Number System
Exponentials

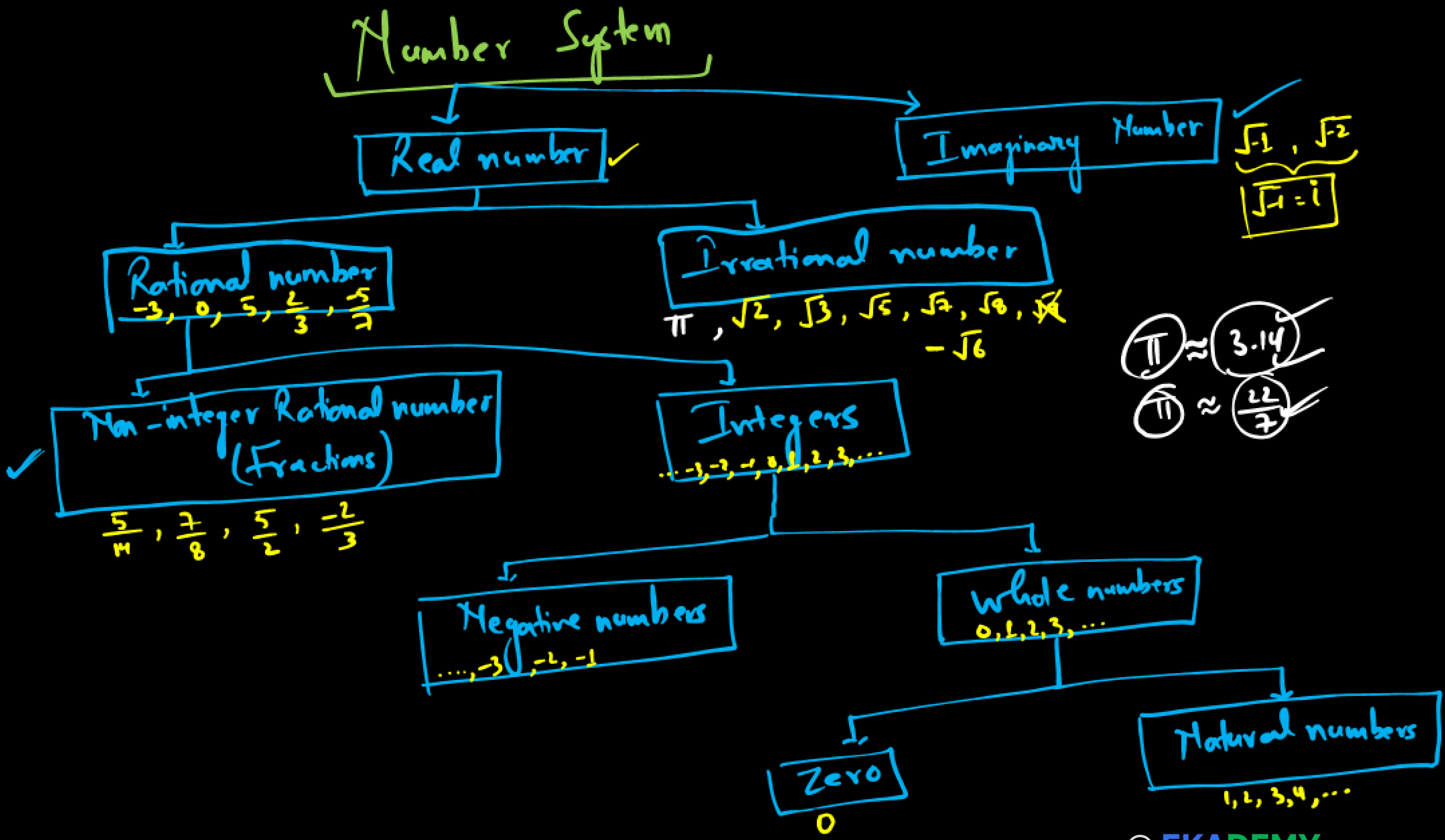
Number System

- Rational number
- Irrational numbers.

→ Representation on number line.

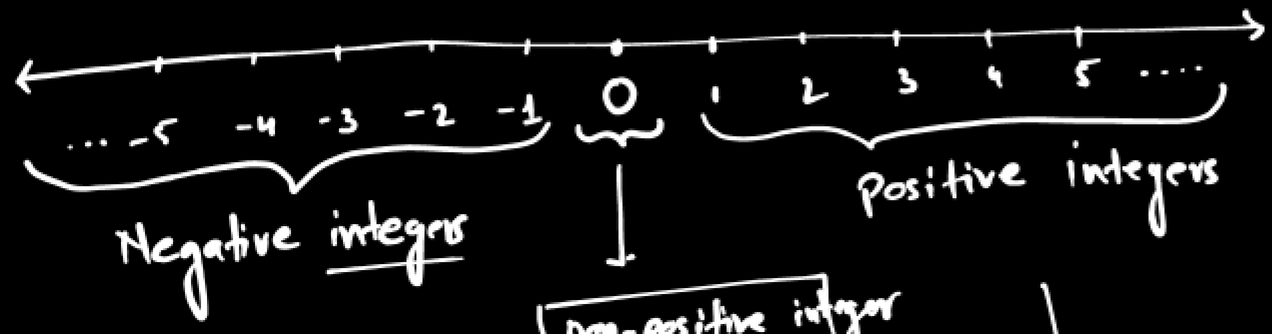
Real numbers.

- → Natural numbers (1, 2, 3, 4, ...) counting nos.
- → whole numbers (0, 1, 2, 3, 4, ...)
- → Integers ($-\infty$, -3, -2, -1, 0, 1, 2, 3, ... ∞)
- → Rational numbers $\left(\frac{p}{q}\right)$ $\frac{2}{3}$, 0.1
- → Irrational nos. (cannot be written as $\frac{p}{q}$) → Non repeating non terminating decimal nos.


Number System
Real number
Imaginary Number
$\sqrt{-1}$, $\sqrt{-2}$
$\sqrt{-1} = i$
Rational number
$-3, 0, 5, \frac{2}{3}, \frac{-5}{7}$
Irrational number
$\pi, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{8}, \cancel{\sqrt{4}}$
$-\sqrt{6}$
$\pi \approx 3.14$
$\pi \approx \frac{22}{7}$
Non-integer Rational number (Fractions)
$\frac{5}{14}, \frac{7}{8}, \frac{5}{2}, \frac{-2}{3}$
Integers
$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Negative numbers
$\ldots, -3, -2, -1$
Whole numbers
$0, 1, 2, 3, \ldots$
Zero
0
Natural numbers
$1, 2, 3, 4, \ldots$

Number Line (Integers)

$\boxed{-2.5}$ $\boxed{3.5}$



$\boxed{\text{non-positive integer}}$
and
non-negative integer

Set of non-positive integers : 0, -1, -2, -3, -4, ...

set of negative integer : -1, -2, -3, -4, ...

Set of non-negative integers : (W)
0, 1, 2, 3, 4, 5, ... ∞

Set of positive integers : (N)
1, 2, 3, 4, 5, ... ∞

Rational Numbers

Any number that can be expressed in the form of $\frac{p}{q}$, where p and q are both integers and $q \neq 0$ is called a rational number.

Rational → Ratio

↳ Ratio of two integers

eg: $\frac{1}{2}$, $\frac{-3}{9}$, $2\left(=\frac{2}{1}\right)$, $0\left(=\frac{0}{1}\right)$, $-3\left(=\frac{-3}{1}\right)$

$3.5\left(=\frac{35}{10}\right)$, $0.1\left(\frac{1}{10}\right)$, $\boxed{-10.25}\left(\frac{-1025}{100}\right)$

$\frac{35}{55}$ is a rational number. $= \left(\frac{p}{q}\right)$

$p = 35$

$q = 55$

5 is common factor of p & q

$\Rightarrow \frac{7}{11} \Rightarrow p$ & q do not have any common factor.

$\therefore \frac{7}{11}$ is called simplest form / irreducible form / lowest form.

$\boxed{\text{Remark: Reciprocal of zero (0) is not allowed}}$

$\frac{p}{q}$

$p \Rightarrow$ negative no., positive no., a zero.

q is taken as any number greater than zero.

eg: $\frac{-2}{3} = \frac{2}{-3}$ ✓ ✗

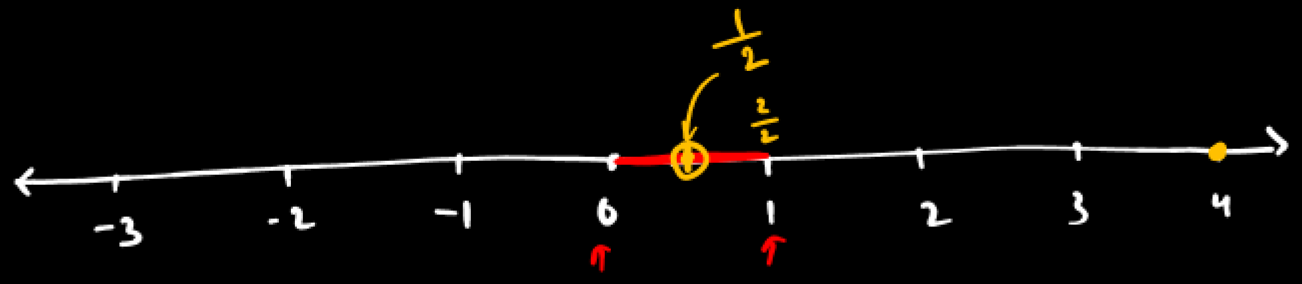
$\frac{5}{-7} \Rightarrow \frac{-5}{7}$

$\frac{+5}{+7} = \frac{5}{7}$

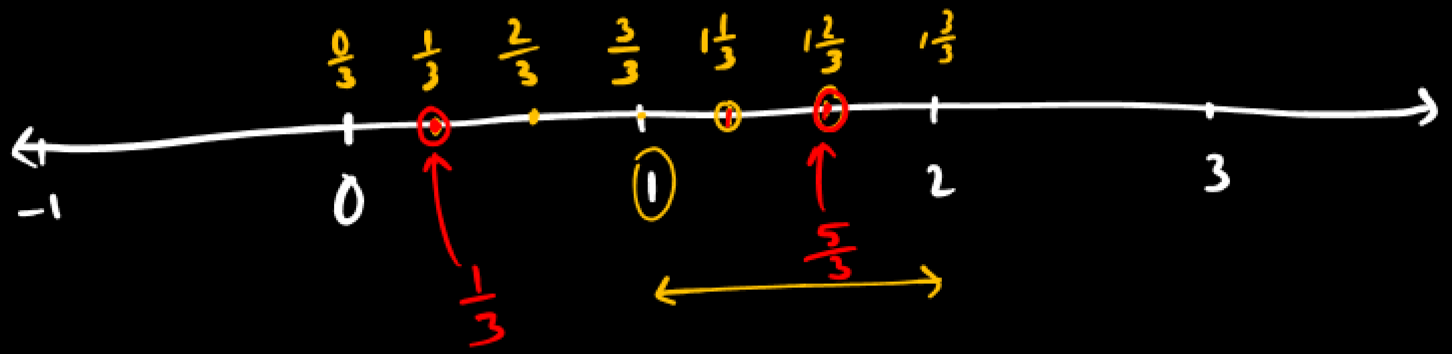
Representation of Rational Numbers on Number line (Real Number line)

- Represent 4 on the number line.
- Represent $\frac{1}{2}$ on the number line

$0 \quad \frac{1}{2} \quad 1$



- Represent $\frac{1}{3}$ on number line.



- Represent $\frac{5}{3}$ on number line.

$$\frac{5}{3} = 1\frac{2}{3}$$

Represent below nos. on number line.

(i) $\frac{7}{5}$

(ii) $-\frac{2}{3}$

(iii) $\frac{5}{-3}$

(iv) $\frac{11}{4}$

$\underline{\text{H.W.}}$

Properties of Rational numbers

If a and b are any two rational number, then

$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ | $\frac{1}{2}+\frac{1}{2}=1$ → rational no.

(i) $a+b$ is also a rational no.

(ii) $a-b$ is also a rational.

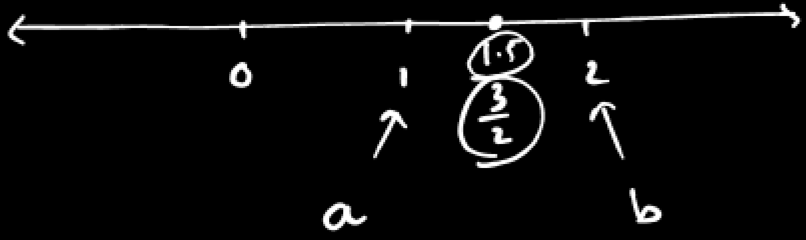
$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

(iii) $a\times b$ is also a rational no.

(iv) $\frac{a}{b}$ is also a rational no if $b\neq 0$. $\frac{1}{2}\div\frac{1}{3}=\frac{1}{2}\times\frac{3}{1}=\frac{3}{2}$

(v) $\frac{a+b}{2}$ is also a rational no. which lies between a and b

"Explation in the next class"



$a = 1 \longrightarrow$ Rational no (Q)

$b = 2 \longrightarrow Q$

$$\boxed{\frac{a+b}{2}} = \frac{1+2}{2} = \boxed{\frac{3}{2}} = \boxed{1.5}$$

$\uparrow$

Q

Q. Insert one rational number between $\frac{5}{7}$ and $\frac{4}{9}$, and arrange them in ascending order.

$a = \frac{5}{7}$

$b = \frac{4}{9}$

$$\frac{a+b}{2} = \frac{\frac{5}{7} + \frac{4}{9}}{2} = \frac{\left(\frac{28+45}{63}\right)}{2}$$

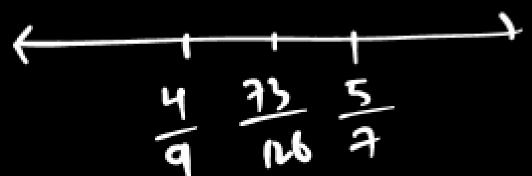
$$= \frac{\left(\frac{73}{63}\right)}{2}$$

$$= \left(\frac{73}{63}\right) \div \frac{2}{1}$$

$$= \frac{73}{63} \times \frac{1}{2} = \frac{73}{126}$$

$\frac{5}{7}, \frac{73}{126}, \frac{4}{9}$

$\frac{4}{9}, \frac{73}{126}, \frac{5}{7}$



Q. Insert three rational numbers between 2 and 3.

a = 2, b = 3

Sol: A rational no. between 2 and 3 $= \frac{2+3}{2} = \frac{5}{2}$

A rational no. between 2 and $\frac{5}{2}$ $= \left(\frac{2}{1} + \frac{5}{2}\right) \div 2$

$= \left(\frac{4+5}{2}\right) \div 2 = \frac{9}{2} \div 2 = \frac{9}{2} \times \frac{1}{2} = \frac{9}{4}$

2, $\frac{9}{4}$, $\frac{5}{2}$, 3

a, b

A rational no. between 2 and $\frac{9}{4}$ $= \left(\frac{2}{1} + \frac{9}{4}\right) \div 2 =$

$= \left(\frac{8+9}{4}\right) \div 2 = \frac{17}{8}$

2 $\boxed{\frac{17}{8} \quad \frac{9}{4} \quad \frac{5}{2}}$ 3

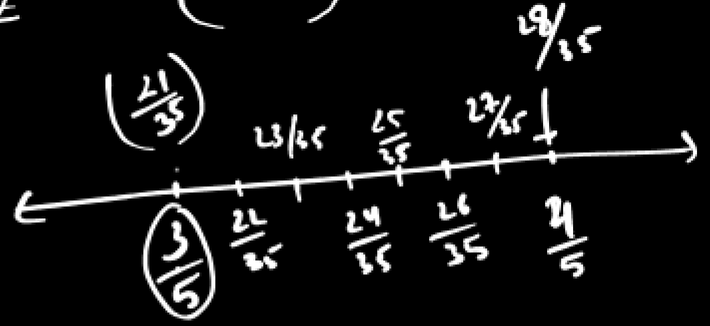
Q. Find six rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution: $\because$ We need 6 rational nos. (n)

$\therefore$ multiply nu. and de. of both the given nos. by $(6+1)$ $(n+1)$

we get, $\frac{3 \times 7}{5 \times 7} = \frac{21}{35}$ and

$\frac{4 \times 7}{5 \times 7} = \frac{28}{35}$



$\frac{21}{35}$

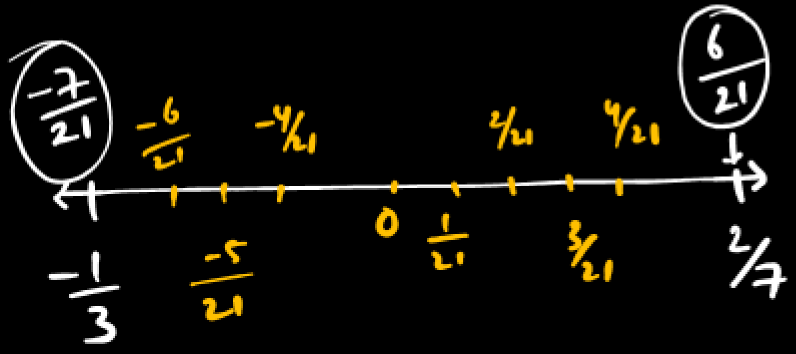
Q. Insert eight rational nos. between $\frac{-1}{3}$ and $\frac{2}{7}$.

Sol: Write the given nos. with same denominator.

LCM of $(3, 7) = 21$

$$\frac{-1}{3}\times\frac{7}{7} = \frac{-7}{21}$$

$$\frac{2\times 3}{7\times 3} = \frac{6}{21}$$



Irrational numbers

A number that cannot be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called irrational number.

eg. $\pi, \sqrt{2}, \sqrt{3}, \sqrt{5}, \cancel{\sqrt{6}, \sqrt{7}, \sqrt{8}}, -\sqrt{6}, -\sqrt{3}, -\sqrt{2}, \frac{1}{\sqrt{5}},$

$\sqrt{111}$

$\boxed{\sqrt{256}} = \boxed{16}$

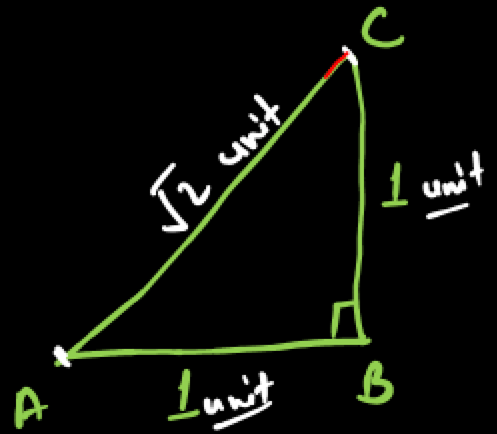
rational

$\sqrt{-1} = i \neq -\sqrt{1}$

imaginary.

Representation of irrational numbers on number line.

Q. Represent $\sqrt{2}$ on number line.



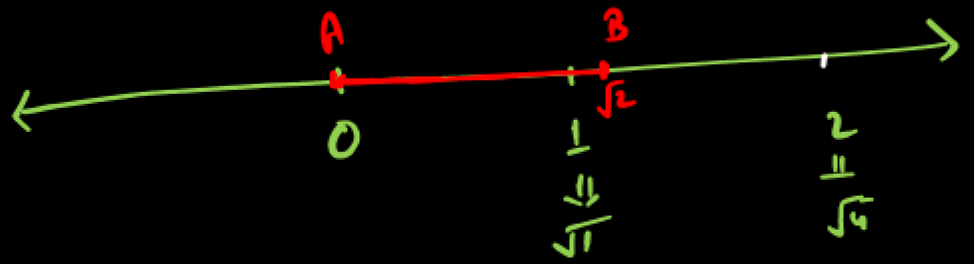
Pythagoras Theorem

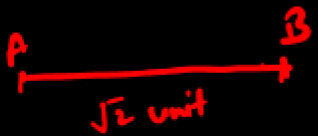
$$AC^2 = AB^2 + BC^2$$

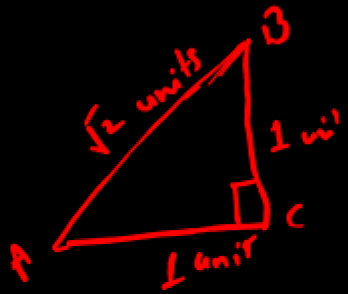
$$AC^2 = 1^2 + 1^2$$

$$AC^2 = 1 + 1$$

$$AC^2 = 2 \quad AC = \sqrt{2}$$

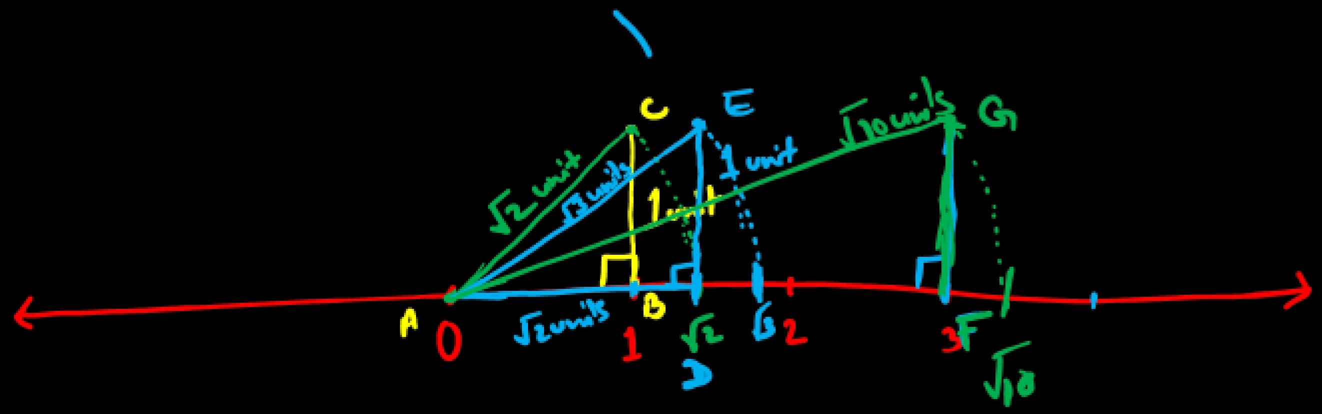


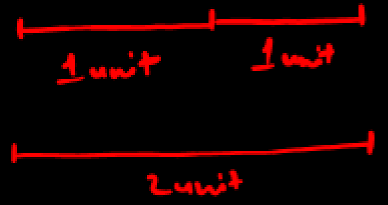




Locate $\sqrt{2}$ on number line.

irrational no.





Using Pythagoras theorem.

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = 1^2 + 1^2$

$AC^2 = 2$

$AC = \sqrt{2}$ unit

In $\triangle ADE$, using Pythagoras theorem,

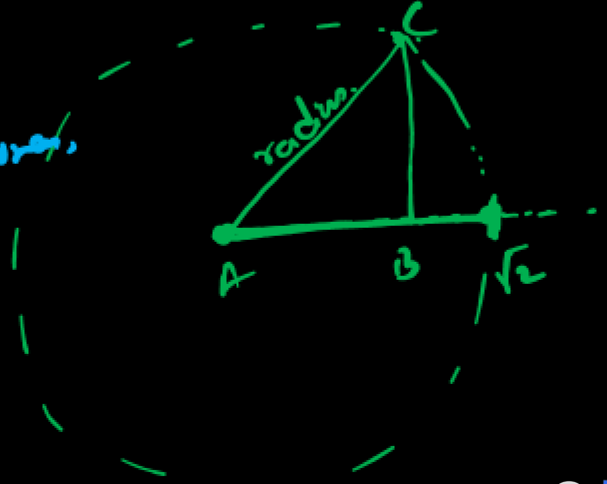
$(AE)^2 = (AD)^2 + (DE)^2$

$AE^2 = (\sqrt{2})^2 + (1)^2$

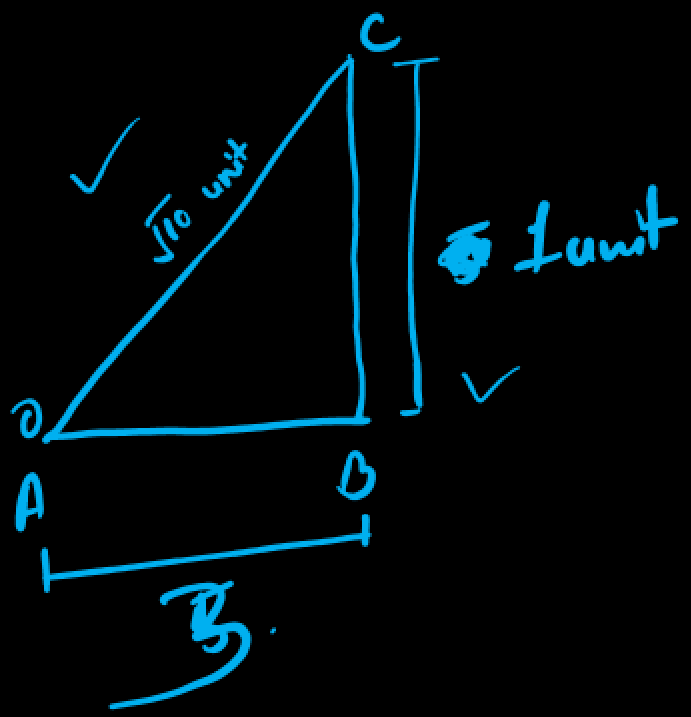
$= 2 + 1$

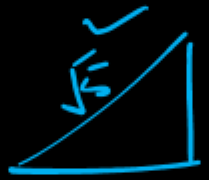
$AE^2 = 3$

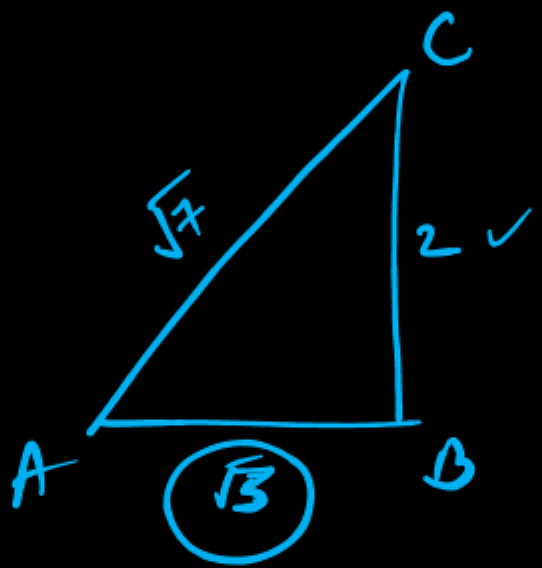
$AE = \sqrt{3}$



Locate $\sqrt{10}$ on ~~unit~~ number line.

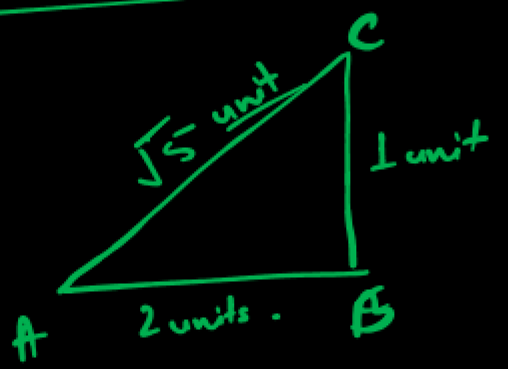


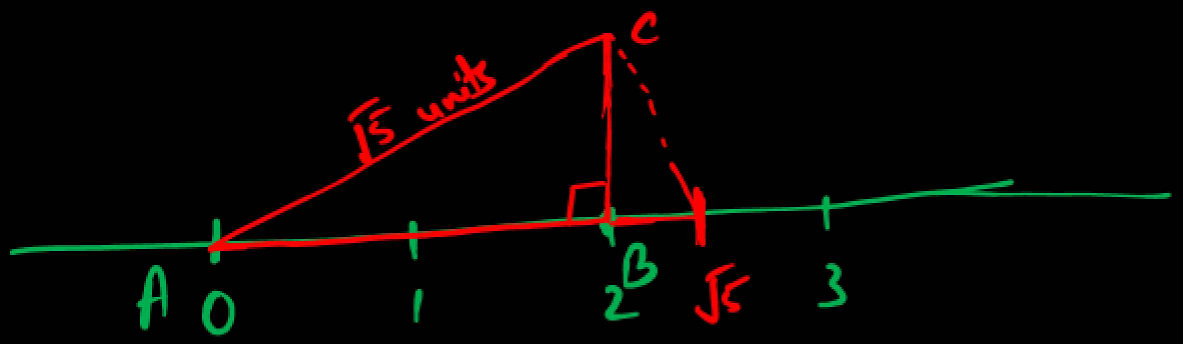




$$AC^2 = 3^2 + 1^2$$
$$= 9 + 1$$
$$AC^2 = 10$$
$$AC = \sqrt{10}$$

Locate $\sqrt{5}$ on number line.





AB = 2 unit

BC = 1 units

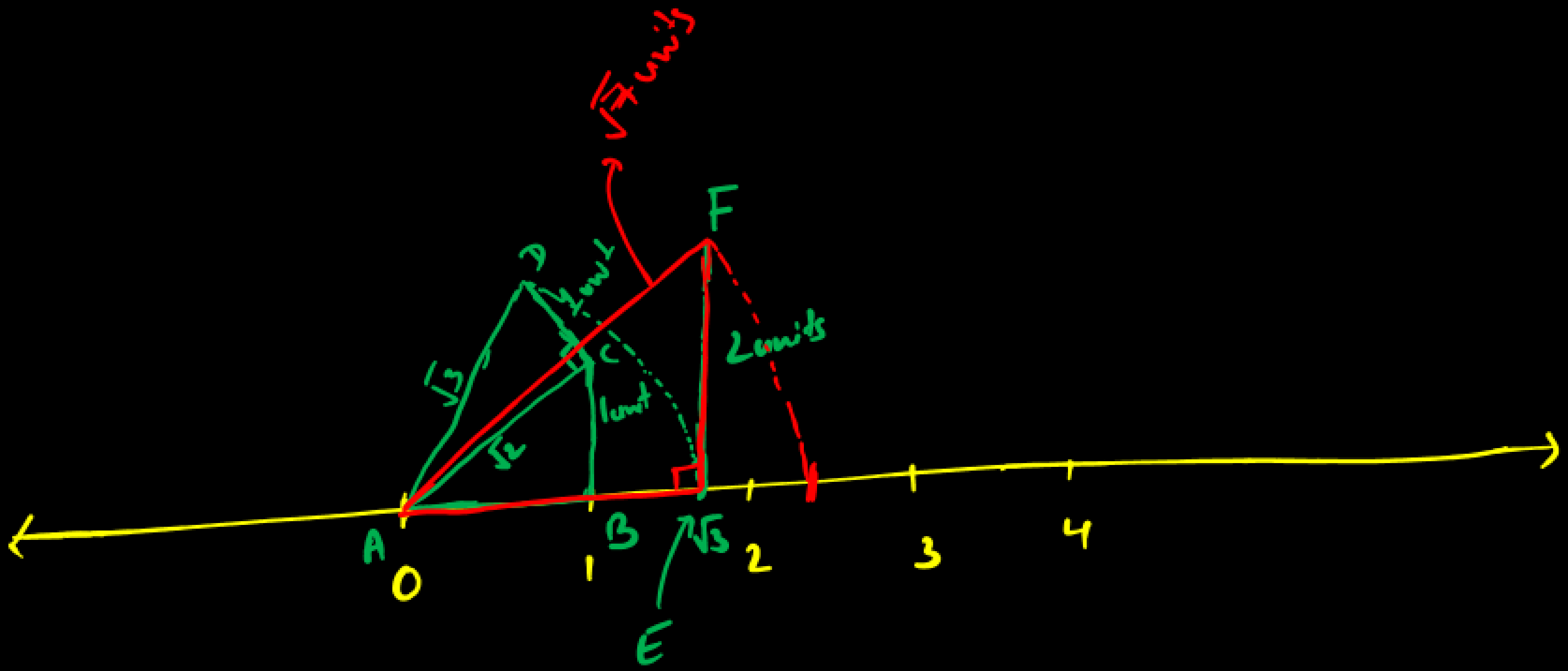
$AB^2 = AB^2 + BC^2$

$AC^2 = 2^2 + 1^2$

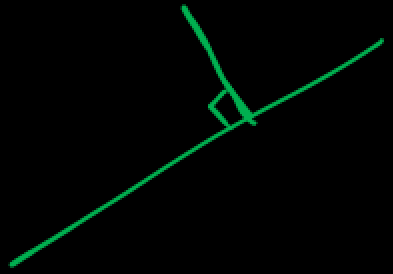
$AC^2 = 5$

$AC = \sqrt{5}$ units

$\sqrt{7}$ on number line:



$AE = \sqrt{3}$ unit



H.W.

Represent, $\sqrt{3}$, $\sqrt{6}$, $\sqrt{8}$, $\sqrt{11}$, $\sqrt{13}$ on number line.

$\sqrt{19}$	$\sqrt{15}$	$\sqrt{21}$

$\boxed{\frac{15}{11}}$ =

Decimal expension

- Termination decimal. ✓
- Non-terminating repeating.

} Rational

$11 \overline{)15} ($

$$\frac{15}{11} = 1.3638\ldots = 1.\overline{36}$$

1.2

Integer : $-\infty \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \infty$

$\boxed{\frac{\sqrt{2}}{7}}$ } ?

1.0100100001...

Non-terminating non-repeating } Irrational.

$\sqrt{2}$

Find the value of $\sqrt{2}$, using long-division method.

dividend

~~non~~terminating non-repeating

$\sqrt{2} \approx 1.414$

$\pi \approx 3.14$

$\pi \approx \frac{22}{7}$

$$
\begin{array}{r|l}
 & 1.41421\ldots \\
\hline
1 & 2.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00} \\
+1 & -1 \\
\hline
24 & 100 \\
+4 & -96 \\
\hline
281 & 400 \\
+1 & -281 \\
\hline
2824 & 11900 \\
+4 & 11296 \\
\hline
28282 & 60400 \\
+2 & -56564 \\
\hline
282841 & 383600 \\
 & -282841 \\
\hline
\end{array}
$$

2824×4

$1^2 = 1 \times 1$

$2^2 = 2 \times 2$

$3^2 = 3 \times 3$

20 21 $\boxed{22}$

28282×3

$\sqrt{3}$

Find value of $\sqrt{3}$ using long division method.

$1.7320\_\_\_\_ \Rightarrow$ Non-terminating

1	3.000000
+1	−1
27	200
+7	−189
343	1100
+3	−1029
3462	7100
+2	−6924
34640	17600
0	00000
34640_	1760000

$$\boxed{\sqrt{4} = 2}$$

$$
\begin{array}{r|l}
 & 2 \\
\hline
2 & 4.\overline{00}\,\overline{00} \\
 & -4 \\
\hline
 & 0
\end{array}
$$

$\sqrt{169.}$

$$\sqrt{256} = 16$$

$$
\begin{array}{r|l}
 & \boxed{1\ 6} \\
\hline
1 & \overline{02}\,\overline{56}. \\
+1 & -1 \\
\hline
26 & 156 \\
 & -156 \\
\hline
 & 000
\end{array}
$$

Find Decimal expansion of :

$$\left\{ \begin{array}{l} \sqrt{11} \\ \sqrt{257} \\ \sqrt{1459} \end{array} \right.$$

direction of pairing

$$\overline{11}.\overline{00}\,\overline{00}$$

$$
\begin{array}{r|l}
 & \quad 3 \\
3 & \overline{14}\,\overline{59}.\overline{00} \\
 & \ 9 \\
\end{array}
$$

00 11 ✓

011 ✓

11 ✓

01

1

$\sqrt{11} = 3.316629...$

$\approx \boxed{3.317}$

$$\sqrt{13492.237} \simeq 116.15\ldots$$

$$\overline{0x}\,\overline{yz}\,\overline{pq}\,.\,\overline{ob}\,\overline{co}\,\overline{00}$$

	116.15--
1	01 34 92 . 23 70 00
+1	−1
21	034
+1	−21
226	1392
+6	−1356
2321	3623
+1	−2321
23225	130270
+5	−116125
23230-	1414500

$$2\,\overline{)\,13492.237}$$

Surds | Radicals

$\sqrt{}$ $\sqrt[3]{}$

$\sqrt{2}$ $\sqrt{3}$ $\sqrt{5}$

$\sqrt[3]{}$

$2^3 = 8$

$2 \times 2 \times 2$

$2^3 = 8$

$\boxed{\sqrt[3]{8}} = 2$

$2^2 = 4$

$\sqrt{4} = 2$

$2^4 = 16$

$\sqrt[4]{16} = 2$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$\boxed{\sqrt[5]{243} = 3}$

Radical.

$\sqrt[n]{}$

$\boxed{n > 1}$ $\Rightarrow$ order of the radical.

$n = 2, 3, 4, 5, 6, \dots$

$\sqrt[n]{a} \Rightarrow$ nth root of a

a radical of 'a' of order n.

1.x

$\sqrt[2]{a} \Rightarrow$ 2nd root of a $\Rightarrow$ Square root of a

$\Downarrow$

$\sqrt{a}$

$\sqrt[3]{a} \Rightarrow$ 3rd root of a

cuberoot of a

$\sqrt[4]{a} \Rightarrow$ 4th root of a

$\sqrt[5]{a} \Rightarrow$ 5th root of a

$\vdots$

$\sqrt[n]{a} =$ nth root of a.

$\boxed{a \text{ cannot be written an } n^{th} \text{ power of any rational number.}}$ $\sqrt[n]{a}$

$\Downarrow$

"Radicals are irrational"

$\sqrt[2]{2}$

$\Rightarrow \sqrt[3]{8} = \sqrt[3]{2^3}$ Not a radical.

$\Rightarrow \sqrt[2]{12} =$ is a radical.

$\Rightarrow \sqrt[3]{9} =$ is a radical.

$$\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$$

$\sqrt{15} \Rightarrow$ order $= \underline{\underline{2}}$

$$\left((2)^2\right)^3 = 2^{2\times 3} = 2^6$$

$$\sqrt{4} = (4)^{\frac{1}{2}} = \left(2^2\right)^{\frac{1}{2}} = 2^{\cancel{2}\times\frac{1}{\cancel{2}}} = 2$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[3]{27} =$$

Rules :

(i) $\left(\sqrt[n]{a}\right)^n = a$

(ii) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

(iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

$\left(\sqrt{2}\right)^2$

$\left(\sqrt[3]{2}\right)^3$

Square root / Cube root (sometime)

Operations:

$\sqrt{3} + \sqrt{5} \neq \sqrt{3+5}$

$\sqrt{3} \times \sqrt{5} = \sqrt{15}$

$1\sqrt{3} + 1\sqrt{3} = 2\sqrt{3}$

$2^2 + 3^2 \neq (5)^2$

$3^5 \times 9^6$

$2^5 \times 2^7$

$3^5 \times 9^5 = (3 \times 9)^5$

$3^5 \times (3^2)^6 = 3^5 \times 3^{12}$

$$2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$$

$$2x + 3x = 5x$$

$$1.a + 3a = 4$$

$$1.\sqrt{6} + 3\sqrt{6} = 4\sqrt{6}$$

$$\sqrt[3]{4} + 3\sqrt[3]{4} = 4\sqrt[3]{4}$$

Exponents

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ times}}$$

base

Laws of exponents :

(i) $a^n \times a^m = a^{n+m}$

(ii) $(a^m)^n = a^{m \cdot n}$

(iii) $\frac{a^n}{a^m} = a^{(n-m)}$

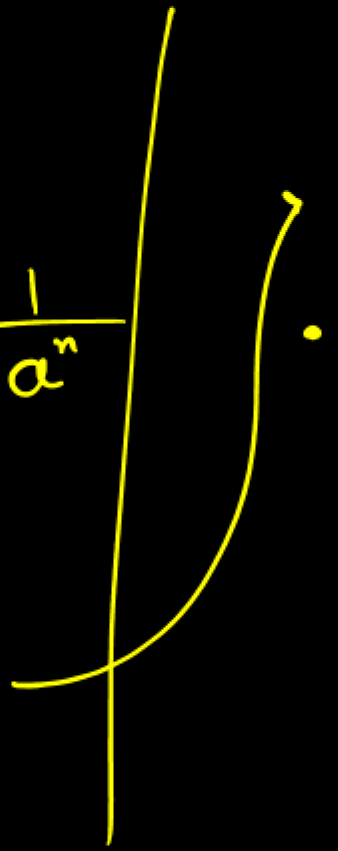
(iv) $a^m \cdot b^m = (ab)^m$

(v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(vi) $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

(vii) $\boxed{a^0 = 1}$

(viii) $a^{\frac{1}{n}} = \sqrt[n]{a}$



$4^{\left(\frac{1}{2}\right)} = \sqrt[2]{4}$

$8^{\frac{2}{3}}$

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$$

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Ex: Simplify:

(i) $\left(-\frac{2x^2}{y^3}\right)^3 = \frac{-2^3 x^6}{y^9} = \frac{-8x^6}{y^9}$

(ii) $\sqrt[3]{27^{-2}} = \left(\sqrt[3]{27}\right)^{-2} = (3)^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$=$

$$\left\{\left[(625)^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^{2}$$

$$\left\{\underbrace{\left[(625)^{-\frac{1}{2}}\right]^{-\frac{1}{4}}}\right\}^{2} = \left[(625)^{-\frac{1}{2}\cdot-\frac{1}{4}}\right]^{2} = \left[(625)^{\frac{1}{8}}\right]^{2} = \underbrace{(625)^{\frac{1}{4}}}_{\text{long division } \times} = \left(25^{2}\right)^{\frac{1}{4}}$$

$$(25)^{2\times\frac{1}{4}} = \underline{(25)^{\frac{1}{2}}} = \underbrace{\left(5^{2}\right)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = \underline{5}}$$

$$\sqrt[2]{25} = \underline{5}$$

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$\Rightarrow \left(\left(\frac{3}{2}\right)^4\right)^{-\frac{3}{4}} \times \left[\left(\left(\frac{5}{3}\right)^2\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$\Rightarrow \left(\frac{3}{2}\right)^{4\times -\frac{3}{4}} \times \left[\left(\frac{5}{3}\right)^{2\times -\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$\Rightarrow \left(\frac{3}{2}\right)^{-3} \times \left[\underbrace{\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}}\right]$$

$$\Rightarrow \left(\frac{3}{2}\right)^{-3} \times \left(\frac{5}{3}\right)^{-3} \times \left(\frac{2}{5}\right)^{-3}$$

$$81 = 9^2 = (3^2)^2 = 3^4$$

$$\Rightarrow \left(\frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{2}}{\cancel{5}}\right)^{-3}$$

$$\Rightarrow (1)^{-3}$$

$$= \frac{1}{1^3} = 1$$

$$a^m \cdot b^m \cdot c^m$$

$$(a \cdot b \cdot c)^m$$

(i) $\left[5\left\{\left(\frac{1}{8}\right)^{-\frac{1}{3}}+\left(\frac{1}{27}\right)^{-\frac{1}{3}}\right\}\right]^{-\frac{1}{2}}$

$\left[5\left\{\left(2^{-3}\right)^{-\frac{1}{3}}+\left((3)^{-3}\right)^{-\frac{1}{3}}\right\}\right]^{-\frac{1}{2}}$

$\Rightarrow \left[5\left\{2+3\right\}\right]^{-\frac{1}{2}}$

$\Rightarrow$

$\Rightarrow \left[5.5\right]^{-\frac{1}{2}} \Rightarrow \left[5^{2}\right]^{-\frac{1}{2}} = 5^{-1} = \frac{1}{5}$

(ii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + 2\sqrt[2]{225}$

$(81)^{\frac{1}{4}} - 8(216)^{\frac{1}{3}} + 15.(32)^{\frac{1}{5}} + 2(225)^{\frac{1}{2}}$

$\Rightarrow (3^4)^{\frac{1}{4}} - 8\{(3\times2)^3\}^{\frac{1}{3}} + 15.(2^5)^{\frac{1}{5}} + 2((15)^2)^{\frac{1}{2}}$

$\Rightarrow 3 - 8(6) + 15(2) + 2.15$

$\Rightarrow 3 - 48 + 30 + 30$

$63 - 48$

$\Rightarrow 63-48$

$\Rightarrow 15$ //

$\Rightarrow (3\times2)^3$

$\sqrt[4]{81} = (81)^{\frac{1}{4}}$

3	216
3	72
3	24
2	8
2	4
2	2
	1

$a^n.b^n$

$216 = 3\times3\times3\times2\times2\times2 = 3^3.2^3 = 6^3$

End of the chapter