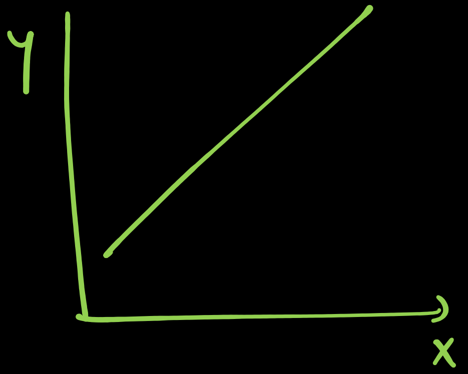
Linear Models

Linear Models

What are Linear models?

Linear $\Rightarrow$ Line (straight)



degree 2

$\boxed{4x^2 + 2y^2 = 7}$ $\rightarrow$ Not a linear equation $\rightarrow$ A quadratic equation

Power of variable(s) = 1

degree 1.

$\boxed{4x + 3y = 5}$

A linear model is an equation (with degree 1) that represents relationship between two variables.

eg. $\boxed{4x + 3y = 5}$

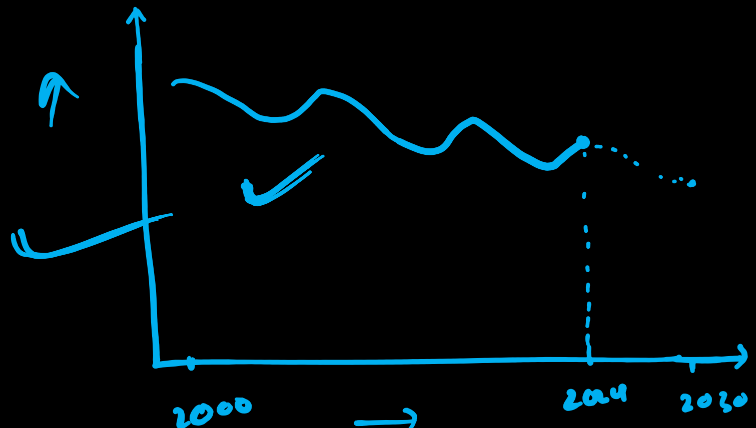
Recap:

$\boxed{4x+2 = 0}$

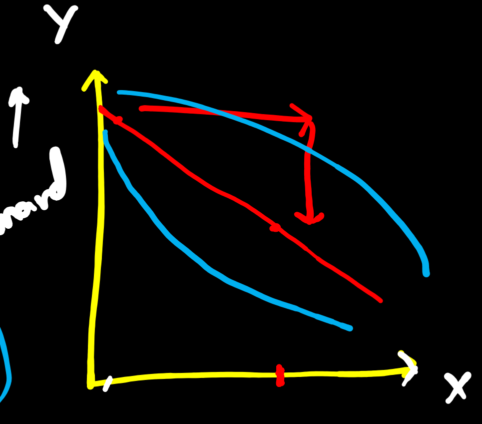
Linear equation (✓)

function (x)

Linear model (x)

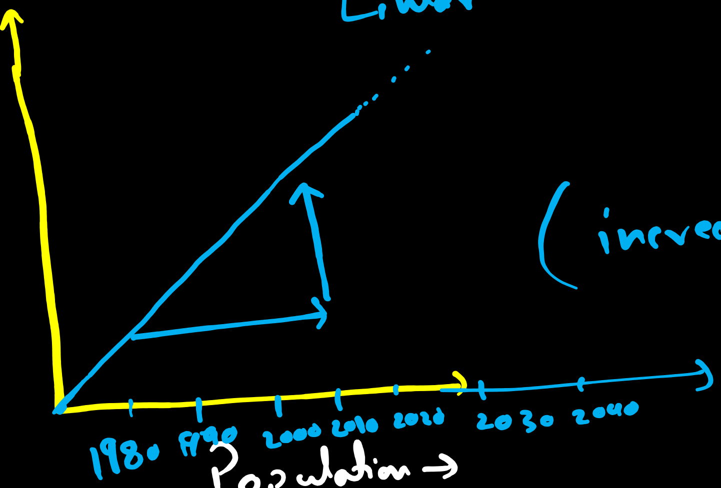


$\boxed{\text{Linear model}}$



(decreasing trend)

(Gold)



(increasing trend)

Models with

- Tables
- Equations
- Graph (Visual representation)

Ex.

Pizza → \$ 6

Toppings → \$ 2 (per topping)

Tabular Representation

# toppings on pizza (x)	Total cost of pizza (y)
0	6
1	8
2	10
3	12
4	14

Representing with an equation

Write an equation for the total cost (y) with x toppings

$\Rightarrow$ Cost of just pizza = \$6

$\Rightarrow$ x toppings (at the rate of \$2 per topping) = \$ $x \cdot 2$

Total cost (y) = \$($6 + 2x$)

100 toppings

$$\boxed{y = 2x + 6}$$ $\Rightarrow$ Linear model in the form of equation

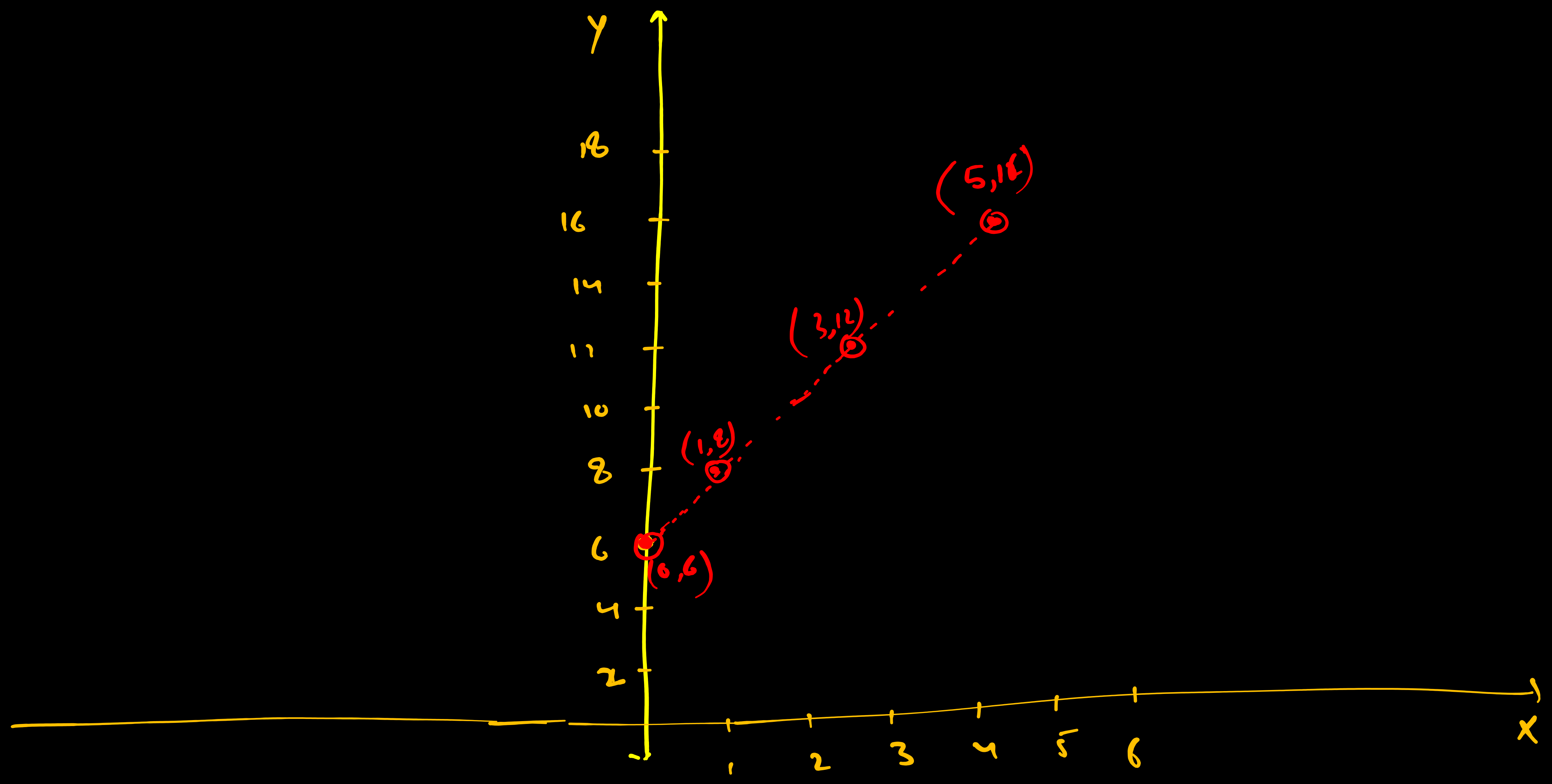
Representing with a graph

Ordered pair

(x, y) $\quad (y, x)$ X

Topping of pizza (x)	Total cost (y)	ordered pair (x, y)
0	6	(0, 6)
1	8	(1, 8)
3	12	(3, 12)
5	16	(5, 16)

$$\boxed{y = 2x + 6}$$


Y
X
18
16
14
12
10
8
6
4
2
1
2
3
4
5
6
(5,18)
(3,12)
(1,8)
(0,6)

Methods:

(I) **Table** : allowed us to see exactly how much a pizza with different number of toppings costs.

(II) **Equation**: gave us a way to find cost of pizza with any no. of toppings.

(III) **Graph** : helped us to visually see the relationship with cost and no. of topping.

[Linear models : Word Problems]

Model with Linear equations

1. On Monday morning there were 12 inches of snow on the ground. The weather warmed up, and by Tuesday morning, 2 inches had melted. 2 more inches melted by Wednesday morning. This pattern continued throughout the week until no more snow was left.

Create an equation and a graph to show the relationship between the day and the amount of snow on the ground.

x	y
0	12
1	10
2	8
3	6
4	4
5	2
6	0

Let x = no. of days after monday (2x)
y = inches of snow on the ground.

equation

$$\boxed{y = 12 - 2x}$$

mx

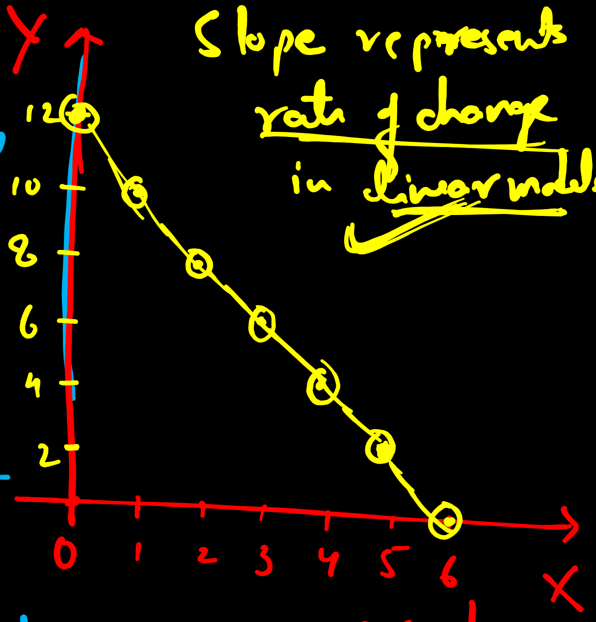
(Linear model)

$m = \boxed{-2}$ slope

Monday
$y = 12 - 2x$
$= 12 - 2\times0$
$= 12$ inches

Monday = day 0
Tue → day 1
W → day 2
Th → day 3
F → day 4
Sd → day 5
Sunday → day 6

Tue → $12 - 2\times1$
$= 12 - 2$
$y = 10$ inch.



Slope represents rate of change in linear models

Linear model / equation

Find the slope of the linear function defined by the table

	Time Worked Hours (x)	Amount of Money Earned Dollars ($) ($y$)
Half-Day	4	54.00
1 Day	8	108.00
2 Days	16	216.00
1 Week	40	540.00
1 Month	180	2,430.00

$\Delta x =$

What does the slope represent in this situation?

$$\text{Slope} = \frac{\text{change in dependent var. } (\Delta y)}{\text{change in independent var. } (\Delta x)} = \frac{108-54}{8-4} = \frac{54}{4} = 13.5$$

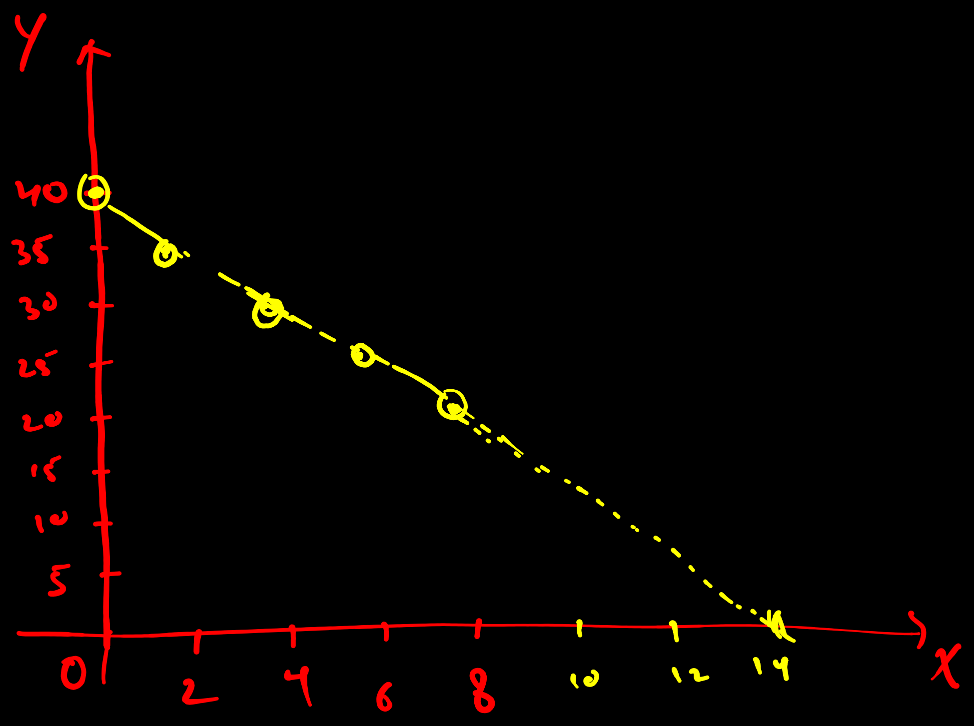
~~Slope~~ Slope represent amount of money earned (13.5) per hour which is \$13.5/hour.

Jill just received \$40. The number of dollars she has left (y) after x days is approximated by the formula $y = 40 - 2.5x$. Graph the equation and use the graph to estimate how much money Jill will have 8 days later.

$y = 40 - 2.5x$

x	y
0	40
2	35
4	30
6	25
8	20
10	15
12	10
13	5
14	0

(0,40)
(2,35)
(4,30)
(6,25)
(8,20)



Q. Priya read a book cover to cover in a single session, at a rate of 55 pages per hour. After reading for 4 hours, she had 330 pages left to read.

How long is the book?

How long did it take Priya to read the entire book?

Linear model

550 pages

(hrs) x	y (#pages read)
1	55
2	110
3	165
4	220

$\boxed{330}$

$\boxed{y = 55x}$

$\boxed{y = 220}$

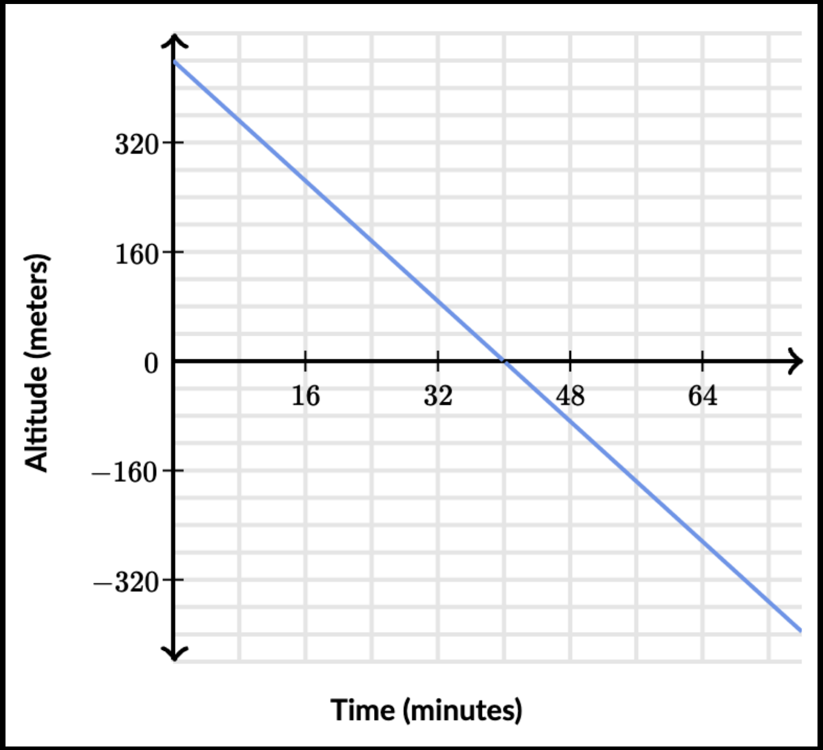
$\boxed{y = 55x + 0}$

Total page = 550

Time = $\frac{550}{55}$ = 10 hours

H.W 1

Amir drove from Jerusalem to the lowest place on Earth, the Dead Sea.
His altitude relative to sea level (in meters) as a function of time (in minutes) is graphed.



How fast did Amir descend?

(a) 10 m/min.

(b) 11 m/min.

(c) 40 m/min.

(d) 41 m/min.

HW.2

A young sumo wrestler goes on a special diet to gain weight. The variable *w* models the wrestler's weight (in kilograms) after the wrestler has been on a special diet for *t* months.

$w = 80 + 5.4t$

How much weight does the wrestler gain every 2 months?

A paintball court charges an initial entrance fee plus a fixed price per ball. The variable p models the total price (in dollars) as a function of n, the number of balls used.

$p = 0.80n + 5.50$

What is the entrance fee?

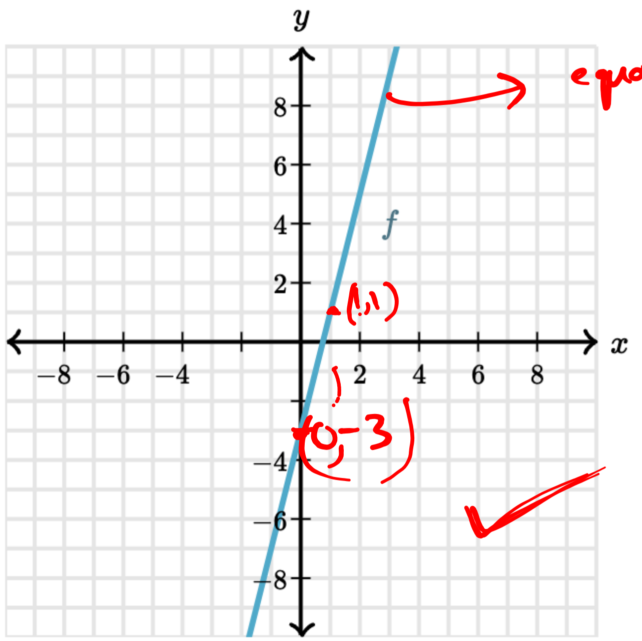
Comparing Linear Functions

Function 1 is defined by the equation $\boxed{y = \frac{7}{2}x - 3}$.

$\rightarrow$ Slope $(m) = \frac{7}{2}$

b

Function 2 is defined by line f, shown on the following graph.



equation ⇒ $\boxed{y = 4x - 3}$

Slope $= \frac{4}{1} = \boxed{4}$

Which function has a greater y-intercept?

Function 1 is defined by the following table.

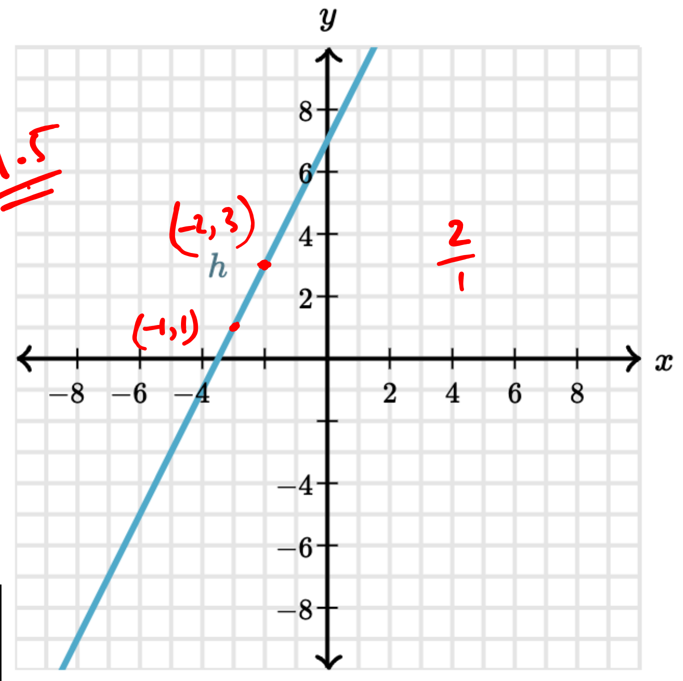
x	y
0	8
3	$\boxed{12.5}$
4	14
6	17

$(0,8)$

$(4,14)$

$m = \frac{\Delta y}{\Delta x} = \frac{14-8}{4-0} = \frac{6}{4} = 1.5$

Function 2 is defined by line h. $\Rightarrow m = 2$



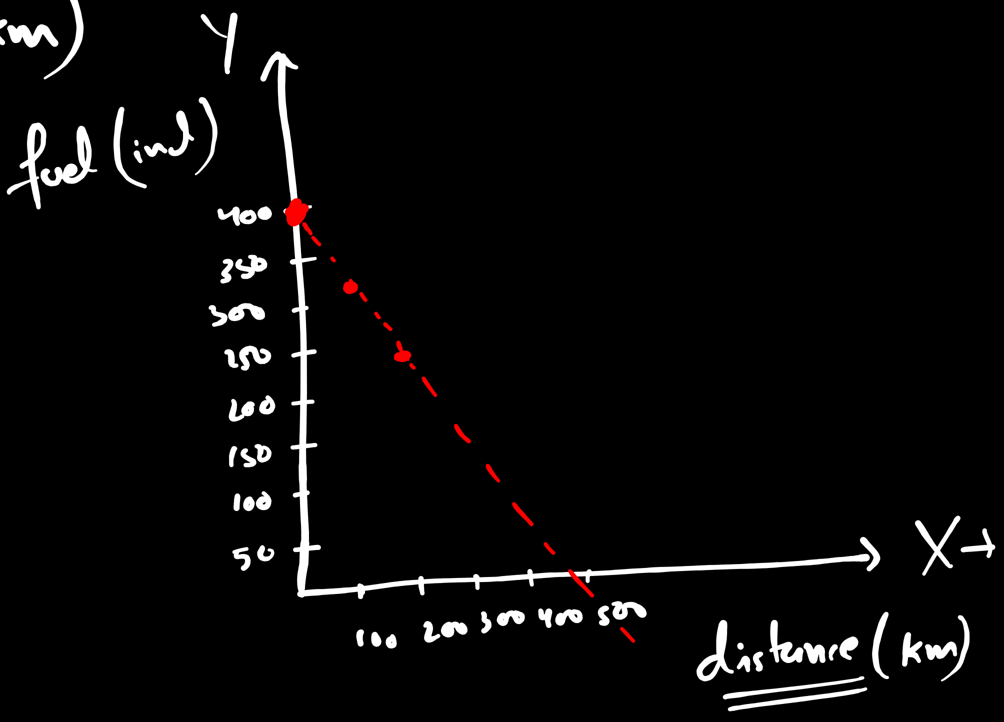
Which function has a greater slope?

Constructing linear models for real-life relationships.

Q. Karl filled up ~~the~~ his truck with 400 litres of fuel and set out to deliver a shipment to Alaska. The truck consumed 0.8 l of fuel for each kilometre.

Graph the amt. of fuel remaining in the truck (in l) as a function of distance (in km)

Distance	Fuel. F(d)	
0	400	
100	320	(400 - 80)
200	240	(320 - 80)



Q. A gym is offering a deal to new members. Customers can sign up by paying a registration fee of \$200 and a monthly fee of \$35. How much will the membership cost a member at the end of the year?

Equation?

$$\boxed{y = 35x + 200}$$

y = Total cost of membership

x = no. of ~~mnth~~ months

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Q. Dominik uses 20 grams of filling for each dumpling he makes. He has 1500 grams of dumpling filling.

The grams F of filling remaining is a function of d, the number of dumplings Dominik makes.

$\boxed{\text{Write the function's formula.}}$ equation

# d	F (g)
0	1500
1	1480
2	1460

1500 → b

$m = \frac{\Delta F}{\Delta d} = \frac{-20}{1}$

$\boxed{m = -20}$

$\boxed{F = -20d + 1500}$

Rei stacks boxes of books on a table. Each box weighs 30 kg and the table with 8 boxes on top weighs a total of 310 kg. The total weight W of the table and boxes is kg is a function of x, the no. of boxes Rei stacks on the table.

Write the function's ~~function~~ formula.

$W = 30 \times 8 + b$

$30 \times 8 + b = 310$

$\boxed{b = 70}$

$\boxed{W = 30x + 70}$

Hiro painted his room at a rate of 8 square metres per hour.
After 3 hours of painting, he has 28 square metres ~~dat~~ left to paint.
Let $A(t)$ denote the area to paint A (measured in sq.m.) as a function of time t (measured in hours).
Write the function's formula.

$$\boxed{A(t) = mt + b}$$

t	$A(t)$
0	52
1	44
2	36
3	28

$\Delta t = 1$

$\Delta A = -8$

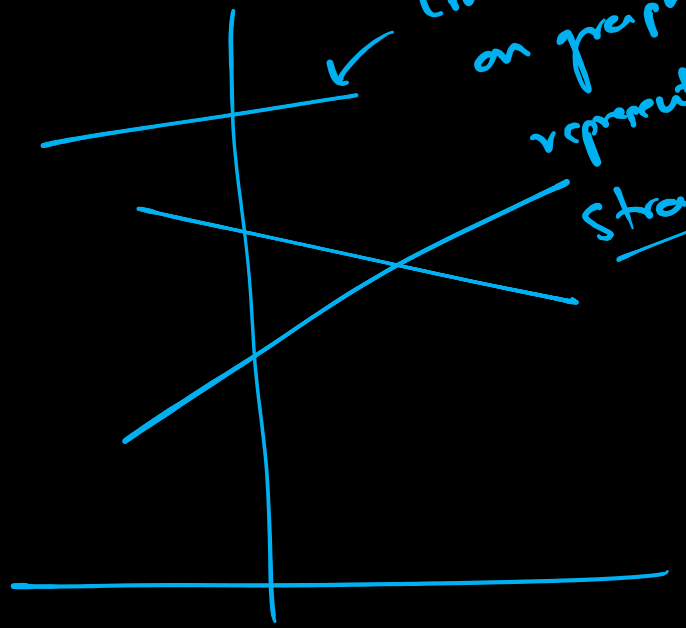
$m = \frac{\Delta A}{\Delta t} = \frac{-8}{1} = -8$

$A(0) = 52 = b$

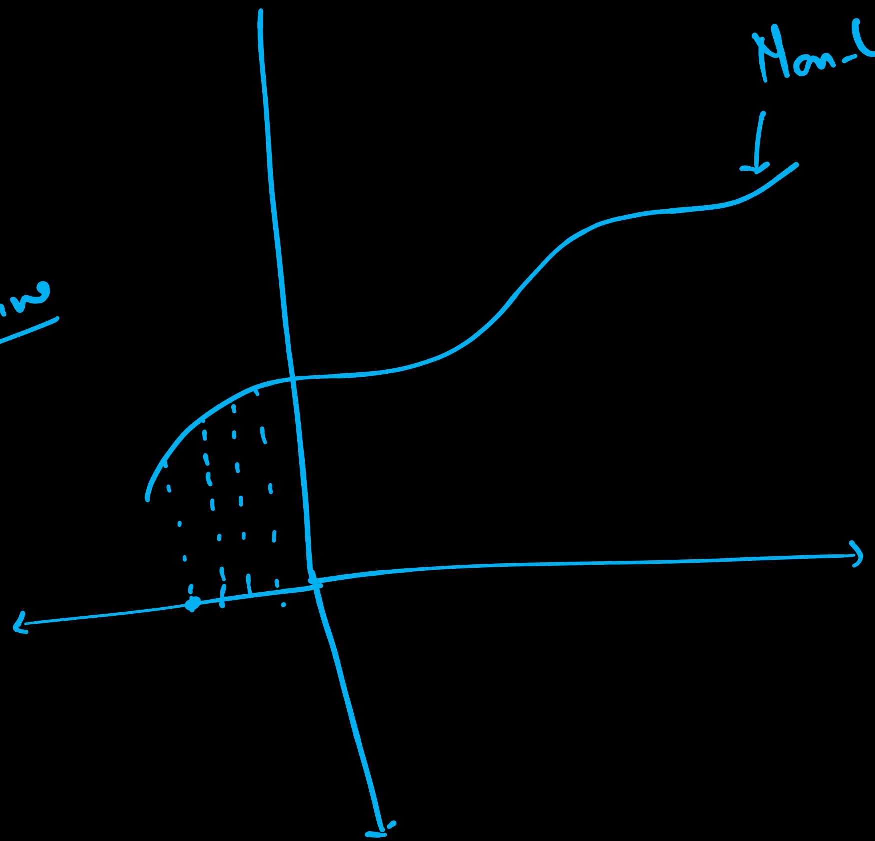
$$\boxed{A(t) = -8t + 52}$$

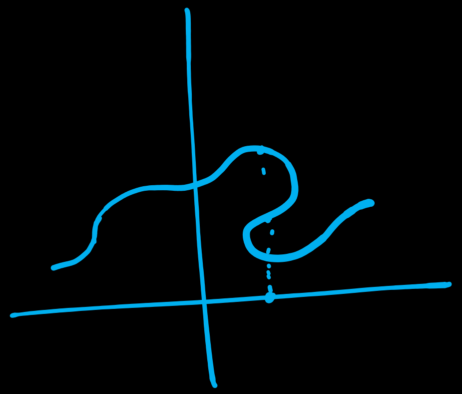
R

Linear function on graph is represented by straight line



Non-Linear function





Not a function

Systems of $\boxed{\text{linear}}$ equations

Two variable

↳ two equations

more than one equation at a time

$\boxed{y_1 = mx_1 + b}$

$\boxed{y_2 = mx_2 + c}$

Solution of systems of equation

⇒ The value of x and y (ordered pair) which belongs to both the equations

Testing a solution to system of equation

$-5y = -5$ ——— (i)

$7x + 6y = 7$ ——— (ii)

$\boxed{(5,1)}$ is a solution of the system

x y

$-5y = -5$

LHS $= (-5 \times 1) = -5 =$ RHS

$\therefore$ (5,1) is a solution of eq. (i).

LHS $= 7x + 6y$

$= (7\times5)+(6\times1)$

$= 41 \neq$ RHS.

$\therefore$ (5,1) is not a solution to eq (ii)

i.e. Hence, (5,1) is not a solution of the system.

Q. → $\boxed{3x - 2y = -1}$ —— (1)

→ $y = -x + 3$ —— (11)

Find the solution of given system.

- elimination
- substitution

Is $\boxed{(1,2)}$ is a solution of the system?

eq (1)

$$\begin{aligned} LHS &= 3x - 2y \\ &= 3(1) - 2(2) \\ &= 3 - 4 \\ &= -1 \\ &= RHS \end{aligned}$$

(11) $y = -x + 3$

$(2) = -1 + 3$

$(2) = 2$

LHS = RHS.

(1,2)

(1,2) is solution of system.

$$3x - 2y = -1$$

$$-2y = \cancel{-1} - 3x - 1$$

$$y = \frac{-3x-1}{-2} = \frac{3x+1}{2} = \frac{3}{2}x + \frac{1}{2}$$

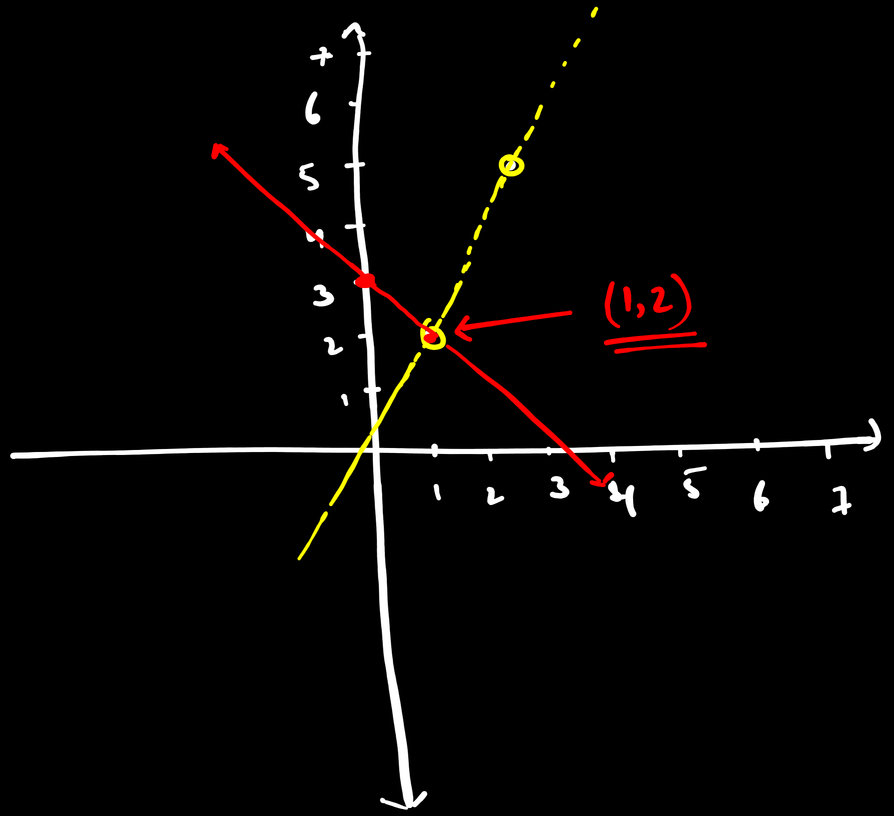
x	y
0	$\frac{1}{2}$
1	2
3	5

(1,2)

(3,5)

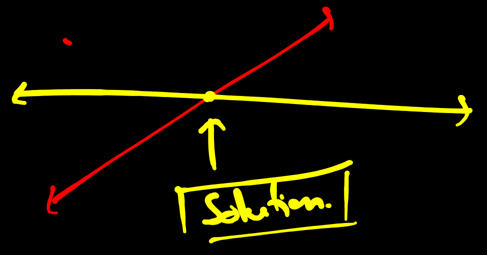
$$y = -x + 3$$

x	y
1	2
0	3

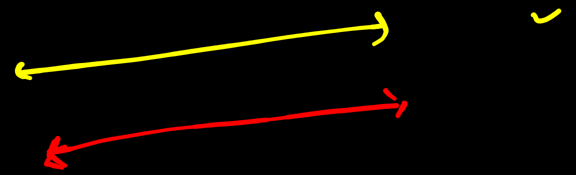


3 kinds of solution

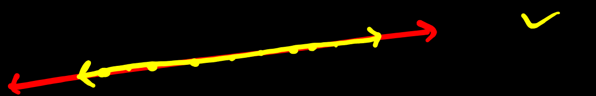
(i) Unique solution (only one solution)
↳ intersecting line.



(ii) No solution, when lines are parallel (same slope)
diff. y intercept



(iii) Infinitly many solutions, when lines are coincident.
(Same slope and same y-intercept)



Finding solutions of system of equations

1. Geometric method. (graph)
   - point of intersection.

2. Algebraic method
   - Substitution
   - Elimination

(1) Find the solution to the system of equations graphically.

$$\begin{cases} y = 2x + 3 & \text{--- (I)} \\ y = -3x + 3 & \text{--- (II)} \end{cases}$$

$\boxed{\text{H.W.}}$

Systems of linear equation using substitution method.

$\boxed{\text{eg.}}$

$y = 2x$ eq. 1 $\rightarrow$ line

$x + y = 24$ eq. 2 $\rightarrow$ line

$2x + 3y + z = x - 4$

We can $\boxed{y}$ of eq. 2 by $\boxed{2x}$ from eq. 1

eq. 2 will be written as.

$x + 2x = 24$

$3x = 24$

$\boxed{x = 8}$

Using $x = 8$ in eq. 1, we get

$y = 2 \cdot 8$

$\boxed{y = 16}$

$\boxed{\begin{array}{l} x = 8 \\ y = 16 \end{array}}$

Solution $(8, 16)$

eq2 $4x+y=28$

$y=3x$

Solve it for x & y using substitution.

eq3

$-3x+y=-9$ — eq.1

$5x+4y=32$ — eq.2

$-3x+y=-9$

$+3x \quad +3x$

$\boxed{y=-9+3x}$

Substi

Substituting $y=-9+3x$ in eq.2 we get

$5x+4(-9+3x)=32$

$5x+(-36)+12x=32$

$17x=68$

$\boxed{x=4}$

$\Rightarrow \boxed{(4,3)}$

~~Putting~~ Substituting $x=4$ in eq.2.

$5.4+4y=32$

$4y=12$

$\boxed{y=3}$

4

$2x - 3y = -5$ — eq 1 ✓

$y = x - 1$ — eq. 2

Sub. $y = x-1$ in eq. 1

$2x - 3(x-1) = -5$

$2x - 3x + 3 = -5$

$-x = -8$

$\boxed{x = 8}$

Subst. $x = 8$ in eq 2.

$y = 8 - 1$

$\boxed{y = 7}$

$(8, 7)$

5

$$7x + 10y = 36 \qquad \text{eq 1}$$

$$-2x + y = 9 \qquad \text{eq 2}$$

$x = -2$

$y = 5$

$(-2, 5)$

$7x + 10y = 36$

$$y = \frac{36 - 7x}{10}$$

$$-2x + \frac{36 - 7x}{10} = 9$$

$-20x + 36 - 7x = 90$

$-27x = 90 - 36$

$\boxed{x = -2}$

$\boxed{-2.7x = 5.4}$

$$x = \frac{5.4}{-2.7}$$

$$= -\frac{54}{27}$$

$= -2$

(6)

$-5x + 4y = 3$ —— eq 1

$x - 2y = -15$ —— eq 2

Sub. $\boxed{x = 2y - 15}$ in eq 1.

$-5(2y-15) + 4y = 3$

$-10y + 75 + 4y = 3$

$-6y = -72$

$6y = 72$

$y = \frac{72}{6} = 12$

{ Addition {
Subtraction {
Division { }

Subst. $y = 12$ in eq 2.

$x - 2(12) = -15$

$x - 24 = -15$

$x = 24 - 15$

$\boxed{x = 9}$

$(9, 12)$

Elimination method :

$-5x + 4y = 3$ — eq 1

$x - 2y = -15$ — eq 2

To Eliminate y. multiply eq 2 by 2, we get.

$(x - 2y = -15) \times 2$

$2x - 4y = -30$ —— eq 3

Adding eq 1 and eq 3

$$\begin{array}{r} -5x + 4y = 3 \\ + \quad 2x - 4y = -30 \\ \hline \end{array}$$

$-5x + 2x + 4y - 4y = 3 + (-30)$

$-3x = 3 - 30$

$-3x = -27$

$\boxed{x = 9}$

$9 - 2y = -15$

$-2y = -15 - 9$

$+2y = +24$

$y = \frac{24}{2} = 12$

$\boxed{9, 12}$

$$5x - 7y = 15 \quad \text{——— eq1} \quad \times 2$$

$$2x - 2y = 6 \quad \text{——— eq2} \quad \times 5$$

$$10x - 14y = 30 \quad \text{——— eq3}$$

$$(-) \quad \underset{(-)}{10x} \underset{(+)}{-} 10y = \underset{(-)}{30} \quad \text{——— eq4}$$

$$0 - 4y = 0$$

$$\boxed{y = 0}$$

eq 2 : $2x = 6$

$$\boxed{x = 3}$$

$$\boxed{3, 0}$$

Q. In 40 years, kushaan will be 11 times as as old as he is right now. How old he is right now.

Present age is x years

Age in 40 years = $(x+40)$ years

$$x \cdot 11 = x + 40$$

$$11x = x + 40$$

$$10x = 40$$

$$\boxed{x = 4 \text{ years}}$$

Solve for x and y:

1.

$3x - 5y = -1$ ——— eq 1

$x - y = -1$ ——— eq 2

$\boxed{\text{eq 2} \times (-3)}$

$3x - 5y = -1$

$-3x + 3y = 3$ ——— (eq 3)

$-5y + 3y = -1 + 3$

$-2y = 2$

$\boxed{y = -1}$

Putting $y = -1$ in eq 2

$x - (-1) = -1$

$x + 1 = -1$

$x = -1 - 1$

$\boxed{x = -2}$

$(-2, -1)$

Substitution

(11) $2x + 3y = 9$ —— eq 1

$3x + 4y = 5$ —— eq 2

eq. 1

$2x + 3y = 9$

$3y = 9 - 2x$

$$\boxed{y = 3 - \frac{2}{3}x}$$

Substitute $y = 3 - \frac{2x}{3}$ in eq 2, we get

$3x + 4\left(3 - \frac{2}{3}x\right) = 5$

$3x + 12 - \frac{8x}{3} = 5$

$3x - \frac{8x}{3} = 5 - 12$

$\frac{9x - 8x}{3} = -7$

$\frac{x}{3} = -7$

$$\boxed{x = -21}$$

Putting $x = -21$ in eq. 1, we get

$2(-21) + 3y = 9$

$-42 + 3y = 9$

$3y = 9 + 42$

$3y = 51$

$\boxed{y = 17}$

$(-21, 17)$

Elimination

(iii) $8x + 5y = 9$ eq 1

$3x + 2y = 4$ eq 2

(iv) $0.5x + 0.7y = 0.74$ — eq.1 $\times 100$

$0.3x + 0.5y = 0.5$ — eq.2 $\times 100$

$50x + 70y = 74$ — 3 $\times 30$

$30x + 50y = 50$ — 4 $\times 50$

$1500x + 2100y = 2220$

$-\;1500x \;(+)\; 2500y = (-)2500$

$-400y = -280$

$y = \frac{28\cancel{0}}{40\cancel{0}} = \frac{28}{40} = \frac{7}{10} = \boxed{0.7}$

Putting $y = \frac{7}{10}$ in eq 3

$50x + 7\cancel{0}\left(\frac{7}{\cancel{10}}\right) = 74$

$50x + 49 = 74$

$50x = 74 - 49$

$50x = 25$

$x = \frac{25}{50} = \frac{1}{2} = \boxed{0.5}$

$(0.5, 0.7)$

Solve, for x and y

(v) $ax + by = 5$ eq(1) $\times 3$

$2ax - 3by = 7$ eq 2

$3ax + 3by = 15$

$+ \quad 2ax - 3by = 7$

$3ax + 2ax = 15 + 7$

$5ax = 22$

$$\boxed{x = \frac{22}{5a}}$$

Putting $x = \frac{22}{5a}$ in eq 1

$$\frac{3}{5} \div \frac{b}{1}$$

$$\frac{3}{5} \times \frac{1}{b}$$

$$\frac{3}{5b}$$

$\cancel{a}\left(\frac{22}{5\cancel{a}}\right) + by = 5$

$\frac{22}{5} + by = 5$

$by = 5 - \frac{22}{5}$

$by = \frac{25 - 22}{5}$

$by = \frac{3}{5}$

$y = \frac{\left(\frac{3}{5}\right)}{b}$

$$\boxed{y = \frac{3}{5b}}$$

$$\left(\frac{22}{5a}, \frac{3}{5b}\right)$$

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$

$\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$

$\sqrt{2} + \sqrt{3} =$

$\sqrt{2} + \sqrt{2} =$

$1a + 1a = 2a$

$1\sqrt{2} + 1\sqrt{2} = 2\sqrt{2}$

$a + b = x$

$\sqrt{2} + \sqrt{3} = x$

$3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

$3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$

(vi)

$\sqrt{2}x - \sqrt{3}y = 0$ eq1

$\sqrt{3}x - \sqrt{8}y = 0$ eq2

$\sqrt{2}x - 0 = 0$

$\sqrt{2}x = 0$

$x = \frac{0}{\sqrt{2}}$

$\boxed{x = 0}$

$-\sqrt{9}y + \sqrt{16}y$

$-3y + 4y$

$\boxed{y = 0}$

$(0, 0)$

(vii) $11x + 15y + 23 = 0$ eq. 1

$7x - 2y - 20 = 0$ eq. 2

$11x + 15y = -23$

$7x - 2y = 20$

$$77x + 105y = -161$$

$$\underset{(-)}{77x} \underset{(+)}{-} 22y = \underset{(-)}{220}$$

$$127y = -381$$

$$y = \frac{\cancel{-381}^{-3}}{\cancel{127}} = \boxed{-3} =$$

$7x - 2y = 20$

$7x - 2(-3) = 20$

$\boxed{7x + 6 = 20}$

$7x = 14$

$\boxed{x = 2}$

$(2, -3)$

Q. William is 4 times as old as Ben. 12 years ago, William was 7 times as old as Ben. How old is Ben now?

Solⁿ: Let us assume that present age of Ben = b years

$\therefore$ William's present age $= 4b$ years

12 years ago,

Ben's age $= (b-12)$ years

William's age $= (4b-12)$ years

As per the problem statement,

$$(4b-12) = 7(b-12)$$

$$4b-12 = 7b-84$$

$$4b-7b = -84+12$$

$$-3b = -72$$

$$\cancel{-}3b = \cancel{-}72$$

$$3b = 72$$

$$b = \frac{72}{3}$$

$$\boxed{b = 24 \text{ years}}$$

Hence, Ben's present age is 24 years..

Q. Present age of Arman and Diya is 18 year and 2 year respectively. How many years will it take for Arman to be 3 times as old as Diya?

~~Let us~~ It will take x years for Arman to be 3 times as old as Diya.

Present age,

Aroman = 18

Diya = 2

Age after x years,

Arman's = $18 + x$

Diya = $2 + x$

As per the problem.

$$18 + x = 3(2 + x)$$

$$18 + x = 6 + 3x$$

$$-2x = -12$$

$$2x = 12$$

$$\boxed{x = 6 \text{ year}}$$

Hence, in 6 years Arman's age will be 3 time as old as Diya.

Q. Ishaan is 2 times as old as Kushaan. 35 years ago, Ishaan was 7 times as old as Kushaan. How old is Kushaan now?

⇓

42 years.

Word problems based of systems of equation.

Q. 4 chairs and 3 tables costs ₹ 2100. and 5 chairs and 2 tables costs ₹ 1750. Find the cost of a table and a chair separately.

Cost of 1 table is ₹ x

Cost of 1 chair is ₹ y

$4y + 3x = 2100$ ———— eq 1

$5y + 2x = 1750$ ———— eq 2

② The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, he ~~buys~~ buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

$$7x + 6y = 3800 \quad \times 3$$

$$3x + 5y = 1750 \quad \times 7$$

$$\cancel{21x} + 18y = 11400$$

$$\cancel{21x} + 35y = 12250$$

$$-17y = -850$$

$$y = \frac{850}{17}$$

$$\boxed{y = 50}$$

$$7x + 300 = 3800$$

$$7x = 3500$$

$$\boxed{x = 500}$$

③ A fraction becomes 4/5, if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator & denominator, the fraction becomes 1/2. What is the fraction?

Numerator $= x$

Denominator $= y$

fraction $= \dfrac{x}{y}$

$$\frac{x+1}{y+1} = \frac{4}{5}$$

$$5(x+1) = 4(y+1)$$

$$5x+5 = 4y+4$$

$$\boxed{5x-4y = -1} \text{ — (I)}$$

$$\frac{x-5}{y-5} = \frac{1}{2}$$

$$2(x-5) = 1(y-5)$$

$$2x-10 = y-5$$

$$\boxed{2x-y = 5} \text{ — (II)}$$

$5x - 4y = -1$

$2x - y = 5$

$\boxed{\frac{7}{9}}$ ✓

If twice the son's age in years is added to father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.

Son's age = s years
father's age = f years

$$2s + f = 70$$

$$(s + 2f = 95) \times 2$$

$$2s + 4f = 190$$
$$2s + f = 70$$

$$3f = 120$$

$f = $ 40 years.

$s = $ 15 years

Q. Father is three times as old as his son. Five years later, he will be two and half times as old as as his son. How old is the father and his son?

Father's age $= x$ years

Son's age $= y$ years.

$\boxed{x = 3y}$ —— ①

$\boxed{x - 3y = 0}$

Five years later,

Father's age $= (x+5)$

Son's age $= (y+5)$

$\boxed{2\frac{1}{2}}$

$(x+5) = \frac{5}{2}(y+5)$

$(x+5)2 = 5(y+5)$

$2x+10 = 5y+25$

$2x-5y = 15$ —— ②

using ①

$2(3y) - 5y = 15$

$6y - 5y = 15$

$\boxed{y = 15}$ years.

from ①:

$x = 3y$

$x = 3 \times 15$

$= 45$ years.

Q. Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Father's present age = x years
Son's ______ = y years.

5 years later :

Father's age = $(x+5)$ ✓
Son's age = $(y+5)$ ✓

$$(x+5) = 3(y+5)$$

$$x+5 = 3y+15$$

$$\boxed{x-3y=10} \text{ — } (1)$$

5 years ago:

Father = $(x-5)$
Son = $(y-5)$

$$x-5 = 7(y-5)$$

$$x-5 = 7y-35$$

$$\boxed{x-7y=-30} \text{ — } (11)$$

$$\boxed{\begin{array}{l} x = 40 \text{ years} \\ y = 10 \text{ years} \end{array}}$$

$42 = \underbrace{40 + 2}$

$xy =$

~~two~~ two digit number 12, 42,

$10 + 2$ $\quad 40+2$

xy

$\Updownarrow$

$\boxed{10x + y}$

Q. In a two digit number, the unit digit is twice the ten's digit. If 27 is added to the number, the digit interchange their places. Find the number.

Number = $10x + y$ = $10 \cdot 3 + 6$ = 36

24 + 18 = 42

~~ten's~~ digit at ten's place is x ✓

digit at unit place = y ✓

3×10

2

$x \cdot y$

number = $\boxed{10x + y}$ ✓

$\boxed{y = 2x}$ —— (1)

$9x - 9(2x) = -27$

$9x - 18x = -27$

$+9x = +27$

$\boxed{x = 3}$

$y = 2 \cdot x$

$y = 2 \cdot 3$

$\boxed{y = 6}$

number + 27 = new number (y is at ten's place & x is unit place)

$(10x + y) + 27 = 10y + x$

$9x - 9y = -27$ —— (2)

Q. In a two digit number, the 10's digit is three time the unit's digit. When the number is decreased by 54, the digits are reversed. Find the number.

ten's digit $= x$

unit's digit $= y$

number $= 10x + y$ ✓

$\boxed{x = 3y}$ —— (I)

~~number − 54~~

$(10x + y) - 54 = 10y + x$

$\boxed{9x - 9y = 54}$ —— (II)

$9(3y) - 9y = 54$

$27y - 9y = 54$

$18y = 54$

$\boxed{y = 3}$

$x = 3(y)$

$x = 3 \cdot 3$

$\boxed{x = 9}$

Number $= 9 \cdot 10 + 3$

$= 93$

Q. The sum of a two digit number and the number formed by interchanging the digit is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

ten's place = x

unit's place = y

Number = $10x + y$ ✓

$(10x + y) + (10y + x) = 132$

$\boxed{11x + 11y = 132}$ —— (I)

$10x + y + 12 = 5(x + y)$

$10x - 5x + y - 5y = -12$

$\boxed{5x - 4y = -12}$ —— (II)

Solve:

$$\frac{3y}{20} - \frac{x}{20} = 7 \qquad \text{—— (1)}$$

$$\frac{x}{20} + \frac{y}{10} = 13 \qquad \text{—— (11)}$$

$$\left[\frac{3y}{20} - \frac{x}{20} = 7\right] \times 20 \qquad \boxed{3y - x = 140} \qquad \text{—— (111)}$$

$$\frac{(3y - x)}{20} = \frac{7}{1}$$

$$(3y - x) = 7 \times 20$$

$$\frac{x}{20} + \frac{y}{10} = 13$$

$$\frac{x + 2y}{20} = 13$$

$$x + 2y = 13 \times 20$$

$$\boxed{x + 2y = 260} \quad \text{— (iv)}$$

$$3y - x = 140 \quad \text{— (iii)}$$

$$
\begin{array}{rl}
 & 2y + x = 260 \\
+ & 3y - x = 140 \\
\hline
 & 5y = 400
\end{array}
$$

$$y = \frac{400}{5}$$

$$\boxed{y = 80}$$

$$x + 2 \cdot 80 = 260$$

$$x + 160 = 260$$

$$\boxed{x = 100}$$

Solve:

$$\frac{x}{5} + \frac{2y}{6} = 9 \rightarrow \frac{6x + 10y}{30} = 9$$

$$\boxed{6x + 10y = 270} \quad \text{(iii)} \times 4$$

$$\frac{x}{3} - \frac{3y}{4} = 10$$

$$\frac{4x - 9y}{12} = 10$$

$$\boxed{4x - 9y = 120} \quad \text{(iv)} \times 6$$

$$24x + 40y = 1080$$

$$\underset{(-)}{24x} \underset{(+)}{-} 54y = \underset{(-)}{720}$$

$$94y = 360$$

$$y = \frac{\cancel{360}^{180}}{\cancel{94}_{47}} = \frac{180}{47}$$

$$\boxed{y = \frac{180}{47}}$$

$$6x + 10 \times \frac{360}{94} = 270$$

$$6x = 270 - \frac{5 \times 360}{47}$$

$$= 270 - \frac{1800}{47}$$

$$6x = \frac{270 \times 47 - 1800}{47}$$

$$= \frac{12690 - 1800}{47}$$

$$6x = \frac{10890}{47}$$

$$x = \frac{10890}{47 \times 6}$$

$$= \frac{10890}{282}$$

$$\boxed{x = \frac{1815}{47}}$$

27 × 47 = 189 + 1080 = 1269

14 × 2 = 28

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Taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is ₹75 and for a journey of 15 km the charge paid is ₹110. What will a person have to pay for travelling a distance of 25 km?

fixed charge + running charge

Let the fixed charge of taxi be ₹ y and the running charge be ₹ x per km

As per the condition given in the problem,

(i) for 10 km journey total amt. paid = ₹75

$$\boxed{y + 10x = 75} \quad \text{--- (1)}$$

(ii) for 15 km journey

$$\boxed{y + 15x = 110} \quad \text{--- (ii)}$$

$$5x = 35$$

$$\boxed{x = 7}$$

$$y + 70 = 75$$

$$\boxed{y = 5}$$

$\therefore$ Fixed charge : ₹5

Running charge = ₹7

Charge for 25 km $\Rightarrow$ ₹$(5 + 25 \times 7)$

$=$ ₹$(5 + 175)$

$=$ ₹ 180

Father's age is three times the sum of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

- Let Father's age be x years, and
- sum of two children's age be y years.

- According to the condition given in the question.

(i) $\boxed{x = 3y}$ ——— (1)

8 + 5

13 + 5

21

5 years

26

Five years later,

- Fathers age will be $(x+5)$ years.
- Sum of childrens age be $(y+10)$ years.

$(x+5) = 2(y+10)$

$\boxed{x - 2y = 15}$ ——— (ii)

$\boxed{\text{Father's age} = 45 \text{ years}}$

End of the chapter