Lines and Angles

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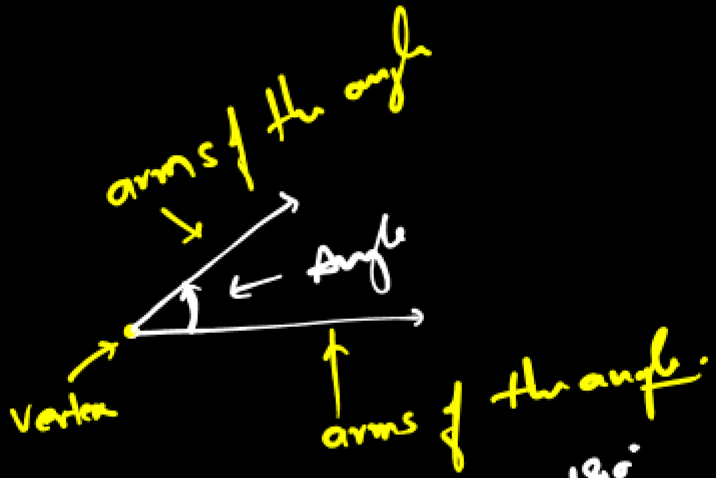
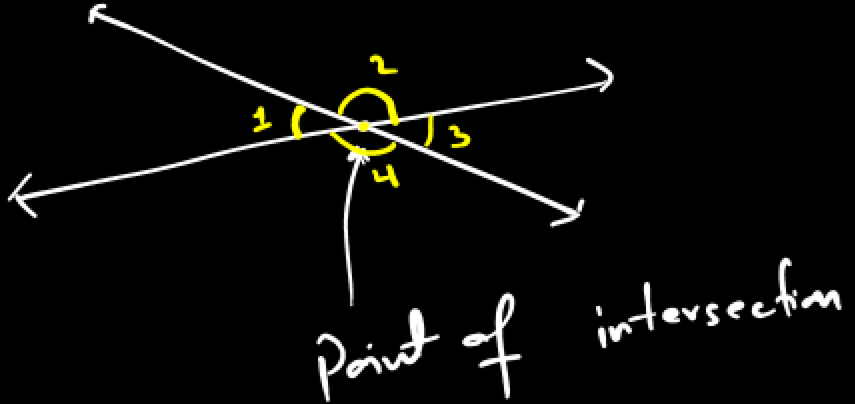
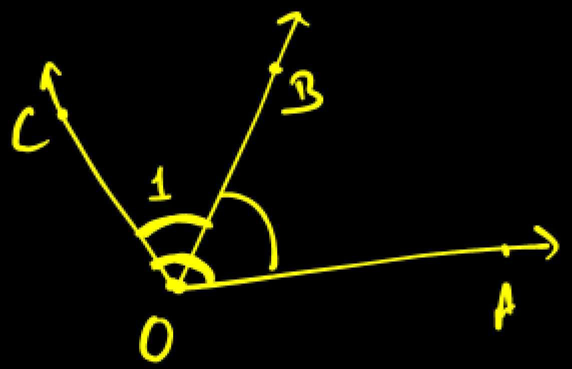




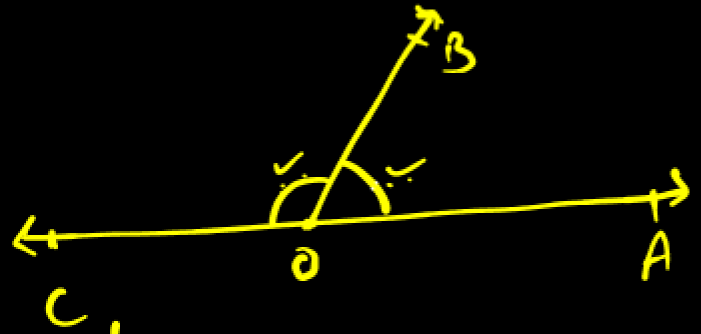
Pairs of Angles

(i) Adjacent angles

$\angle COB$ and $\angle BOA$ are adjacent



(ii) Linear Pair:

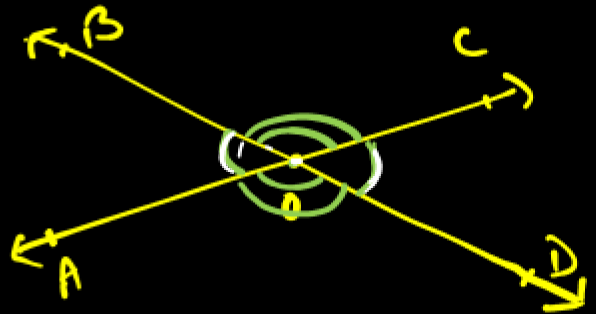


Two adjacent angles are said to form a linear pair, if their non-common arms are two opposite rays.

$$\boxed{\angle COB + \angle BOA = 180^\circ}$$

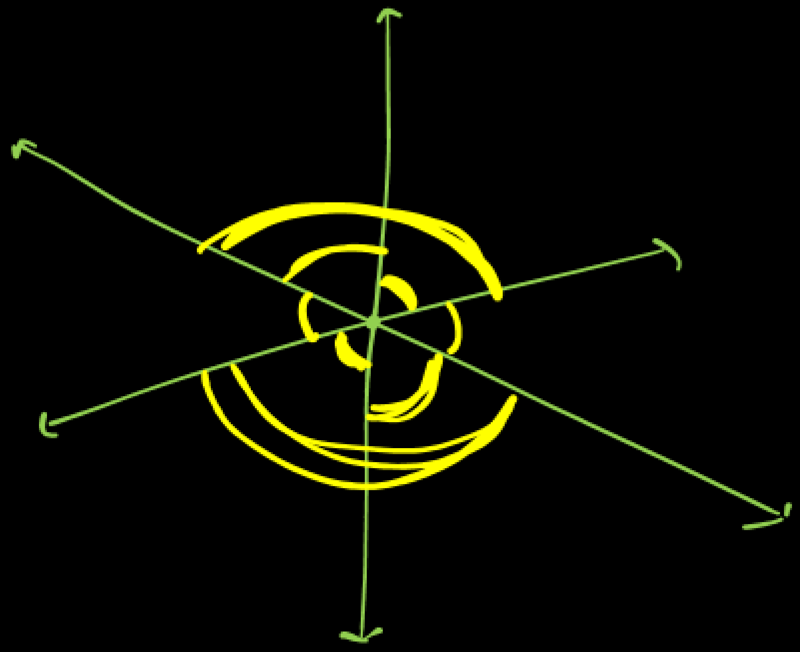
(14) Vertically opposite angles

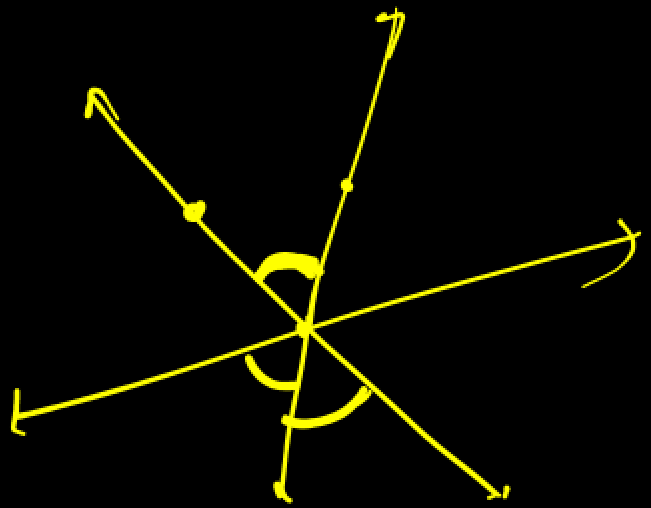
Two angles formed by two intersecting lines having no common arm are called vertically opposite angles

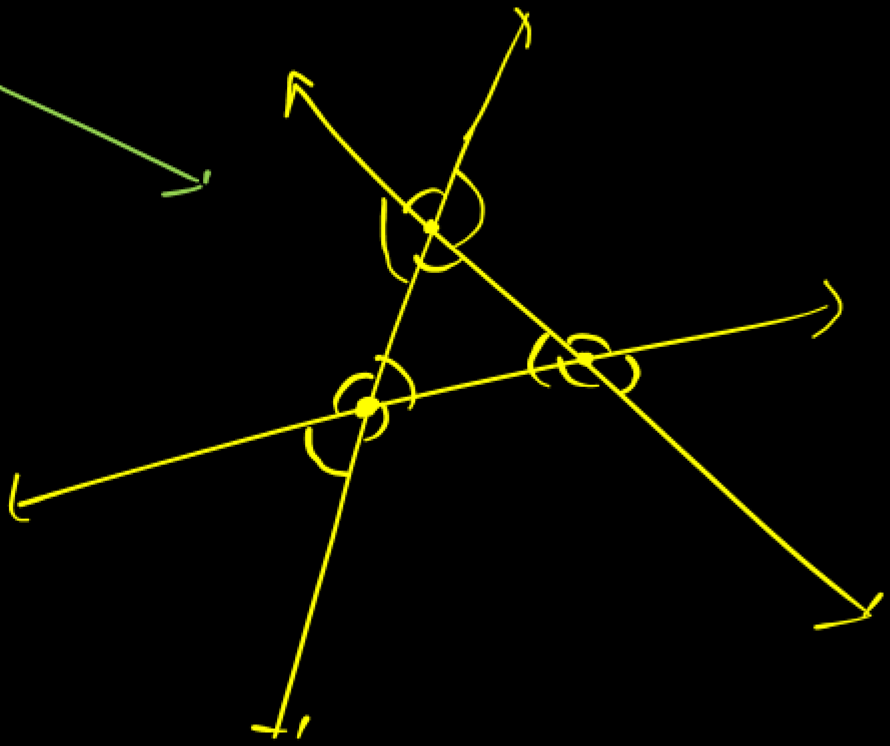


eg: • $\angle AOB$ and $\angle COD$ are vertically opposite.

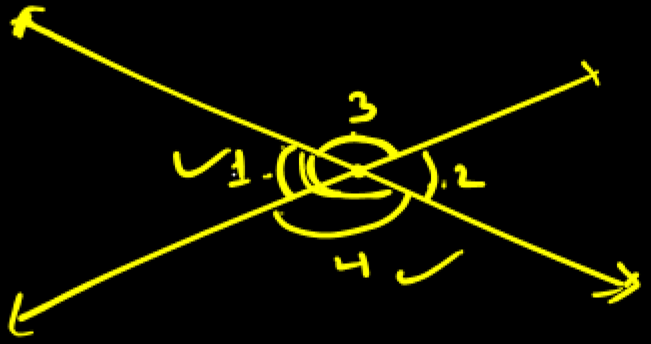
• $\angle BOC$ and $\angle AOD$ are also vertically opposite.







* measure of pair of $\angle$ vertically opposite angles are ~~at~~ always equal.



$$\boxed{\angle 1 = \angle 2}$$

$\angle 1 + \angle 3 = 180^\circ$ (Linear pair)

$$\boxed{\angle 1 = 180^\circ - \angle 3}$$

$\angle 1 + \angle 4 = 180^\circ$

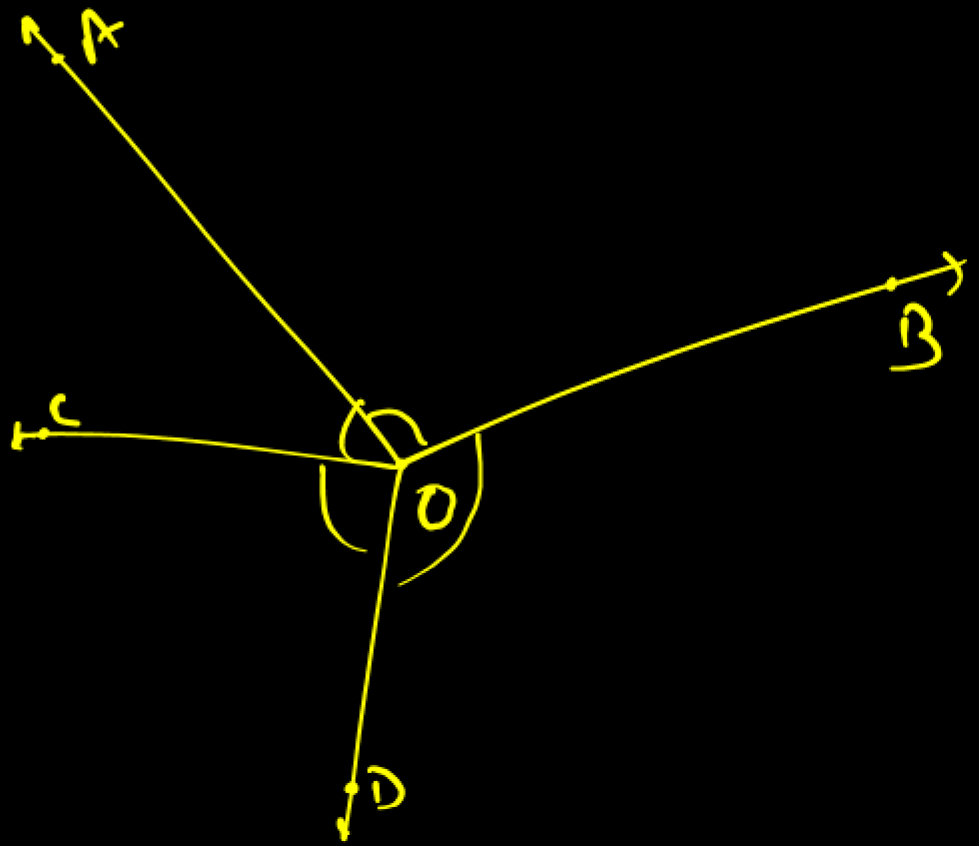
$$\boxed{\angle 1 = 180^\circ - \angle 4} \quad \text{(ii)}$$

$\angle 2 + \angle 4 = 180^\circ$

$$\boxed{\angle 2 = 180^\circ - \angle 4} \quad \text{(i)}$$

from eq. (i) and eq. (ii)

$$\boxed{\angle 2 = \angle 1}$$

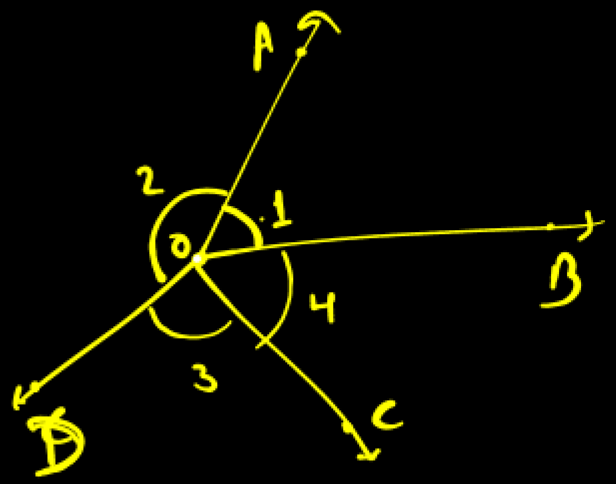


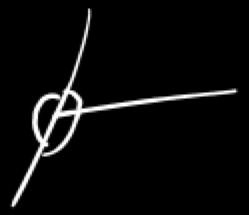
Non of the angles are vertically opposite.

Angles at a point :

Angles formed by a number of rays having common initial point.

$$\boxed{\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ}$$





Complementary angles

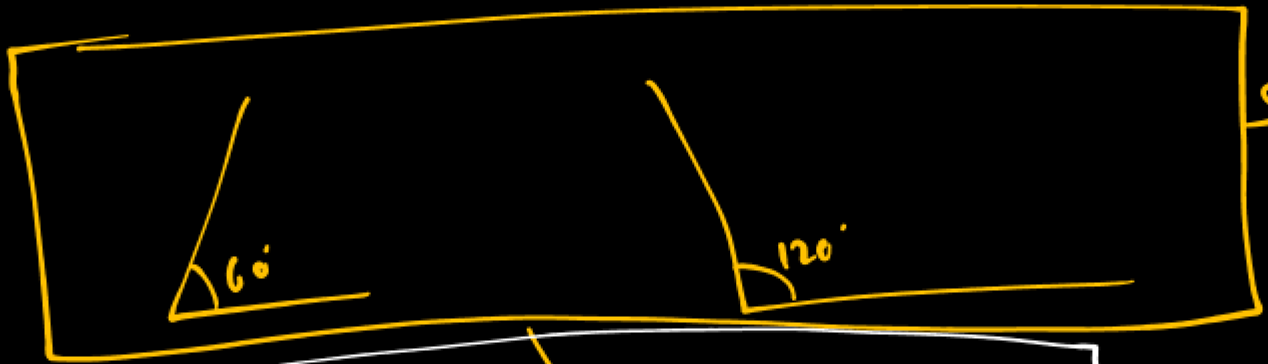
- If the sum of the measures of two angles is $\underline{\underline{90^\circ}}$, then the angles are called complementary angles, and each angle is called complement of other.

eg. ① $\underline{35^\circ}$ and $\underline{55^\circ}$ are complementary angles.

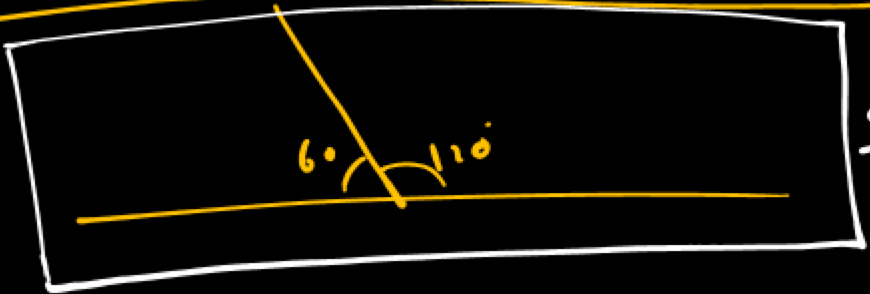
- 55° is complement of 35°.

Supplementary angle.

Two angles are said to be supplementary angles if the sum of their measures is 180°, and each of them is called supplement of the other.



Supplementary angles.



Supplementary angles.

Q. Two supplementary angles differ by 34°. Find the angles.

Sol.

one angle $= x^\circ$

other angle $= (x+34)^\circ$

$\because$ these angles are supplementary,

$\therefore \quad x^\circ + (x+34)^\circ = 180^\circ$

$\Rightarrow x^\circ + x^\circ + 34^\circ = 180^\circ$

$\Rightarrow 2x^\circ + 34^\circ = 180^\circ$

$2x = 180 - 34$

$2x = 146$

$x = \frac{146}{2} = 73^\circ$

one angle $= 73^\circ$

other angle $= 73^\circ + 34^\circ$

$= 107^\circ$

$\therefore$ therefore

$\because$ since/because

Q. An angle is equal to five times its complement. Find the measure of the angle.

Sol. Let the measure of angle $= x^\circ$.

Then the complement of $x^\circ = (90 - x)^\circ$

As per condition given in the problem.

$$x = 5(90 - x)$$

$$x = 450 - 5x$$

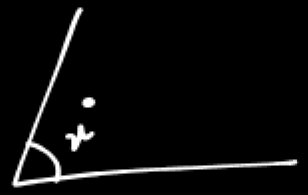
$$+5x \qquad +5x$$

$$\boxed{6x = 450}$$

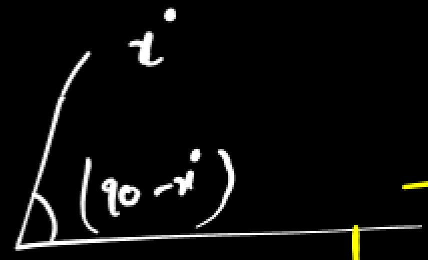
$$x = \frac{450}{6} = 75$$

$$\frac{\overset{7}{4\overset{3}{5}0}}{6}$$

Hence, the measure of req. angle is 75°



$\therefore (90-x)^\circ$ is complement of



$$2x + 9 = 15 - x$$

$$2x + x = 15 - 9$$

$$3x = 6$$

$$\boxed{x = 2}$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$\boxed{x = 5}$$

$$x - 3 = 7$$

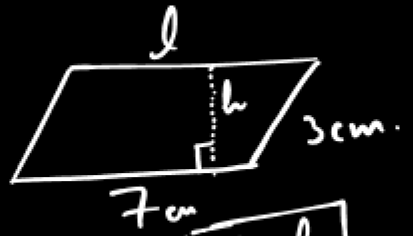
$$x = 7 + 3 = 10$$

$$\frac{450}{6} = 75$$

$$\frac{850}{5} = 170$$

$l \times b$

$7 \times 3 \ cm^2$

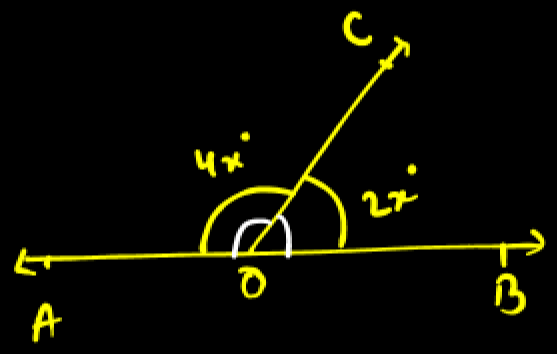


$\boxed{l \times h}$

$5 \overline{)850} ($

Q. $\angle AOC$ and $\angle COB$ forms a linear pair as shown in the fig. Find x.

linear pair $\Rightarrow 180^\circ$



Sol: $\because \angle AOC$ and $\angle COB$ are linear pair.

$\therefore \angle AOC + \angle COB = 180^\circ$

$4x + 2x = 180^\circ$

$6x = 180^\circ$

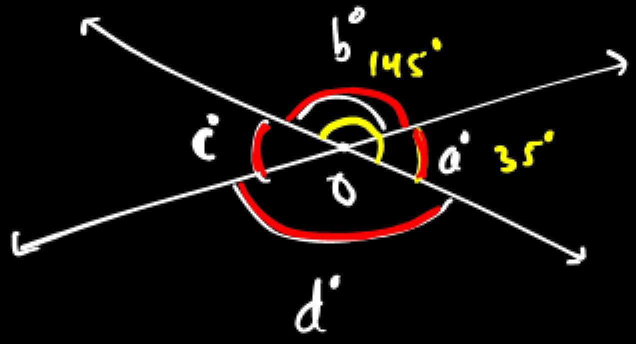
$x = \frac{180}{6}$

$\boxed{x = 30^\circ}$

$\therefore$ Therefore

$\because$ Since / because

3 - Two lines intersecting at O. If angle a = 35°, find the values of b, c and d.



Sol:

$\because$ $\angle a$ and $\angle b$ are linear pair.

$\therefore$ $\angle a + \angle b = 180°$

$35 + \angle b = 180°$

$\angle b = 180° - 35°$

$= \underline{\underline{145°}}$

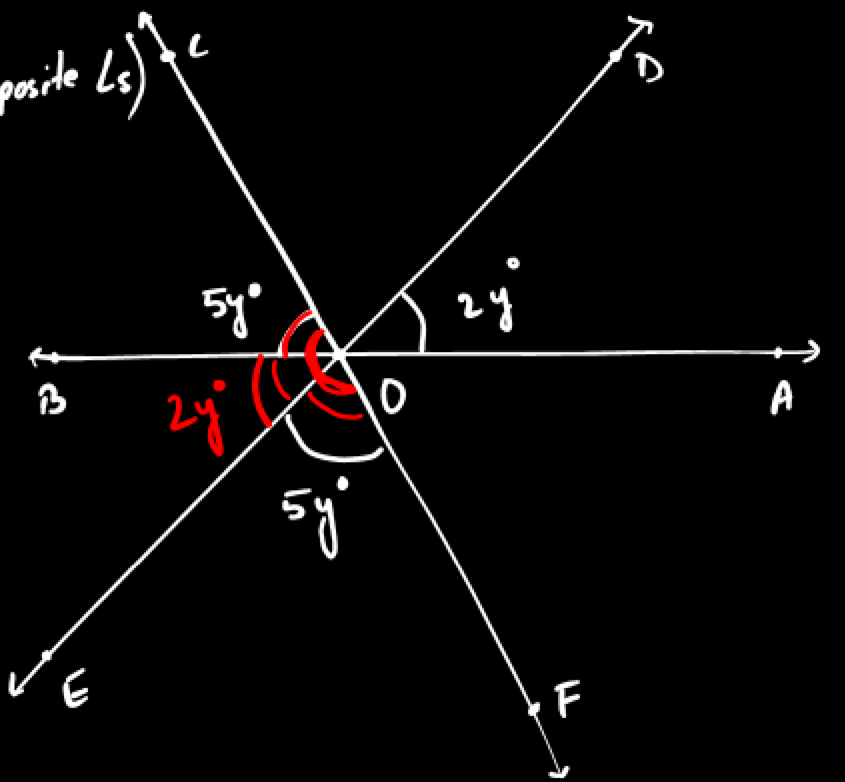
$\because$ $\angle a$ and $\angle c$ are vertically opposite angles

$\therefore$ $\angle c = \angle a = 35°$

$\because$ $\angle b$ and $\angle d$ are vertically angles.

$\therefore$ $\angle d = \underline{\underline{145°}}$.

Q. Find y in the given fig.



Sol:

$\because \angle AOD = \angle BOE$ (vertically opposite $\angle$s)

$\therefore \angle BOE = 2y^\circ$

Also, OC and OF are opposite rays.

$\therefore \angle COB + \angle BOE + \angle EOF = 180^\circ$

$\Rightarrow 5y^\circ + 2y^\circ + 5y^\circ = 180^\circ$

$\Rightarrow 12y^\circ = 180^\circ$

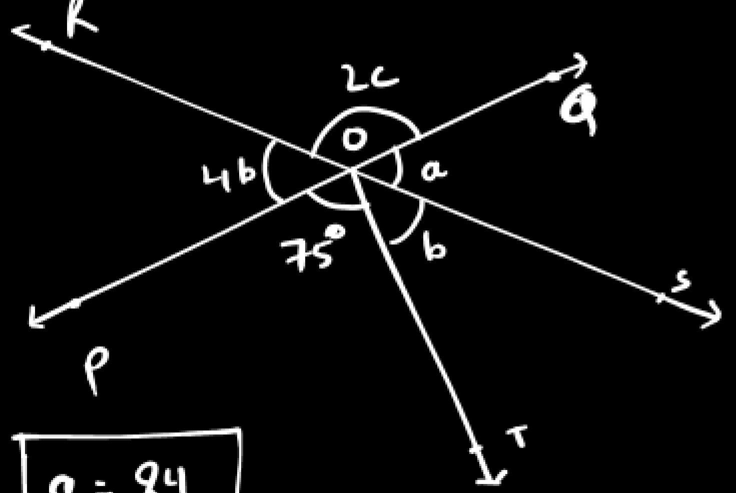
$\Rightarrow y = \dfrac{\cancel{180}^{\cancel{30}\ 15}}{\cancel{12}_{\cancel{2}}}$

$\Rightarrow \boxed{y = 15}$

angles ($\Rightarrow \angle$s)

$\boxed{\text{Hence } y = 15.}$

Q. In the given fig. find the values of a, b and c.



Sol: Since, OR and OS are opposite rays

$\therefore \angle ROP + \angle POT + \angle TOS = 180°$

$\Rightarrow 4b + 75° + b = 180°$

$\Rightarrow 5b + 75° = 180°$

$\Rightarrow 5b = 180° - 75$

$\Rightarrow 5b = 105°$

$\Rightarrow b = \frac{105°}{5} = 21$

$$\boxed{b = 21}$$

Since, $\angle QOS = \angle POR$ (Vert. Opp. $\angle$s)

$\Rightarrow a = 4b$

$\Rightarrow a = 4 \times 21$

$\Rightarrow \boxed{a = 84}$

Since, $\angle ROQ = \angle POS$ (Vert. opp. $\angle$s)

$\Rightarrow 2c = 75 + b$

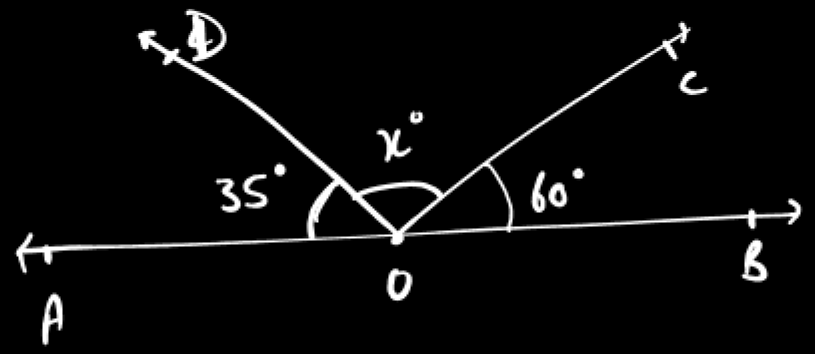
$\Rightarrow 2c = 75 + 21$

$\Rightarrow 2c = 96$

$\Rightarrow c = \frac{96}{2} = 48$

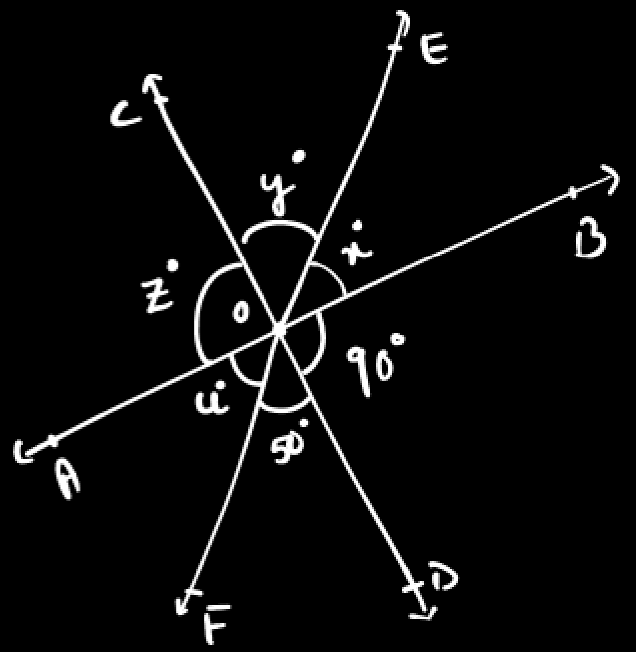
Hence, $a = 84$, $b = 21$ and $c = 48$.

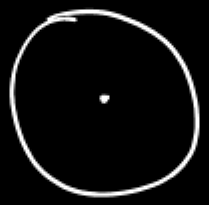
Find x



Find x, y, z and u

$\boxed{\text{H.W.}}$

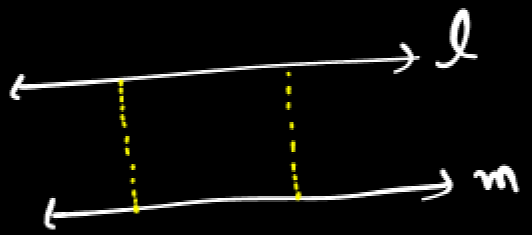




Parallel lines

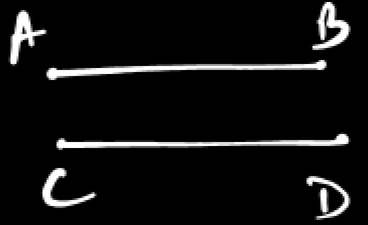
Two lines l and m in the same plane are said to be ~~pall~~ parallel if l and m never intersects.

$\Rightarrow$ $\underbrace{l \text{ is parallel to } m}$



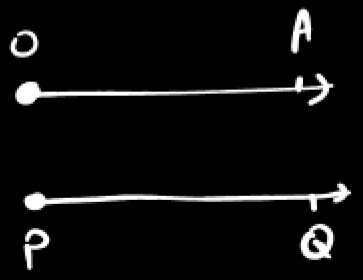
Symbol $\Rightarrow$ $l \parallel m$] Symbolic representation

$\parallel$: is parallel to.



$AB \parallel CD$

Parallel line segments



Parallel rays.

$\overrightarrow{OA} \parallel \overrightarrow{PQ}$

$\boxed{\dot{3}}$ ✗

Non terminating recurring decimal nos. $\rightarrow \boxed{\frac{1}{3}}$

$0.333\ldots\ldots$

$\boxed{0.\dot{3}}$

$\frac{10}{3}$

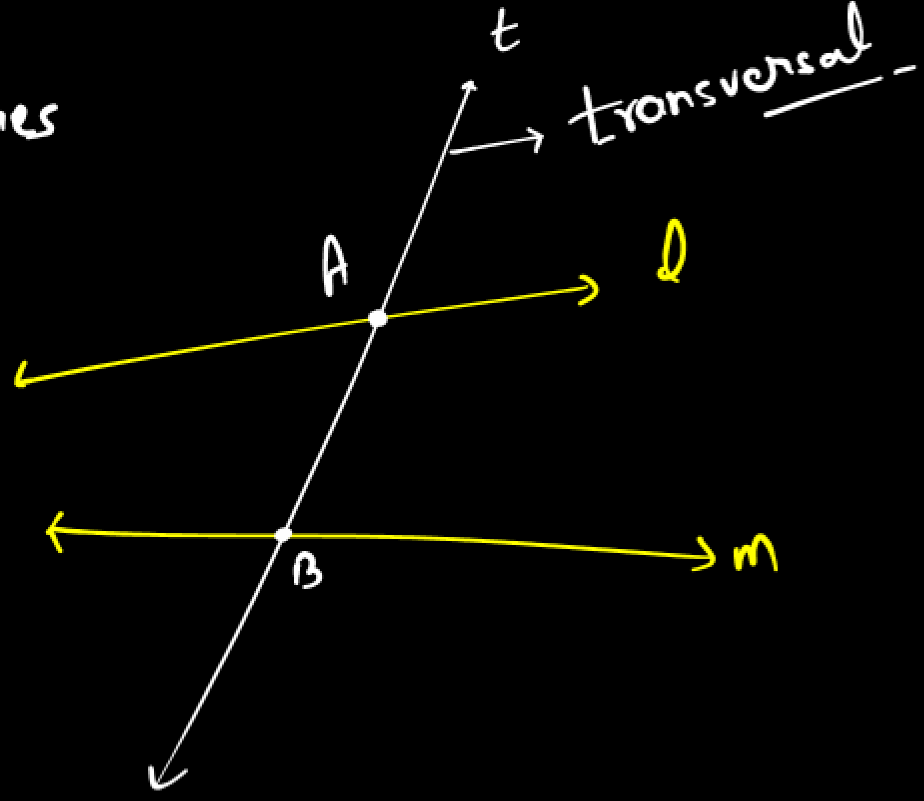
$0.\bar{3}$

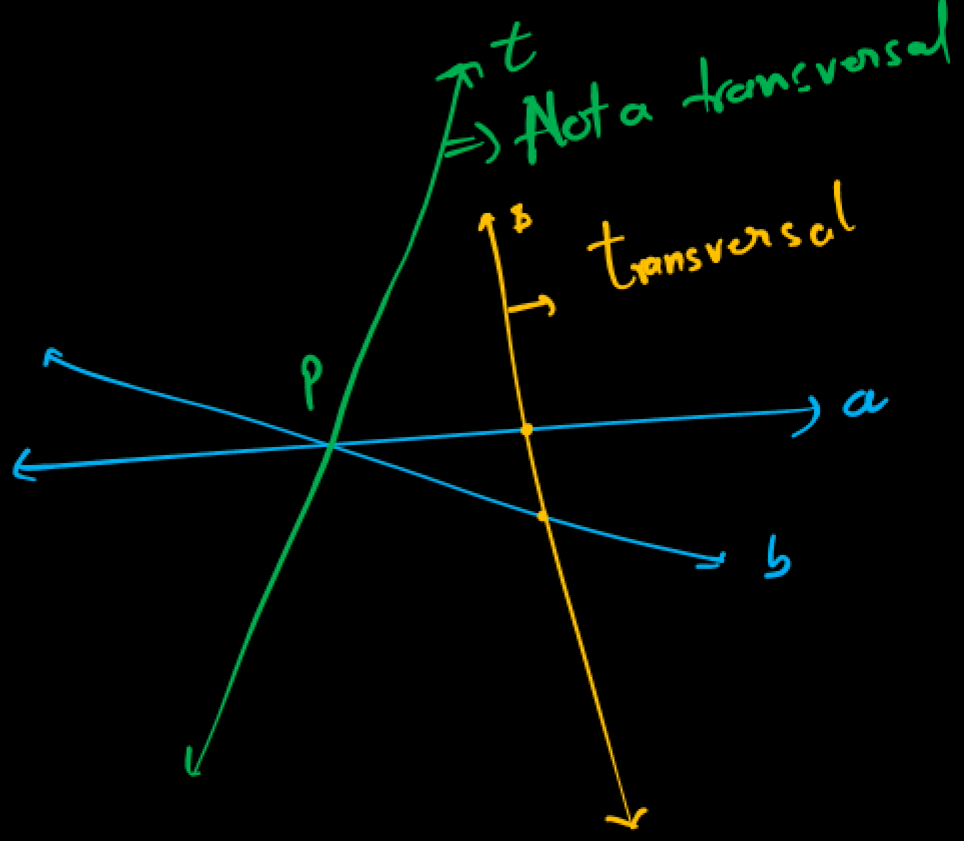
$\boxed{\boxed{\dot{3}}}$

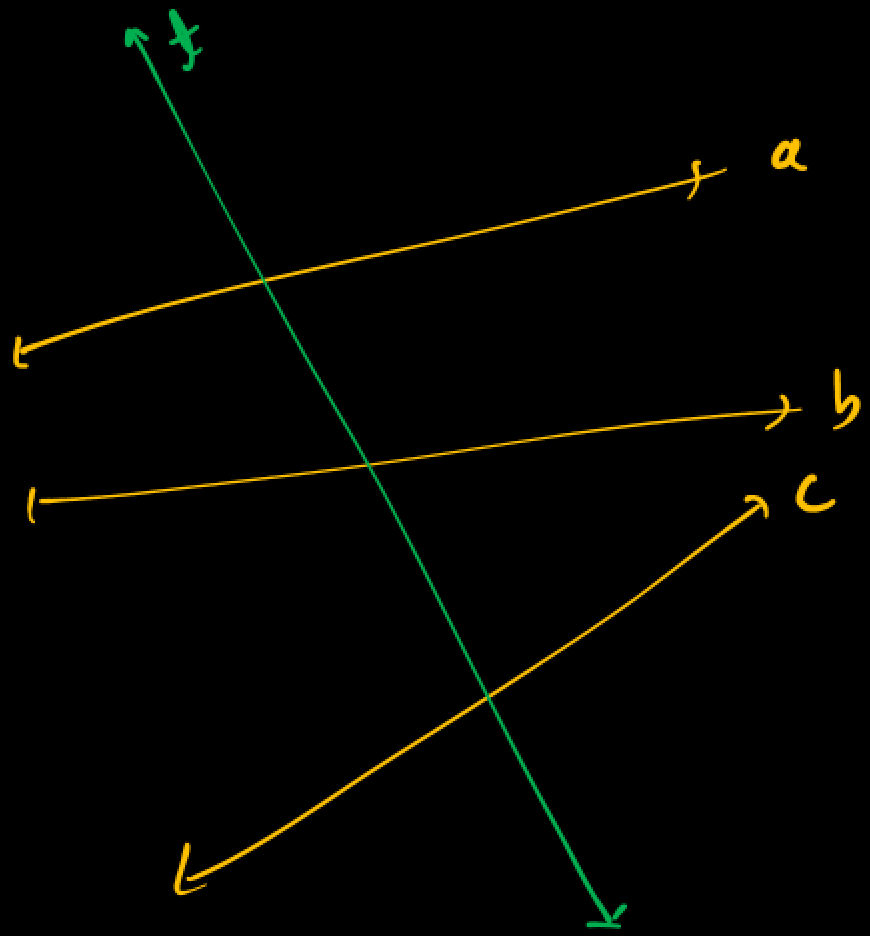
33333

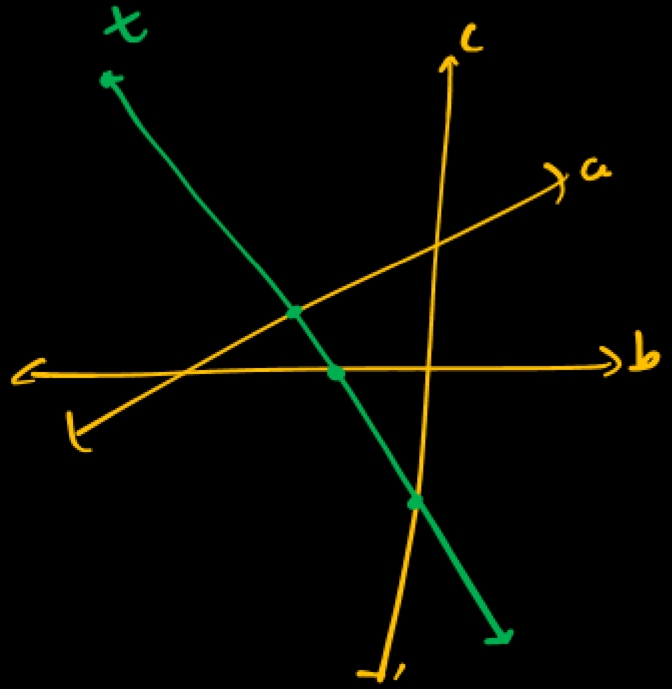
Transversal :

⇒ A line intersecting two or more lines in a plane at distinct points.

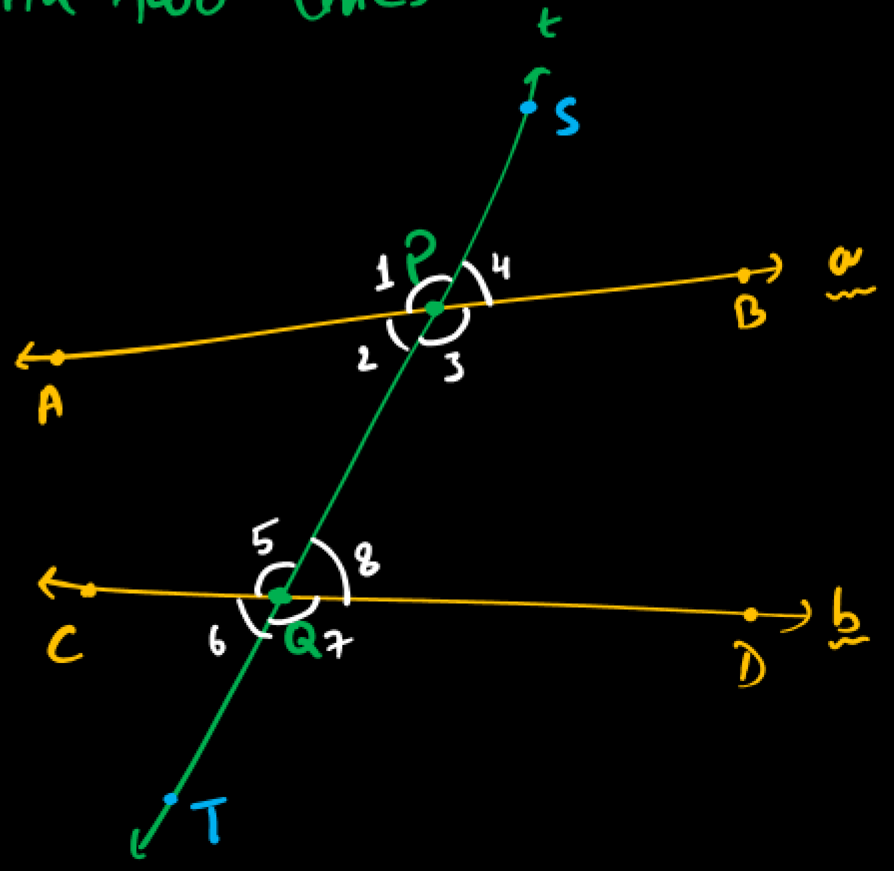





t
a
b
c


t
c
a
b

Angles made by a transversal with two lines.



① **Exterior angles:**

$\angle 1$, $\angle 4$, $\angle 6$ and $\angle 7$ are exterior angles.

② **Interior angles:**

$\angle 2$, $\angle 3$, $\angle 5$ and $\angle 8$ are interior angles

③ **Corresponding angles (Pair)**

$\angle 1$ and $\angle 5$
$\angle 4$ and $\angle 8$
$\angle 2$ and $\angle 6$
$\angle 3$ and $\angle 7$
} Pair of corresponding angles.

(4) Alternate interior angles (Pair)

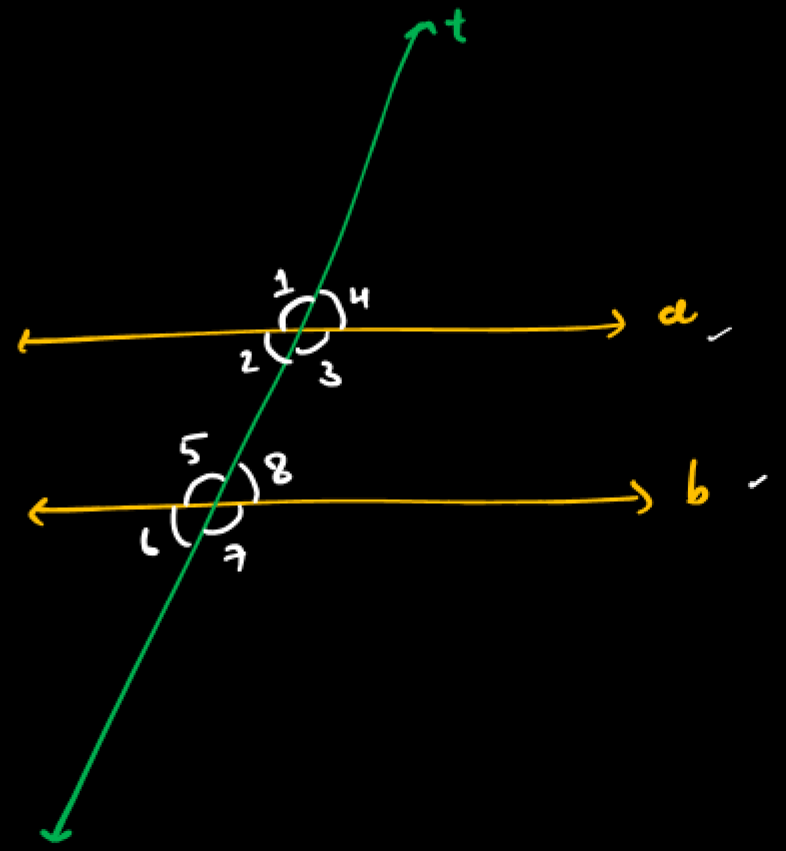
$\angle 2$ & $\angle 8$ } Pair of alternate interior angles
$\angle 3$ & $\angle 5$ }

(5) Alternate exterior angles (Pair)

$\angle 1$ and $\angle 7$ } Pair of alternate exterior angles.
$\angle 6$ and $\angle 4$ }

If the lines (a and b) are parallel

$a \parallel b$



Property 1: Pairs of corresponding angles are equal.

$\angle 1 = \angle 5$ | $\angle 2 = \angle 6$

$\angle 4 = \angle 8$ | $\angle 3 = \angle 7$

Property 2: Pairs of alternate angles are equal.

$\underbrace{\angle 2 = \angle 8,\ \angle 3 = \angle 5}_{\text{alternate interior angles}}$, $\underbrace{\angle 1 = \angle 7,\ \angle 4 = \angle 6}_{\text{alternate exterior angles}}$

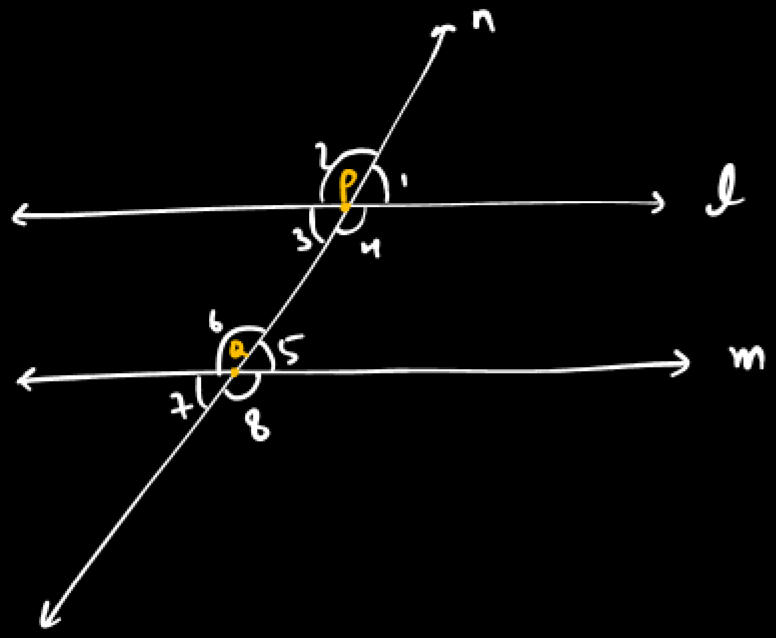
Property 3: Sum of interior (or exterior) angles on the same side of the transversal is 180°.

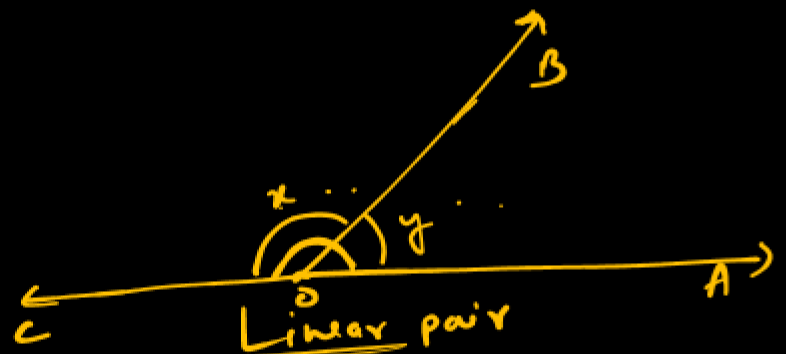
$\underbrace{\angle 2 + \angle 5 = 180^\circ \ ; \ \angle 3 + \angle 8 = 180^\circ}_{\text{Interior angles on the same side of transversal}}$; $\underbrace{\angle 1 + \angle 6 = 180^\circ \ ; \ \angle 4 + \angle 7 = 180^\circ}_{\text{exterior angles}}$

1. Given:

$l \parallel m$, $\angle 1 = 40^\circ$

Find remaining angles.



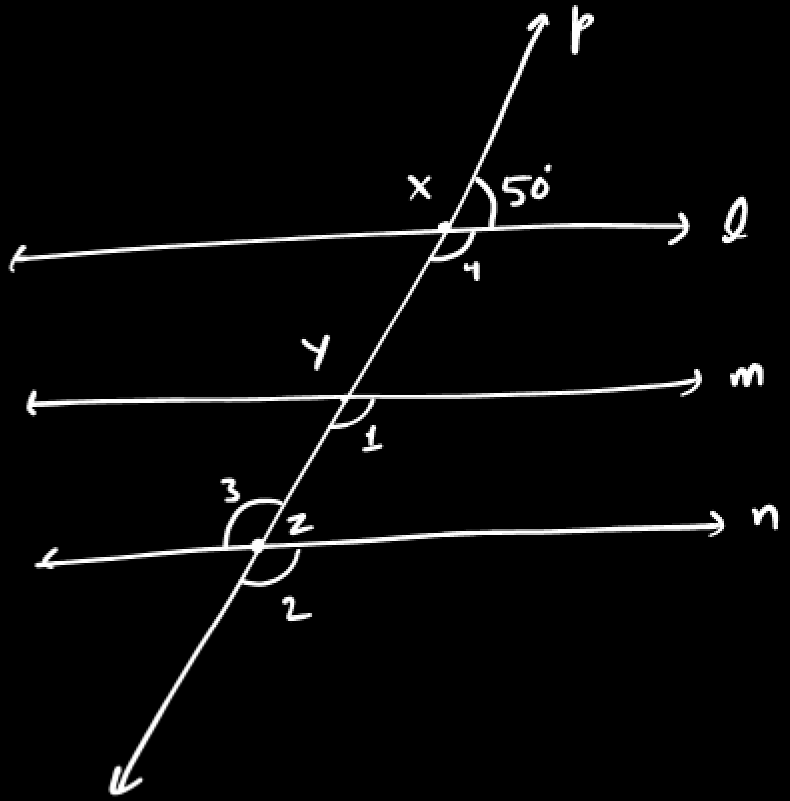


Linear pair

$x + y = 180^\circ$

2. $l \parallel m \parallel n$

find $\angle 1$, $\angle 2$, $\angle 3$. Give reasons.



$\angle 1 = 130°$ (

$\boxed{\angle 4 = 130°}$ ($\because \angle 4 + 50° = 180°$)

$\Rightarrow \boxed{\angle 1 = \angle 4}$ (corres. $\angle$s)

$\Rightarrow \therefore \angle 1 = 130°$

(alt. $\angle$s)

$\Rightarrow$ Also, $\angle 1 = \angle 3$

$\therefore \angle 3 = 130°$

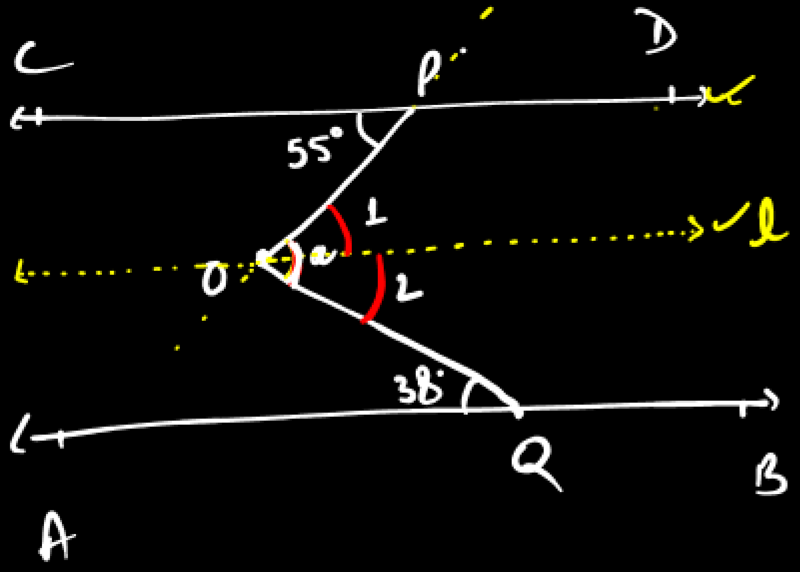
$\Rightarrow \angle 2 = 130°$ ($\because \angle 2$ & $\angle 3$ are alt. $\angle$s)

③ AB || CD

Find ∠a.

Construction:

Draw a line $l \parallel AB$



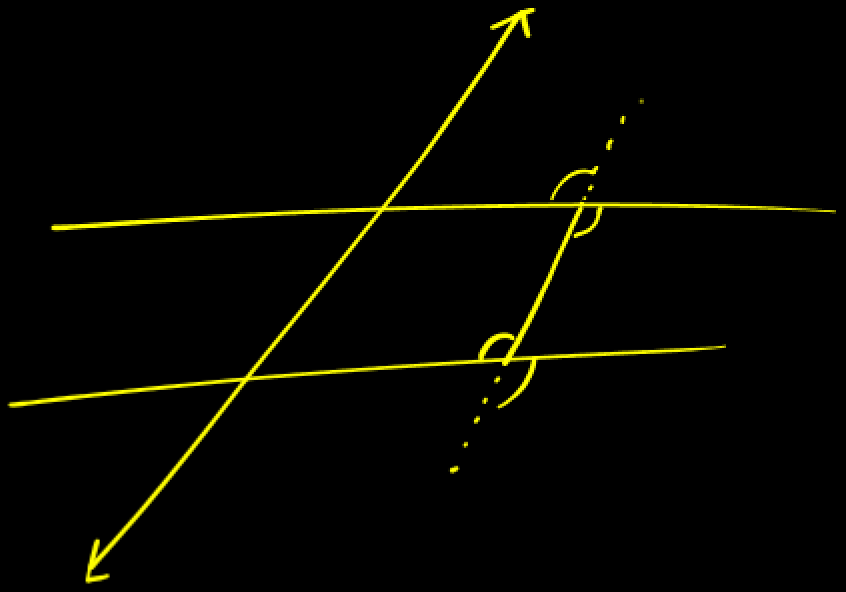
$\angle 1 = 55^\circ$ (alt. ∠s)

$\angle 2 = 38^\circ$ (alt. ∠s)

$\angle a = \angle 1 + \angle 2$

$= 55^\circ + 38^\circ$

$= 93^\circ$



$\frac{\cancel{200}}{\cancel{300}_1} \times \cancel{540}^{180}$

$= 2 \times 180$

$= 360$ €

$66\frac{2}{3}$ of $\quad 33\frac{1}{3}$ of 24

$\Rightarrow \frac{200}{3}$ of $\frac{100}{3}$ of 24

$\boxed{\frac{2}{3}\% \text{ of } 540}$

$\Rightarrow$

$\boxed{2\% \text{ of } 540}$

$\boxed{20\% \text{ of } 540}$

$\leftarrow$

$\frac{2}{\boxed{100}} \times 540$

$\frac{20}{100} \times 540$

$66\frac{2}{3}\%$ of $\left(\frac{9}{7} \text{ of } 420\right)$ €

$66\frac{2}{3}\% \left(\frac{9}{\cancel{7}_1} \times \frac{\cancel{420}^{60}}{1}\right)$ €

$66\frac{2}{3}\%$ of (540) €

$\boxed{\frac{200}{3}\% \text{ of } 540}$

$\left(\frac{200}{3} \div 100\right) \times 540$

$\left(\frac{200}{3} \times \frac{1}{100}\right) \times 540$

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$66\frac{2}{3}\%$

$\frac{200}{3}\%$

$\boxed{66\frac{2}{3}\% = \frac{2}{3}}$ ✗

$64\frac{2}{3}\%$ ✗ $\frac{2}{3}$

$2\% = \frac{2}{100}$

$100\% = \frac{100}{100}$

$5\% = \frac{5}{100}$

$\frac{4}{5}\% = \frac{4}{500}$

$\boxed{6\frac{2}{3} = \frac{20}{3}}$

$\boxed{6\frac{2}{3}\% = \frac{20}{300}}$

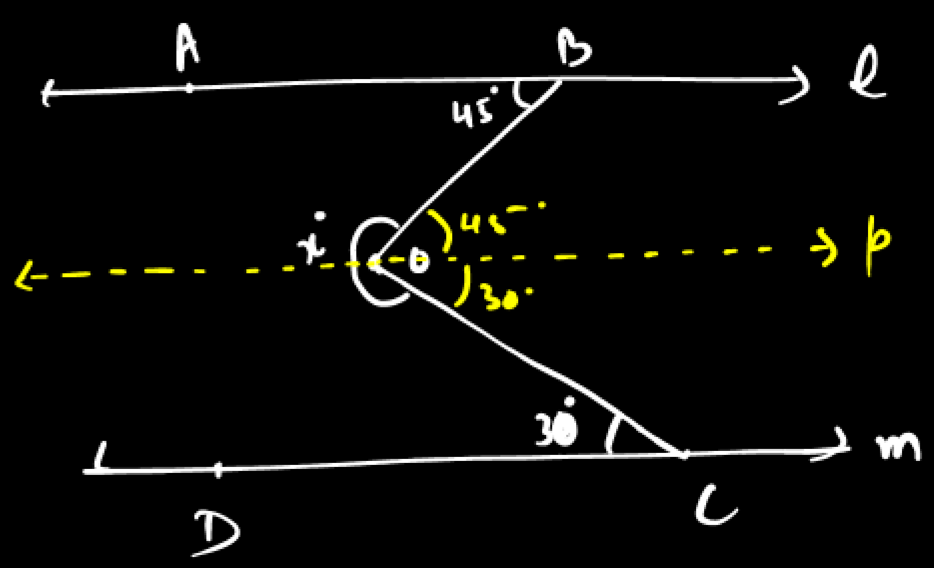
$66\frac{2}{3} = \frac{200}{3}$

$\boxed{66\frac{2}{3}\% = \frac{200}{300} = \frac{2}{3}}$ ✓

$\boxed{2\% = \frac{2}{100}}$

$\Rightarrow \boxed{\frac{3}{2}\% = \frac{3}{2} \times \frac{1}{100}} = \frac{3}{200}$

Q. Given $l \parallel m$ $\quad x^\circ = ?$



Draw a line $p \parallel l$

$x = 360^\circ - 75^\circ$

$\boxed{x = 285^\circ}$

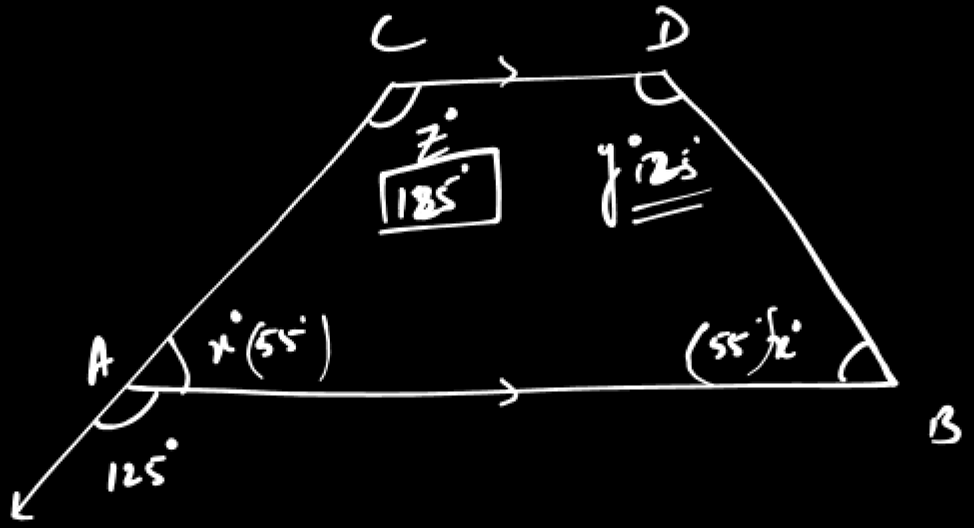
Q. AB || CD, find x, y & z.

Sol:

$x = 55°$

$x = 180° - 125°$

$= 55°$



$\boxed{\text{Sum of angles in a polygon} = (n-2) \times 180°}$

$\boxed{n = \text{no. of sides}}$

$n = 6$

Sum of ∠s $= 4 \times 180°$

$= \underline{720°}$

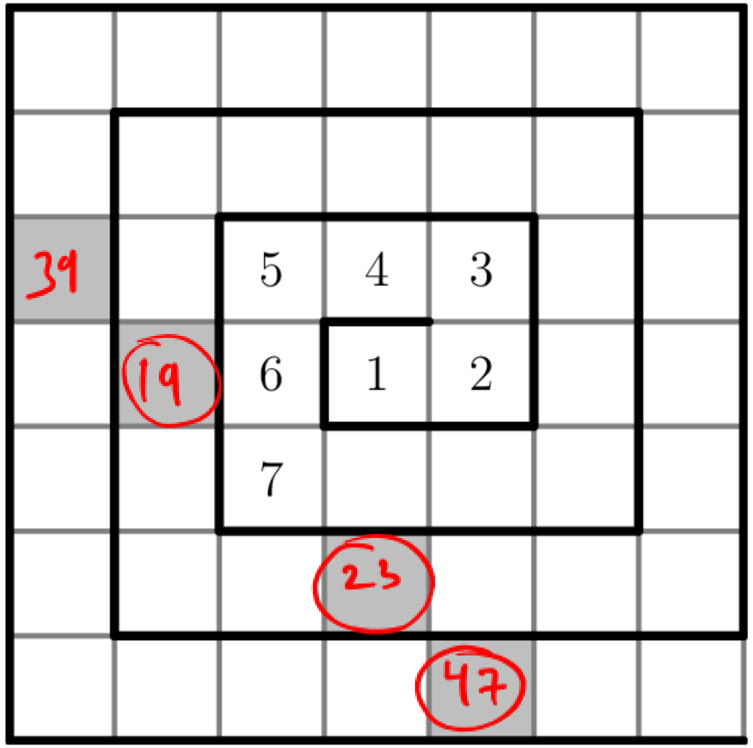
$n = 5$

Sum of all angles of pentagon $= (5-2) \times 180°$

$= 3 \times 180° = \underline{540°}$

Practice Problems for AMC

The numbers from 1 to 49 are arranged in a spiral pattern on a square grid, beginning at the center. The first few numbers have been entered into the grid below. Consider the four numbers that will appear in the shaded squares, on the same diagonal as the number 7. How many of these four numbers are prime?



(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

The digits $2, 0, 2,$ and 3 are placed in the expression below, one digit per box. What is the maximum possible value of the expression?

$$\boxed{3^{2}} \times \boxed{2^{0}}$$

(A) 0 **(B)** 8 **(C)** 9 **(D)** 16 **(E)** 18

$\boxed{2^0 = 1}$

$\boxed{a^0 = 1}$

$\boxed{2^3}$ $\boxed{3^2}$

$= 8$ 9

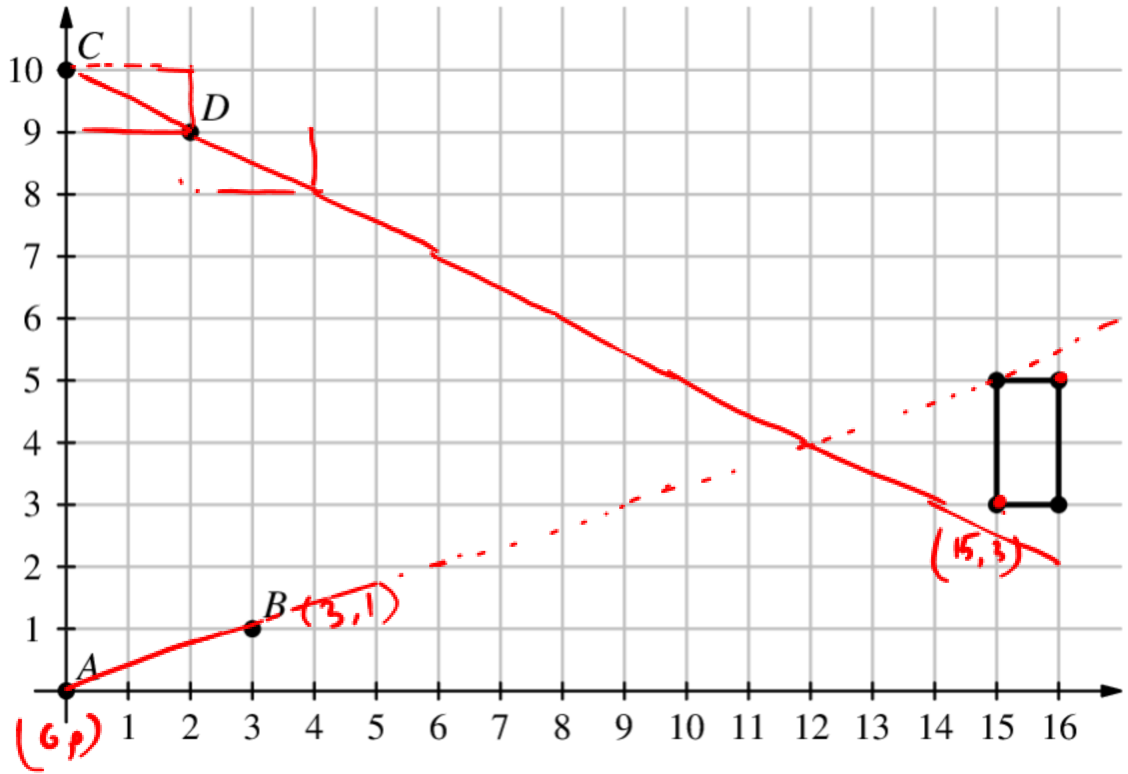
0^2

$a^0 = 1$

2^0 $\boxed{0^2} = 0$

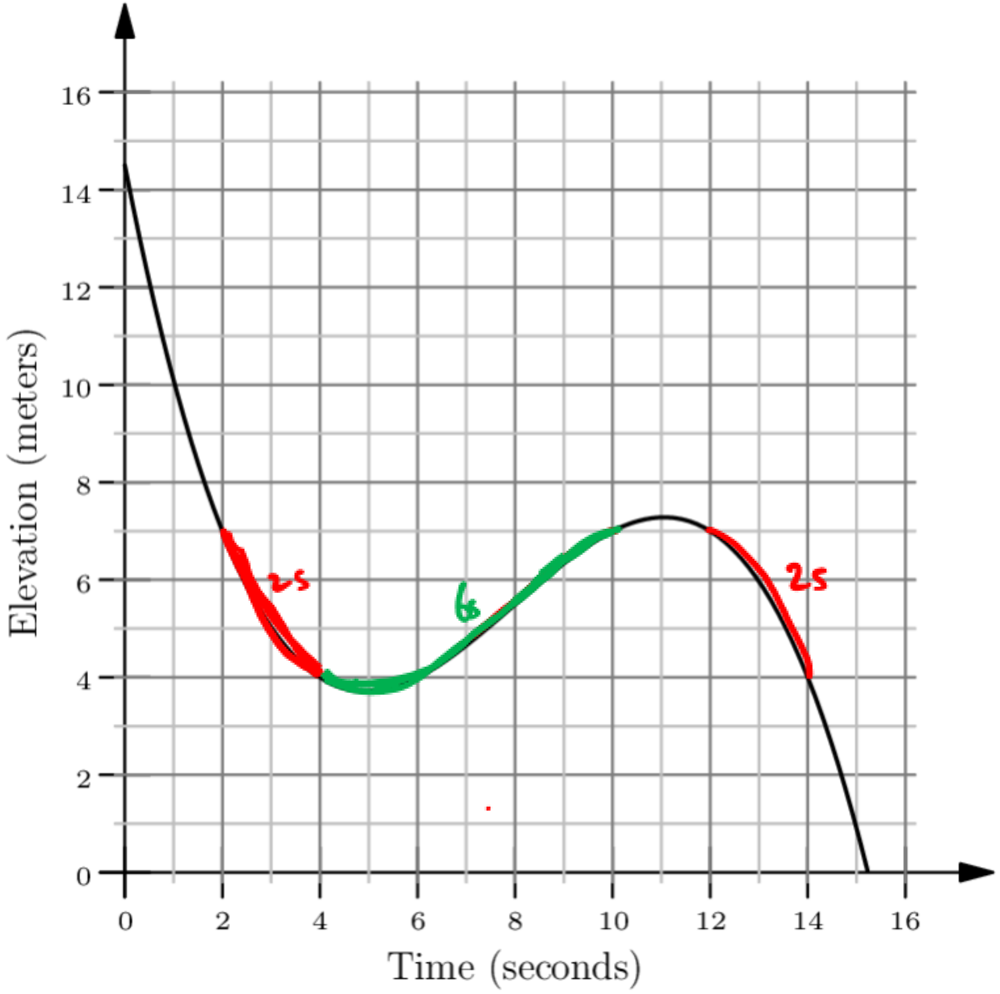
$5^0 = 1$ $\boxed{2^0 = 1}$

A rectangle, with sides parallel to the x-axis and y-axis, has opposite vertices located at $(15, 3)$ and $(16, 5)$. A line is drawn through points $A(0, 0)$ and $B(3, 1)$. Another line is drawn through points $C(0, 10)$ and $D(2, 9)$. How many points on the rectangle lie on at least one of the two lines?



(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

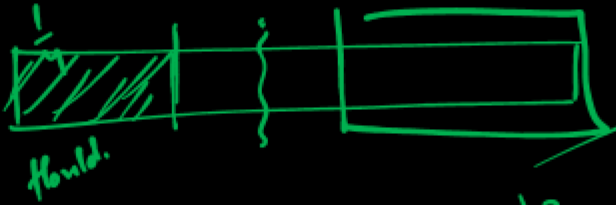
Malaika is skiing on a mountain. The graph below shows her elevation, in meters, above the base of the mountain as she skis along a trail. In total, how many seconds does she spend at an elevation between 4 and 7 meters?



(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

Harold made a plum pie to take on a picnic. He was able to eat only $\frac{1}{4}$ of the pie, and he left the rest for his friends. A moose came by and ate $\frac{1}{3}$ of what Harold left behind. After that, a porcupine ate $\frac{1}{3}$ of what the moose left behind. How much of the original pie still remained after the porcupine left?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{5}{12}$



$\frac{3}{4}$ left.

Moose $\Rightarrow$ $\boxed{\frac{1}{3} \text{ of } \frac{3}{4}}$

$\Rightarrow \frac{1}{\cancel{4}} \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{1}{4}$

$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

PP $= \frac{1}{3}$ of $\frac{1}{2}$

$= \left(\frac{1}{6}\right)^{th}$

left $= \frac{1}{2} - \frac{1}{6}$

$= \frac{3-1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

The figure below shows a large white circle with a number of smaller white and shaded circles in its interior. What fraction of the interior of the large white circle is shaded?

Area of circle $= \pi r^2$ unit^2

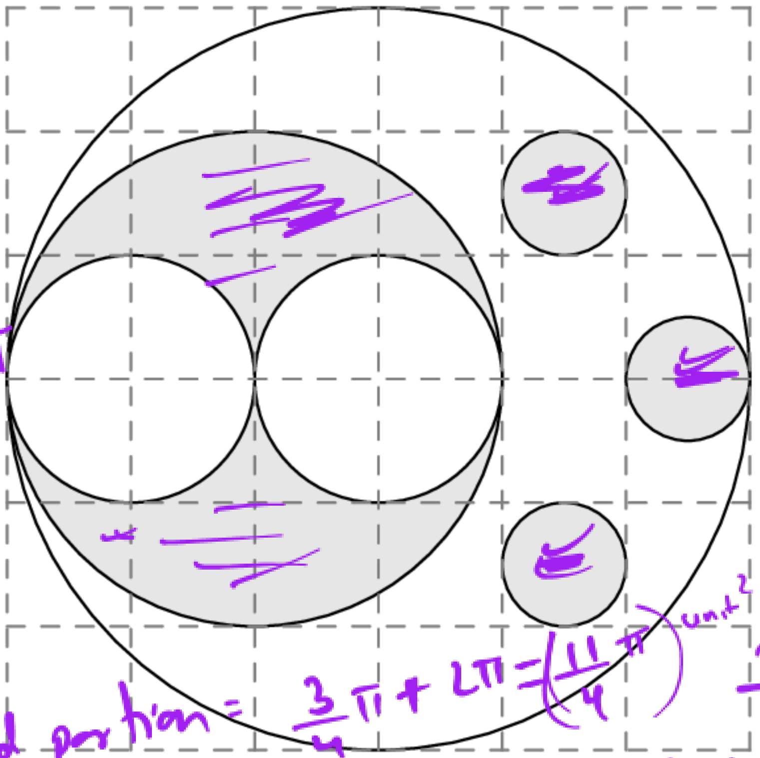
Area of biggest circle $= \pi 3^2 = \boxed{9\pi}$

Area of 3 smaller circles $= 3\cdot\pi\left(\frac{1}{2}\right)^2 = 3\pi\frac{1}{4} = \frac{3\pi}{4}$

Area of shaded portion of 2 unit circle $= \boxed{\pi(2)^2 - 2(\pi\cdot 1^2)}$

$= 4\pi - 2\pi$

$= 2\pi$



Area of shaded portion $= \frac{3}{4}\pi + 2\pi = \left(\frac{11}{4}\pi\right)$ unit^2

$\frac{3}{4} + \frac{2}{1}$

4.5 × 9.1

$\pi\cdot 4 \quad -2\pi\cdot 1$

$4\pi - 2\pi$

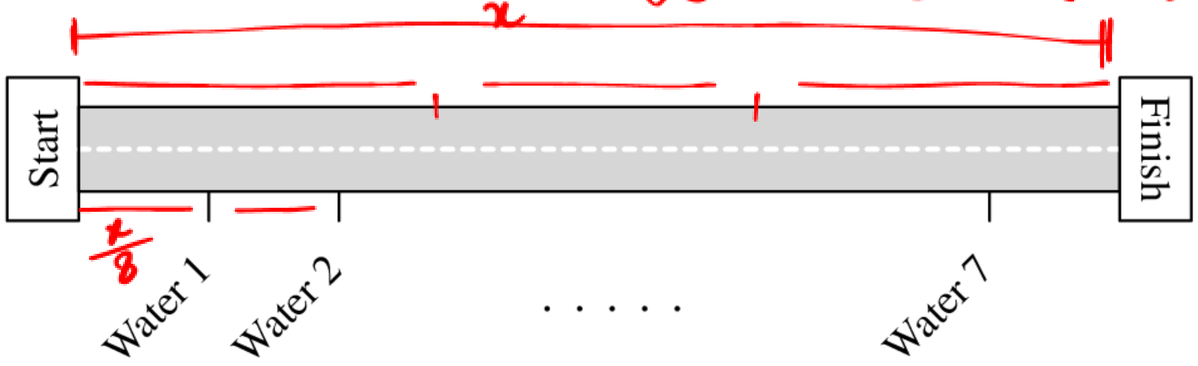
$\boxed{2\pi \text{ cm}^2}$

$\frac{Nu}{De} = \frac{11}{4}\pi \div \frac{9\pi}{1}$

$= \frac{11}{4}\pi \times \frac{1}{9\pi} = \frac{11}{36}$

(A) $\frac{1}{4}$ (B) $\frac{11}{36}$ (C) $\frac{1}{3}$ (D) $\frac{19}{36}$ (E) $\frac{5}{9}$

Along the route of a bicycle race, 7 water stations are evenly spaced between the start and finish lines, as shown in the figure below. There are also 2 repair stations evenly spaced between the start and finish lines. The 3rd water station is located 2 miles after the 1st repair station. How long is the race in miles?



(A) 8 **(B)** 16 **(C)** 24 **(D)** 48 **(E)** 96

$W_1 = \frac{x}{8}$ mile.

$W_2 = 2\frac{x}{8}$

$\boxed{W_3 = \frac{3x}{8} \text{ miles}}$

$\boxed{R_1 = \frac{x}{3} \text{ mile.}}$

$R_2 = 2\frac{x}{3}$ mile

$\frac{3x}{8} - 2 = \left(\frac{x}{3}\right)$

$\frac{3x}{8} - \frac{x}{3} = 2$

$\frac{9x - 8x}{24} = 2$

$\frac{1x}{24} = 2$

$\frac{x}{24} = 2$

$\frac{x}{\cancel{24}} \times \cancel{24} = 2 \times 24$

$\boxed{x = 48}$

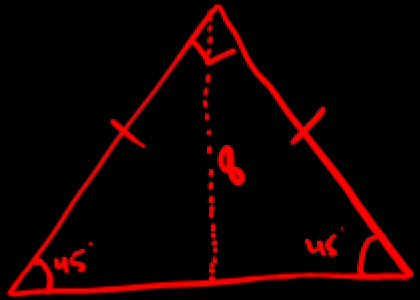
25 Questions → & ~~60 minutes~~ 40 minutes

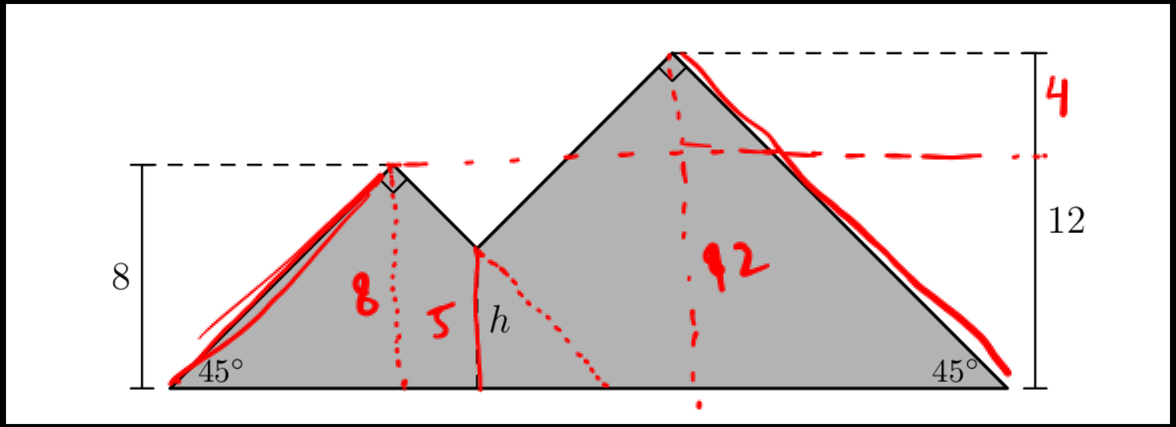
$$1Q = \frac{40}{25}$$

$$\sim \boxed{1:20} \text{ second}$$

10 Question (within min)

Area of Δ = $\frac{1}{2} \cdot$ height $\times$ base





Calculus

Trigonometry

$\boxed{22}$ $\boxed{21}$

$\boxed{20}$ $\boxed{21}$

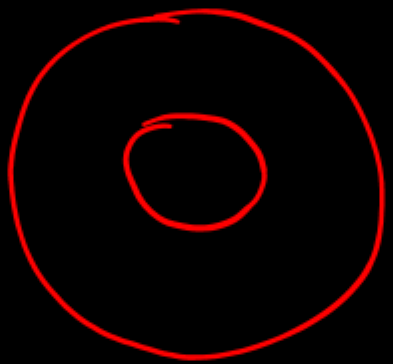
$(0,4)$ $(2,8)$

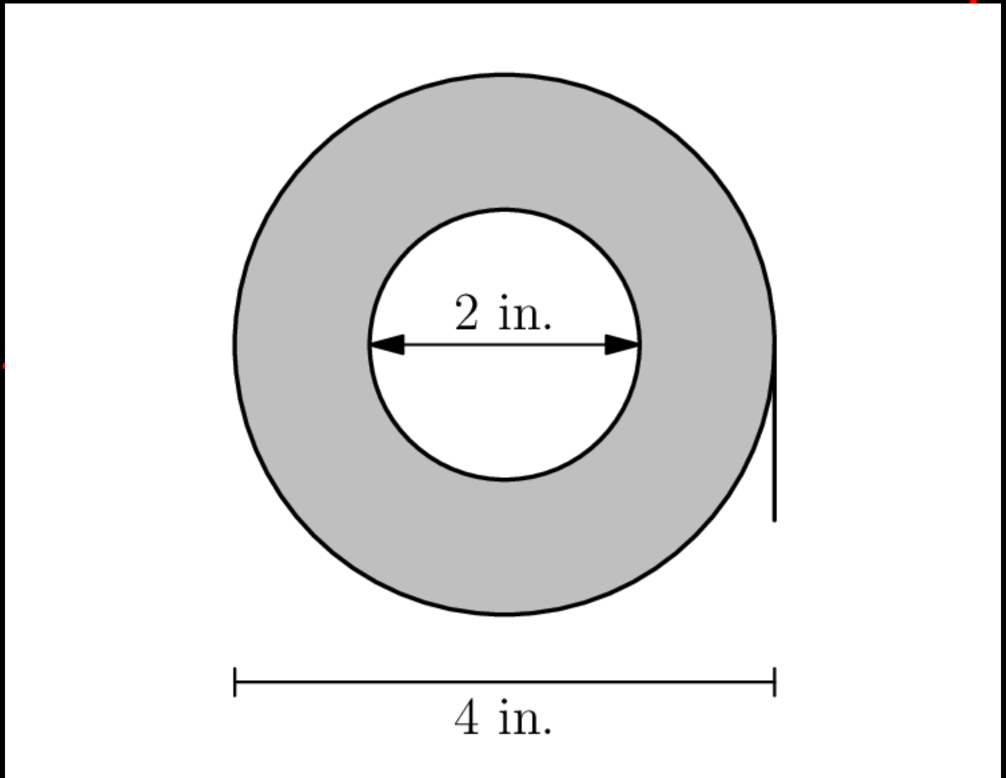
$\boxed{\text{4 cells.}}$

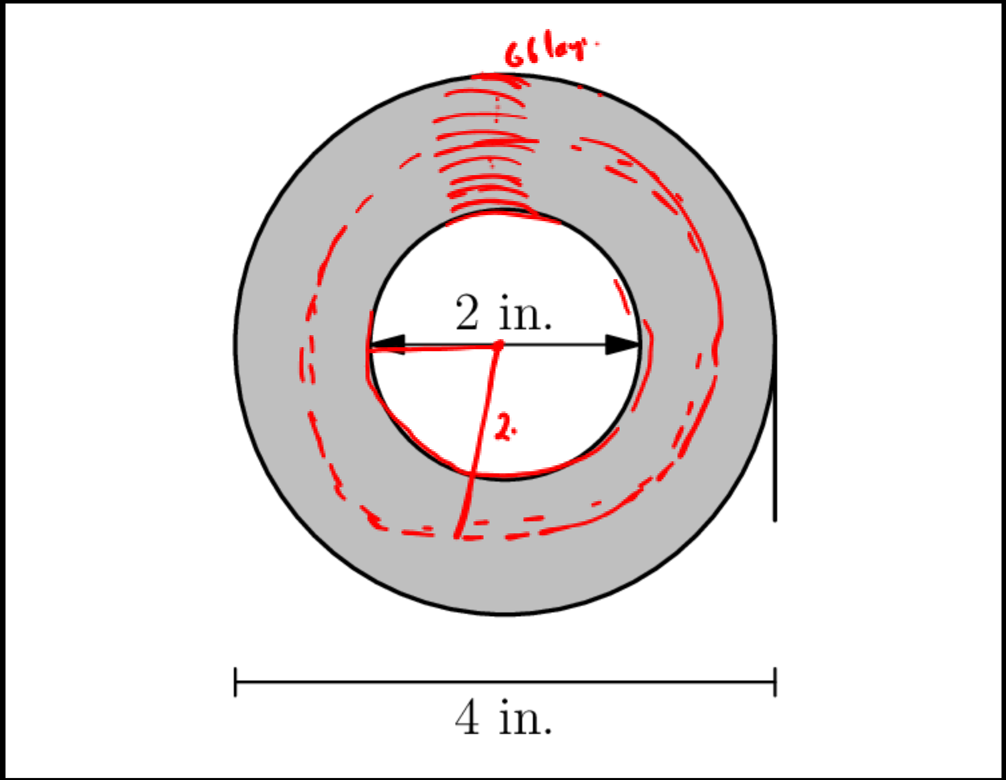
$(2000, 3000)$ $(5000, 8000)$

$(5,8) \times \boxed{1000}$

$$\frac{1}{0.015} = 200 \quad \boxed{\frac{\cancel{1000}}{\cancel{15}\,3}}$$







$\boxed{2\pi}$

$\boxed{4\pi}$

$\boxed{3\pi}$

Area circum: $= 3 \times 3.14 \times 66$

$= \sim 10 \times 66$

~ 660

$\sim \boxed{600}$

End of the chapter