

$y = \frac{-3}{4}x + b$

$y = \frac{-3}{4}x + 8$

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Slope-intercept equation from two points:

Ex. A line goes through the points $(-1,6)$ and $(5,-4)$. What is the equation of the line?

(x_1, y_1) $(-1,6)$ x_1 y_1

(x_2, y_2) $(5,-4)$ x_2 y_2

$y = mx + b$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-6}{5-(-1)} = \frac{-10}{6} = \frac{-5}{3}$$

$y = \frac{-5}{3}x + b$

Because this line is passing through $(-1,6)$, hence.

$(y - y_1) = m(x - x_1) + b$ $\quad y = \frac{-5}{3}x + b$

~~with slope~~ can be written as, $\quad 6 = \frac{-5}{3}(-1) + b$

$$b = 6 - \frac{5}{3} = \frac{18}{3} - \frac{5}{3} = \frac{13}{3}$$

$6 = \frac{5}{3} + b$

$y = \frac{-5}{3}x + \frac{13}{3}$

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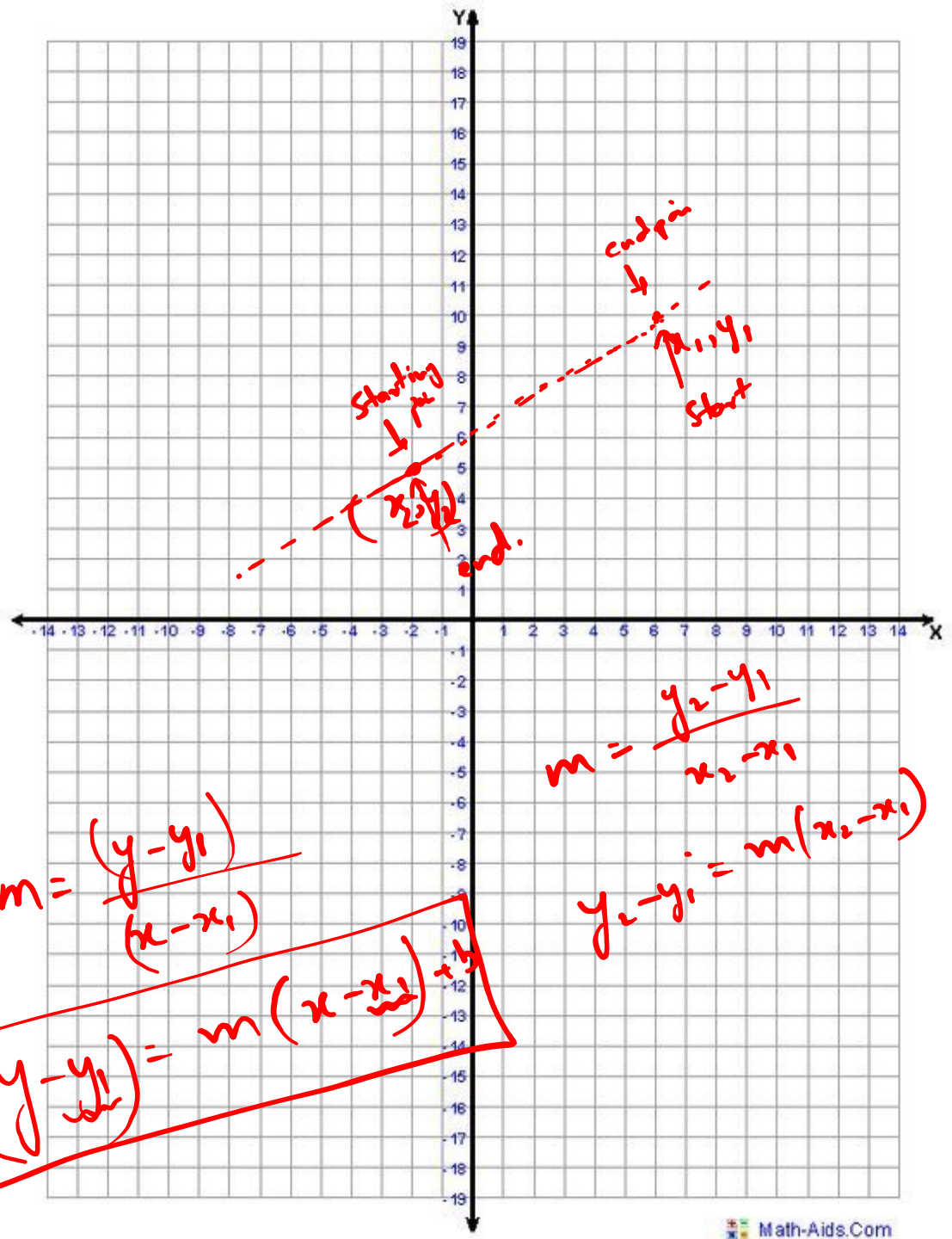
(x_1, y_1)

(x, y)

$$m = \frac{(y_1 - y)}{(x_1 - x)}$$

$$(y_1 - y) = m(x_1 - x)$$



$$m = \frac{(y - y_1)}{(x - x_1)}$$

$$(y - y_1) = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y_2 - y_1 = m(x_2 - x_1)$$

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End of the chapter

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