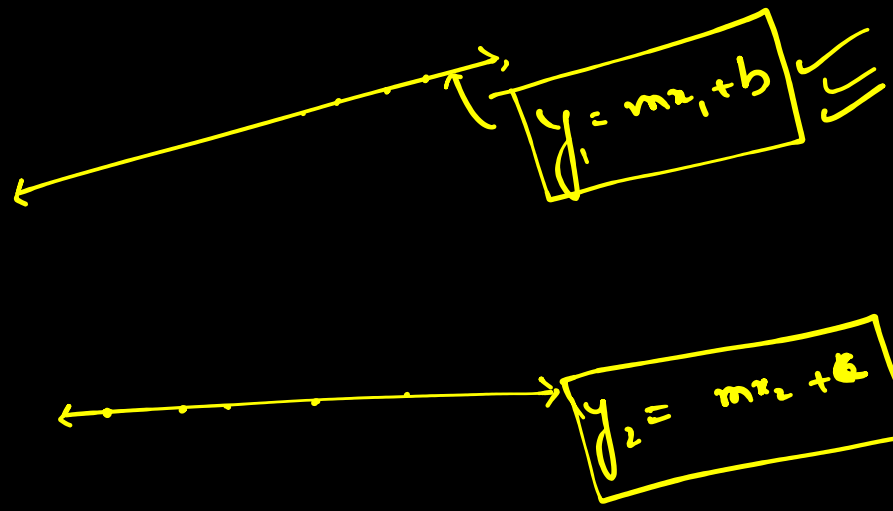
Systems of Equations

Systems of linear equations

Two variable

↳ two equations

more than one equation at a time



Solution of systems of equation

⇒ The value of x and y (ordered pair) which belongs to both the equations

Testing a solution to system of equations

$-5y = -5$ ——— (i)

$\hookrightarrow \quad 7x + 6y = 7$ ——— (ii)

$(5, 1)$ is a solution of the system

x y

$-5y = -5$

$\text{LHS} = (-5 \times 1) = -5 = \text{RHS}$

$\therefore (5,1)$ is a solution of eq. (i).

LHS $= 7x + 6y$

$= (7 \times 5) + (6 \times 1)$

$= 41 \neq$ RHS.

$\therefore (5,1)$ is not a solution to eq (ii)

$\therefore$ Hence, $(5,1)$ is not a solution of the system.

Q. → $3x - 2y = -1$ — (i)

→ $y = -x + 3$ — (ii)

Find the solution of given system.
↓
{ • elimination
• substitution }

?, $(1,2)$ is a solution of the system?

eq: (i)

LHS = $3x - 2y$

$= 3(1) - 2(2)$

$= 3 - 4$

$= -1$

$= RHS$

$(1,2)$

(ii) $y = -x + 3$

$(2) = -1 + 3$

$(2) = 2$

LHS = RHS.

$(1,2)$ is solution of system.

$$3x - 2y = -1$$

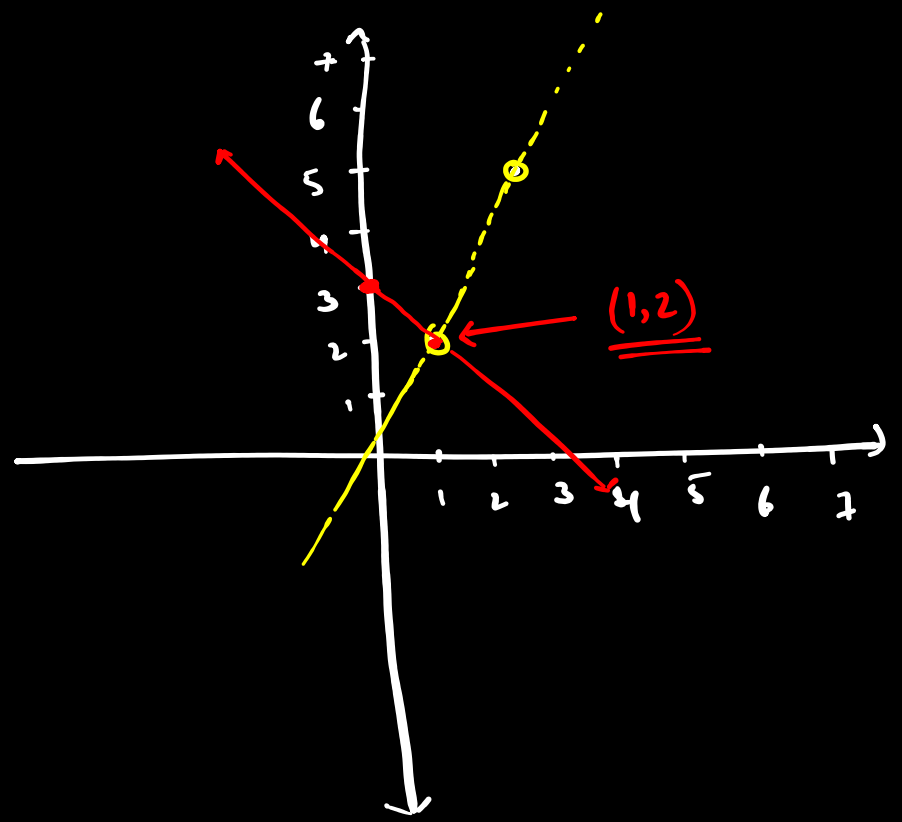
$$-2y = -1 - 3x - 1$$

$$y = \frac{-3x-1}{-2} = \frac{3x+1}{2} = \frac{3}{2}x + \frac{1}{2}$$

x	y	
0	$\frac{1}{2}$	
1	2	(1,2)
3	5	(3,5)

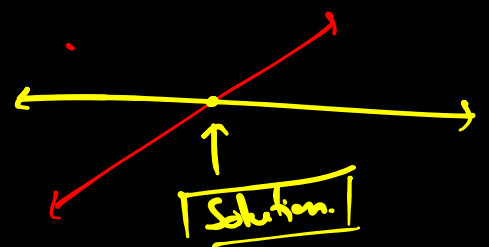
$$y = -x + 3$$

x	y
1	2
0	3

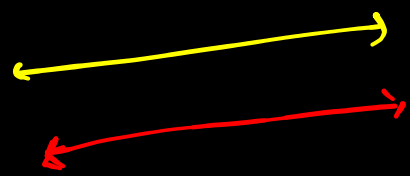


3 kinds of solution

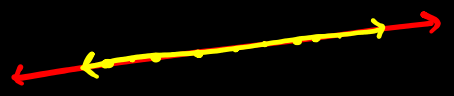
(i) Unique solution (only one solution)
by intersecting line.



(ii) No solution, when lines are parallel (same slope)
diff. y intercept



(iii) Infinitely many solutions, when lines are coincident.
(Same slope and same y-intercept)



Finding Solutions of System of equations

(I) Geometric method. (graph)

↳ point of intersection.

(II) Algebraic method

↳ Substitution }

↳ Elimination }

① Find the solution to the system of equations graphically.

$$\begin{cases} y = 2x + 3 & \text{(I)} \\ y = -3x + 3 & \text{(II)} \end{cases}$$

H.W.

Systems of linear equation using substitution method.

eg. $y = 2x$ — eq. 1 → line

$x + y = 24$ — eq. 2 → line

$2x + 3y + z = x - 4$

We can substitute y of eq. 2 by $2x$ from eq. 1

eq. 2 will be written as.

$$x + 2x = 24$$

$$3x = 24$$

$$\boxed{x = 8}$$

Using $x = 8$ in eq. 1, we get

$$y = 2 \cdot 8$$

$$\boxed{y = 16}$$

$$\boxed{\begin{matrix} x = 8 \\ y = 16 \end{matrix}}$$

Solution $(8, 16)$.

Solve it for x & y using substitution.

eg2 $\quad 4x + y = 28$

$y = 3x$

eg3 $\quad -3x + y = -9$ — eq.1

$5x + 4y = 32$ — eq.2

$-3x + y = -9$

$+3x \qquad +3x$

$y = -9 + 3x$

Subsktuting $y = -9 + 3x$ in eq.2 we get

$5x + 4(-9 + 3x) = 32$

$5x + (-36) + 12x = 32$

$17x = 68$

$x = 4$

$\Rightarrow (4, 3)$

~~Putting~~ Substituting $x = 4$ in eq.2.

$5.4 + 4y = 32$

$4y = 12$

$y = 3$

(4)

$2x - 3y = -5$ — eq 1 ✓

$y = x - 1$ — eq. 2

Sub. $y = x - 1$ in eq. 1

$2x - 3(x-1) = -5$

$2x - 3x + 3 = -5$

$-x = -8$

$\boxed{x = 8}$

Subst. $x = 8$ in eq 2.

$y = 8 - 1$

$\boxed{y = 7}$

$(8, 7)$

(5)

$7x + 10y = 36$ — eq 1

$-2x + y = 9$ — eq 2

$x = -2$

$y = 5$

$(-2, 5)$

$7x + 10y = 36$

$y = \frac{36 - 7x}{10}$

$-2x + \frac{36 - 7x}{10} = 9$

$-20x + 36 - 7x = 90$

$-27x = 90 - 36$

$x = -2$

$-2.7x = 5.4$

$x = \frac{5.4}{-2.7}$

$= \frac{54}{-27}$

$= -2$

(6)

$-5x + 4y = 3$ ——— eq 1

$x - 2y = -15$ ——— eq 2

Sub. $\boxed{x = 2y - 15}$ in eq 1.

$-5(2y - 15) + 4y = 3$

$-10y + 75 + 4y = 3$

$-6y = -72$

$6y = 72$

$y = \frac{72}{6} = 12$

{Addition
Subtraction
Division}

Subst. $y = 12$ in eq 2.

$x - 2(12) = -15$

$x - 24 = -15$

$x = 24 - 15$

$\boxed{x = 9}$

$(9, 12)$

Elimination method :

$$-5x + 4y = 3 \quad \text{— eq 1}$$
$$x - 2y = -15 \quad \text{— eq 2}$$

To Eliminate y, multiply eq 2 by 2, we get.

$$2(x - 2y = -15) \times 2$$

$$2x - 4y = -30 \quad \text{— eq 3}$$

Adding eq 1 and eq 3

$$\begin{array}{r} -5x + 4y = 3 \\ +\ 2x - 4y = -30 \\ \hline -5x + 2x + 4y - 4y = 3 + (-30) \end{array}$$

$$-3x = 3 - 30$$

$$-3x = -27$$

$$x = 9$$

$$9 - 2y = -15$$

$$-2y = -15 - 9$$

$$+2y = +24$$

$$y = \frac{24}{2} = 12$$

$(9, 12)$

$5x - 7y = 15$ —— eq1) × 2

$2x - 2y = 6$ —— eq2) × 5

$10x - 14y = 30$ —— eq3

$10x - 10y = 30$ —— eq4

(−) (−) (+) (−)

$0 - 4y = 0$

$y = 0$

eq 2 :

$2x = 6$

$x = 3$

3, 0

Q. In 40 years, Kushaan will be 11 times as old as he is right now. How old he is right now.

Present age is x years

Age in 40 years = $(x + 40)$ years

$$x \cdot 11 = x + 40$$

$$11x = x + 40$$

$$10x = 40$$

$$x = 4 \text{ years}$$

Solve for x and y:

(1)

$$3x - 5y = -1 \quad \text{——— eq1}$$

$$x - y = -1 \quad \text{——— eq2}$$

eq2 x(3)

$$3x - 5y = -1$$

$$-3x + 3y = 3 \quad \text{——— (eq3)}$$

$$-5y + 3y = -1 + 3$$

$$-2y = 2$$

$$y = -1$$

Putting $y = -1$ in eq 2

$$x - (-1) = -1$$

$$x + 1 = -1$$

$$x = -1 - 1$$

$$x = -2$$

$$(-2, -1)$$

Substitution

(11) $2x + 3y = 9$ — eq1

$3x + 4y = 5$ — eq2

eq:1

$2x + 3y = 9$

$3y = 9 - 2x$

$$\boxed{y = 3 - \frac{2}{3}x}$$

Substitute $y = 3 - \frac{2x}{3}$ in eq 2, we get

$$3x + 4\left(3 - \frac{2}{3}x\right) = 5$$

$$3x + 12 - \frac{8x}{3} = 5$$

$$3x - \frac{8x}{3} = 5 - 12$$

$$\frac{9x - 8x}{3} = -7$$

$$\frac{x}{3} = -7$$

$$\boxed{x = -21}$$

Putting $x = -21$ in eq.1, we get

$2(-21) + 3y = 9$

$-42 + 3y = 9$

$3y = 9 + 42$

$3y = 51$

$\boxed{y = 17}$

$(-21, 17)$

Elimination

(iii) $8x + 5y = 9$ — eq 1

$3x + 2y = 4$ — eq 2

(iv) $0.5x + 0.7y = 0.74$ — eq.1 $\times 100$

$0.3x + 0.5y = 0.5$ — eq.2 $\times 100$

$50x + 70y = 74$ — (3) $\times 30$

$30x + 50y = 50$ — (4) $\times 50$

$1500x + 2100y = 2220$

$-1500x \pm 2500y = (-)2500$

$-400y = -280$

$y = \frac{280}{400} = \frac{28}{40} = \frac{7}{10} = \boxed{0.7}$

Putting $y = \frac{7}{10}$ in eq 3

$50x + 70\left(\frac{7}{10}\right) = 74$

$50x + 49 = 74$

$50x = 74 - 49$

$50x = 25$

$x = \frac{25}{50} = \frac{1}{2} = \boxed{0.5}$

$(0.5, 0.7)$

Solve for x and y

(v)

$$ax + by = 5$$

$$2ax - 3by = 7$$

eq① × 3

eq②

$$3ax + 3by = 15$$

$$+ \quad 2ax - 3by = 7$$

$$3ax + 2ax = 15 + 7$$

$$5ax = 22$$

$$\boxed{x = \frac{22}{5a}}$$

Putting $x = \frac{22}{5a}$ in eq①

$$\frac{22}{5} + by = 5$$

$$by = 5 - \frac{22}{5}$$

$$by = \frac{25 - 22}{5}$$

$$by = \frac{3}{5}$$

$$y = \frac{\left(\frac{3}{5}\right)}{b}$$

$$\boxed{y = \frac{3}{5b}}$$

$$\frac{3}{5} \div \frac{b}{1}$$

$$\frac{3}{5} \times \frac{1}{b}$$

$$\frac{3}{5b}$$

$$\left(\frac{22}{5a}, \frac{3}{5b}\right)$$

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$

$\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$

$\sqrt{2} + \sqrt{3} =$

$\sqrt{2} + \sqrt{2} =$

$1a + 1a = 2a$

$1(\sqrt{2}) + 1(\sqrt{2}) = 2\sqrt{2}$

$a + b = x$

$\sqrt{2} + \sqrt{3} = x$

$3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

$3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$

(11)

$\sqrt{2}x - \sqrt{3}y = 0$ eq1

$\sqrt{3}x - \sqrt{8}y = 0$ eq2

$\sqrt{2}x - 0 = 0$

$\sqrt{2}x = 0$

$x = \frac{0}{\sqrt{2}}$

$\boxed{x = 0}$

$-\sqrt{9}y + \sqrt{11}y$

$-3y \neq 4y$

$\boxed{y = 0}$

$(0, 0)$

(VII) $11x + 15y + 23 = 0$ — eq. ①

$7x - 2y - 20 = 0$ — eq. ②

$11x + 15y = -23$

$7x - 2y = 20$

$77x + 105y = -161$

$(-)\ 77x \mp 22y = (-)\ 220$

$127y = -381$

$y = \frac{-381}{127} = \boxed{-3}$ ✓

$7x - 2y = 20$

$7x - 2(-3) = 20$

$\boxed{7x + 6 = 20}$

$7x = 14$

$\boxed{x = 2}$

$(2, -3)$

Q. William is 4 times as old as Ben. 12 years ago, william was 7 times as old as Ben. How old is Ben now?

Sol^n: Let us assume that present age of Ben be = b years

∴ William's present age = 4b years

12 years ago,

Ben's age = (b − 12) years

William's age = (4b − 12) years

As per the problem statement,

$$(4b-12) = 7(b-12)$$

$$4b-12 = 7b-84$$

$$4b-7b = -84+12$$

$$-3b = -72$$

$$+3b = +72$$

$$3b = 72$$

$$b = \frac{72}{3}$$

$$\boxed{b = 24 \text{ years}}$$

Hence, Ben's present age is 24 years.

Q. Present age of Arman and Diya is 18 years and 2 years respectively. How many years will it take for Arman to be 3 times as old as Diya?

Let, it will take x years for Arman to be 3 times as old as Diya.

Present age,

Arman = 18

Diya = 2

Age after x years,

Arman's = $18 + x$

Diya = $2 + x$

As per the problem,

$$18 + x = 3(2 + x)$$

$$18 + x = 6 + 3x$$

$$-2x = -12$$

$$2x = 12$$

$$\boxed{x = 6 \text{ years}}$$

Hence, in 6 years Arman's age will be 3 time as old as Diya.

Q. Ishaan is 2 times as old as Kushaan. 35 years ago, Ishaan was 7 times as old as Kushaan. How old is Kushaan now?

11,

42 years.

Word problems based of systems of equation.

Q. 4 chairs and 3 tables costs ₹ 2100. and 5 chairs and 2 tables costs ₹ 1750. Find the cost of a table and a chair separately.

cost of 1 table is ₹ x

cost of 1 chair is ₹ y

$4y + 3x = 2100$ ——— eq1

$5y + 2x = 1750$ ——— eq2

② The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, he ~~buys~~ buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

$7x + 6y = 3800 \quad \times 3$

$3x + 5y = 1750 \quad \times 7$

$21x + 18y = 11400$

$\ominus \quad 21x + 35y = 12250$

$-17y = -850$

$y = \frac{850}{17}$

$y = 50$

$7x + 300 = 3800$

$7x = 3500$

$x = 500$

③ A fraction becomes 4/5, if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator & denominator, the fraction becomes 1/2. What is the fraction?

Numerator = x

Denominator = y

Fraction = $\frac{x}{y}$

$$\frac{x+1}{y+1} = \frac{4}{5}$$

$$5(x+1) = 4(y+1)$$

$$5x+5 = 4y+4$$

$$\boxed{5x-4y = -1} \quad \text{——} (1)$$

$$\frac{x-5}{y-5} = \frac{1}{2}$$

$$2(x-5) = 1(y-5)$$

$$2x-10 = y-5$$

$$\boxed{2x-y = 5} \quad \text{——} (2)$$

$$5x - 4y = -1$$
$$2x - y = 5$$

$$\boxed{\frac{7}{9}}$$

If twice the son's age in years is added to father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.

Son's age = s years

father's age = f years

$2s + f = 70$

$(s + 2f = 95)$ $\times 2$

$2s + 4f = 190$

$2s + f = 70$

$3f = 120$

$f = 40$ years.

$s = 15$ years

Q. Father is three times as old as his son. Five years later, he will be two & and half times as old as his son. How old is the father and his son?

Father's age = x years

Son's age = y years.

$x = 3y$ ——— (1)

$x - 3y = 0$

Five years later,

Father's age = $(x+5)$

Son's age = $(y+5)$

$2\frac{1}{2}$

$(x+5) = \frac{5}{2}(y+5)$

$(x+5)2 = 5(y+5)$

$2x + 10 = 5y + 25$

$2x - 5y = 15$ ——— (II)

Using (1)

$2(3y) - 5y = 15$

$6y - 5y = 15$

$y = 15$ years.

from (1):

$x = 3y$

$x = 3 \times 15$

$= 45$ years.

Q. Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Father's present age = x years

Son's ——— = y years.

5 years later:

Father's age = $(x+5)$ ✓

Son's age = $(y+5)$ ✓

$$(x+5) = 3(y+5)$$

$$x+5 = 3y+15$$

$$\boxed{x-3y = 10} \quad \text{—— (i)}$$

5 years ago:

Father = $(x-5)$

Son = $(y-5)$

$$x-5 = 7(y-5)$$

$$x-5 = 7y-35$$

$$\boxed{x-7y = -30} \quad \text{—— (ii)}$$

$$\boxed{x = 40 \text{ years}}$$

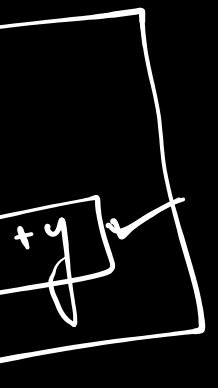
$$\boxed{y = 10 \text{ years}}$$

$\overline{40} = 40 + 2$

$\overline{xy} =$

Do two digit number $\overline{12}$, $\overline{42}$,

$10 + 2$ $\quad$ $40 + 2$



Q. In a two digit number, the unit digit is twice the ten's digit. If 27 is added to the number, the digit interchange their places. Find the number.

digit at ten's place is x

digit at unit place = y

number = $10x + y$

$y = 2x$ — (1)

number + 27 = new number

$(10x + y) + 27 = 10y + x$

$9x - 9y = -27$ — (11)

$9x - 9(2x) = -27$

$9x - 18x = -27$

$+9x = +27$

$x = 3$

$y = 2x$

$y = 2 \cdot 3$

$y = 6$

Number = $10x + y = 10 \cdot 3 + 6 = 36$

$24 + 18 = 42$

3×10

2

$x \cdot y$

(y is at ten's place & x is unit place)

Q. In a two digit number, the 10's digit is three time the unit's digit. When the number is decreased by 54, the digits are reversed. Find the number.

ten's digit $= x$

unit's digit $= y$

number $= 10x + y$ ✓

$x = 3y$ — (I)

number $= 54$

$(10x + y) - 54 = 10y + x$

$9x - 9y = 54$ — (II)

$9(3y) - 9y = 54$

$27y - 9y = 54$

$18y = 54$

$y = 3$

$x = 3(y)$

$x = 3 \cdot 3$

$x = 9$

Number $= 9 \cdot 10 + 3$

$= 93$

Q. The sum of a two digit number and the number formed by interchanging the digit is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

ten's place = x

unit's place = y

Number = $10x+y$ ✓

$(10x+y)+(10y+x)=132$

$11x+11y=132$ — (I)

$10x+y+12=5(x+y)$

$10x-5x+y-5y=-12$

$5x-4y=-12$ — (II)

Solve:

$$\frac{3y}{20} - \frac{x}{20} = 7 \quad \text{——— (I)}$$

$$\frac{x}{20} + \frac{y}{10} = 13 \quad \text{——— (II)}$$

$$\left[\frac{3y}{20} - \frac{x}{20} = 7\right] \times 20$$

$$3y - x = 140 \quad \text{——— (III)}$$

$$\frac{(3y - x)}{20} = \frac{7}{1}$$

$$(3y - x) = 7 \times 20$$

$$\frac{x}{20} + \frac{y}{10} = 13$$

$$\frac{x + 2y}{20} = 13$$

$$x + 2y = 13 \times 20$$

$$\boxed{x + 2y = 260} \quad \text{(IV)}$$

$$3y - x = 140 \quad \text{(III)}$$

$$2y + x = 260$$

$$+ \quad 3y - x = 140$$

$$5y = 400$$

$$y = \frac{400}{5}$$

$$\boxed{y = 80}$$

$$x + 2 \cdot 80 = 260$$

$$x + 160 = 260$$

$$\boxed{x = 100}$$

Solve:

$$\frac{x}{5} + \frac{2y}{6} = 9 \rightarrow \frac{6x + 10y}{30} = 9$$

$$6x + 10y = 270 \quad \text{(III)} \times 4$$

$$\frac{x}{3} - \frac{3y}{4} = 10$$

$$\frac{4x - 9y}{12} = 10$$

$$4x - 9y = 120 \quad \text{(IV)} \times 6$$

$$24x + 40y = 1080$$

$$(-)\ 24x \ (+)\ 54y = (-)\ 720$$

$$94y = 360$$

$$y = \frac{360}{94} = \frac{180}{47}$$

$$y = \frac{180}{47}$$

$$6x + 10 \times \frac{360}{94} = 270$$

$$6x = 270 - \frac{5360}{47}$$

$$= 270 - \frac{1800}{47}$$

$$6x = \frac{270 \times 47 - 1800}{47}$$

$$= \frac{12690 - 1800}{47}$$

$$6x = \frac{10890}{47}$$

$$x = \frac{10890}{47 \times 6}$$

$$= \frac{10890}{282}$$

$$x = \frac{1815}{47}$$

Taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is ₹75 and for a journey of 15 km the charge paid is ₹110. What will a person have to pay for travelling a distance of 25 km?

fixed charge + running charge

Let the fixed charge of taxi be ₹ y and the running charge be ₹ x per km

As per the condition given in the problem,

(i) for 10 km journey total amt. paid = ₹ 75

$$y + 10x = 75 \quad \text{---(I)}$$

(ii) for 15 km journey

$$y + 15x = 110 \quad \text{---(II)}$$

$5x = 35$

$x = 7$

$y + 70 = 75$

$y = 5$

$\therefore$ Fixed charge : ₹5

Running charge = ₹7

Charge for 25 km $\Rightarrow$ ₹(5 + 25 × 7)

= ₹(5 + 175)

= ₹ 180

Father's age is three times the sum of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

- Let father's age be x years, and
  - Sum of two children's age be y years.
- According to the condition given in the question.

(i) $\boxed{x = 3y}$ ——— (1)

- Five years later,
  - Father's age will be $(x+5)$ years.
  - Sum of children's age be $(y+10)$ years.

$$(x+5) = 2(y+10)$$

$$\boxed{x - 2y = 15} \quad \text{——— (ii)}$$

8	+5
13	+5

21

5 years

26

$\boxed{\text{Father's age} = 45 \text{ years}}$

End of the chapter