Volume

Cylinder, Cone and Sphere

Volume

3D shapes (Regular)

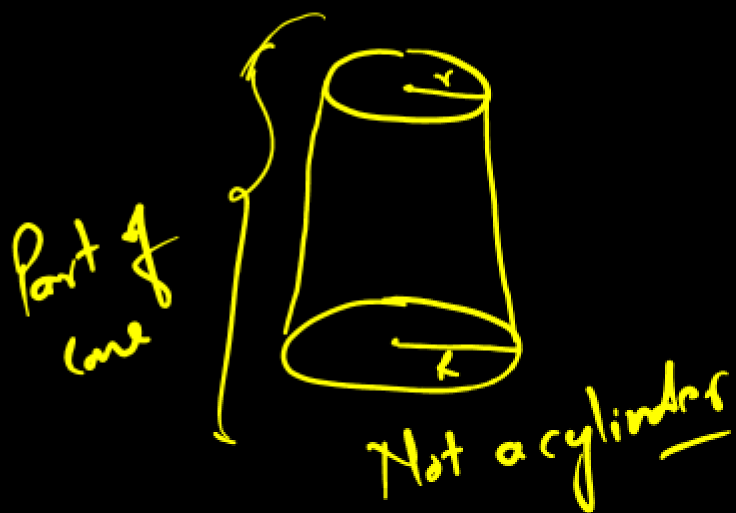
Cube, Cuboid, Cylinder, Cone, Sphere

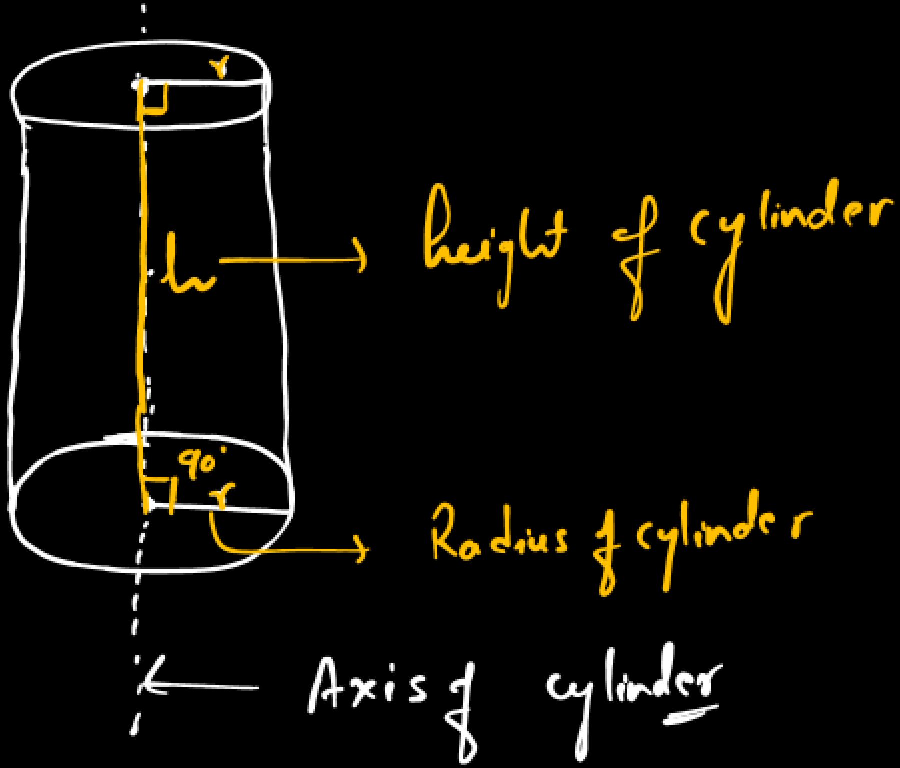
Volume of cube: $l \times l \times l = l^3$ $(\text{unit})^3$

$\boxed{\text{SI unit} = (m)^3}$

Volume of cuboid = $l \times b \times h$ $(\text{unit})^3$

Cylinder (Right circular cylinder)

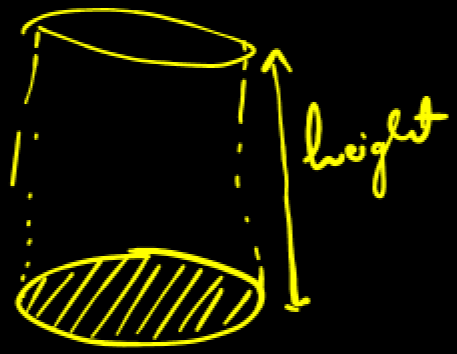


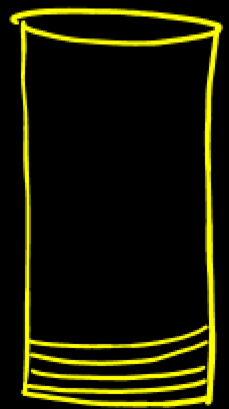


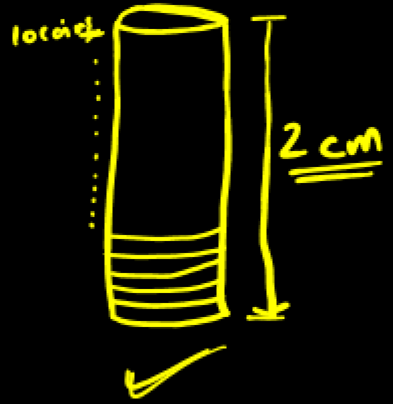
Volume of cylinder $\equiv$ Space occupied by the cylinder.

10 coins $\Rightarrow$ 0.2cm $\Rightarrow$ thickness of coin/disc

$r \to$ 1cm







Area of 1 ~~disc~~ coin $= \pi r^2$

$= \pi (1)^2$

$= \pi$

Total space occupied by 10 coins stacked one above other

$= \pi r^2 \times h$

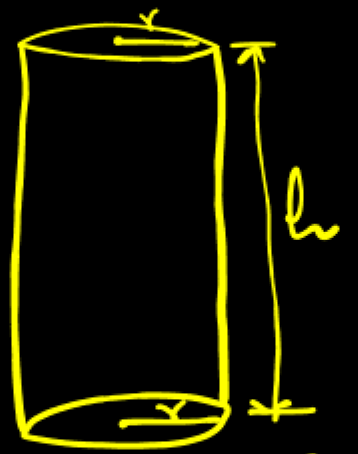
$$\boxed{\text{Volume of cylinder} = \pi r^2 h}$$

Volume of cylinder = area of base x height

$$\boxed{\text{Volume} = \pi r^2 \times h}$$

$(\text{unit})^3$

Solid cylinder



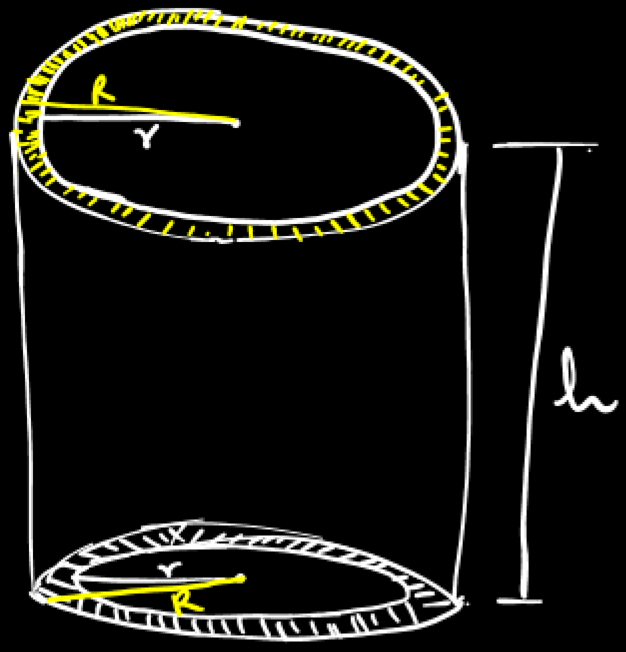
$\boxed{1\,cm^3 = 1\,ml}$

Volume of Hollow cylinder

$= \text{Area of base} \times \text{height}$

$= (\pi R^2 - \pi r^2) \times h.$

$$\boxed{\text{Volume of H. Cylinder} = \pi(R^2 - r^2)\,h.}$$



$a^2 - b^2 =$

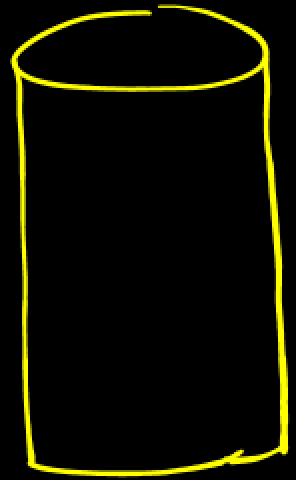
$(a-b)^2 = a^2 + b^2 - 2ab$

$R^2 - r^2 = (R+r)(R-r)$

$(a+b)^2 = a^2 + b^2 + 2ab$

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

Diameter of its base is 4 cm.
Height of the cylinder is 35 cm.
Find the volume of solid cylinder.



Volume of cylinder $= \pi r^2 h$

$= \pi (2)^2 \times 35$

$= \frac{22}{\cancel{7}} \times 4 \times \cancel{35}^{5}$

$= \underline{\underline{22 \times 4 \times 5}} \quad \underline{\underline{cm^3}}$

$= 22 \times 20$

$= \underline{\underline{440 \; cm^3}}$

$\begin{cases} \pi = 3.14 \\ \pi = \frac{22}{7} \end{cases}$

A metal pipe is 77 cm long. The inner diameter of the cross section is 4 cm and the outer diameter is 4.4 cm. Find the volume of the material with which the pipe is made of.

Volume of hollow cylinder $= \pi(R^2 - r^2)h$.

$= \frac{22}{7}(2.2^2 - 2^2)\, 77$

$= \frac{22}{\cancel{7}}(4.2)(0.2)\, \cancel{77}^{11}$

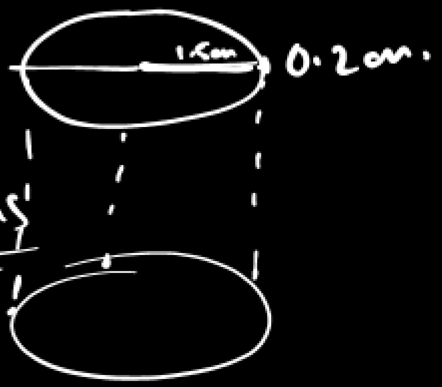
$= 11 \times 22 \times 0.84$

$= 242 \times 0.84 \quad cm^3$

$= \underline{203.28} \quad cm^3$

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

Q. Find the no. of 1.5 cm in diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 5 cm ~~15 cm~~ and diameter ~~4.5 cm.~~ 4.5 cm.

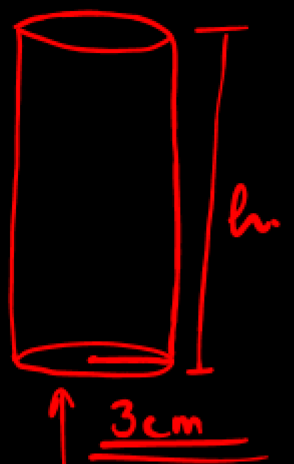


$\Rightarrow x$ coins

volume of x coins = volume of new cylinder

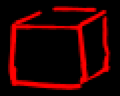
$x \times \cancel{\pi}\underbrace{(0.75)^2 \times 0.2}_{\text{vol. of 1 coin}} = \cancel{\pi} \times (2.25)^2 \times 5$

$$\boxed{x = \frac{(2.25)^2 \times 5}{(0.75)^2 \times 0.2}}$$



height = ?

→ melt → 11 ~~■~~ cubes → each cube dimension is ~~1cm~~ 1.5 cm. = l



volume of cylinder = $\pi r^2 h$.

vol. of cylinder = vol. of 11 cubes.

$\pi r^2 h = 11 \cdot l^3$

$\pi\, 3^2 \times h = 11 \cdot (1.5)^3$

$\cancel{\frac{22}{7}}^{\,2} \times 3^2 \times h = \cancel{11} \cdot (1.5)^3$

$\frac{2}{7} \times 9 \times h = (1.5)^3$

$h = \frac{(1.5)^3 \times \cancel{7}^{\,3.5}}{\cancel{2} \times 9}$

$h = \frac{\cancel{1.5}^{\,0.5} \times \cancel{1.5}^{\,0.5} \times 1.5 \times 3.5}{\cancel{3} \times \cancel{3}}$

vol. of 1 cube = l^3

vol.

$\pi \approx 3.14 \approx \frac{22}{7}$

5.25
0.25

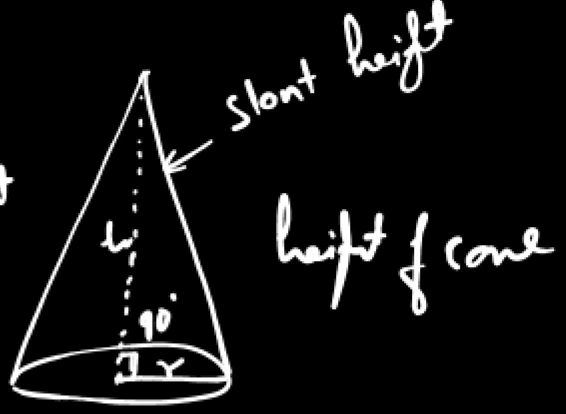
1.5
3.5
75
45×
5.25

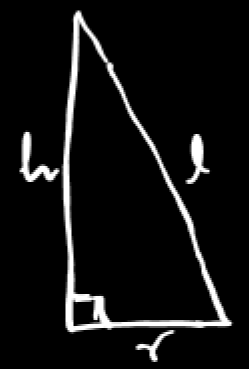
$0.5 \times 0.5 \times 1.5 \times 3.5$

$= 0.25 \times 1.5 \times 3.5$

$\boxed{h = 1.3125 \text{ cm}}$

Cone

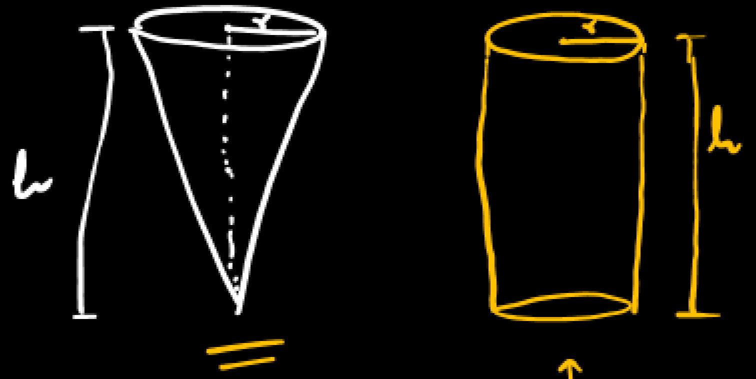




$$l^2 = h^2 + r^2$$

$$\boxed{l = \sqrt{h^2 + r^2}}$$ slant height

Volume of cone :



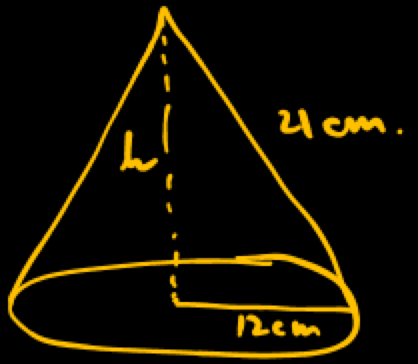
vol. of 3 cones = vol. of 1 cylinder. vol. = $\pi r^2 h$

$$\boxed{\text{vol. of cone} = \frac{1}{3}\pi r^2 h}$$

r = radius of cone
h = height of cone.

If the slant height of cone is 21 cm and diameter of its base is 24 cm. find its height and its volume.

$a^2-b^2 = (a+b)(a-b)$



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{\cancel{3}}\pi (12)^2\, \cancel{3}\sqrt{33} \quad cm^3$$

$$= 3.14 \times 144 \times \sqrt{33} \quad cm^3$$

$$= 451.36\sqrt{33} \quad cm^3$$

$$h^2 + 12^2 = 21^2$$

$$h^2 = 21^2 - 12^2$$

$$h^2 = (33)(9)$$

$$h^2 = 297$$

$$h = \sqrt{297} = 3\sqrt{33}\ cm$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Slant height of cone $= \sqrt{r^2+h^2}$

$\boxed{\text{Base of cone is circular}}$

↳ Area of base of cone $= \pi r^2$

The volume of a conical tent is 1232 m^2, and the area of the base of floor is 154 m^2. Calculate the following:

(i) Radius of the floor. (ii) height of the tent.

Sol:

Area of the circular floor = 154 m^2

$$\boxed{\pi r^2 = 154\ m^2}$$

$$r^2 = \frac{154}{\pi}$$

$$r^2 = \frac{154}{1} \div \frac{22}{7}$$

$$= 154 \times \frac{7}{22}$$

$$r^2 = 49$$

$$r = \sqrt{49} = 7\,m$$

Volume of tent = 1232 m^2

$$\frac{1}{3}\boxed{\pi r^2}h = 1232$$

$$\frac{1}{3} \times 154 \times h = 1232$$

$$h = \frac{1232 \times 3}{154}\ m$$

$$\boxed{h = 24\ m}$$

Volume of Sphere

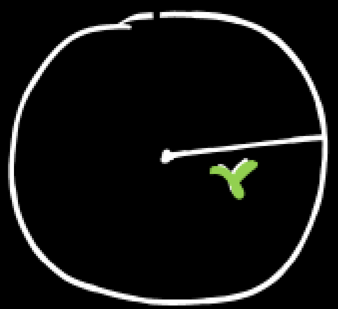
Volume of sphere $= 4 \times \frac{1}{3}\pi r^3$

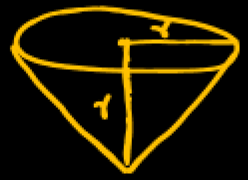
$$\boxed{\text{vol. of sphere} = \frac{4}{3}\pi r^3}$$

vol. of solid sphere $(r) = \frac{4}{3}\pi r^3$

to fill the given sphere completely

4 x complete cones





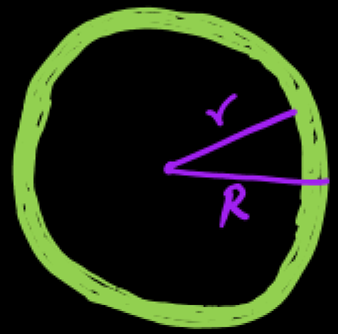
Volume of cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 \cdot r$

$= \frac{1}{3}\pi r^3$

Volume of a spherical shell.



Volume of spherical shell $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

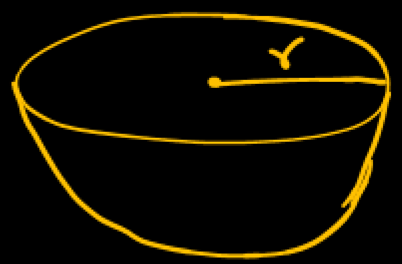
$= \frac{4}{3}\pi (R^3 - r^3)$

Volume of the material of spherical shell $= \frac{4}{3}\pi (R^3 - r^3)$

Volume of hemisphere $= \frac{1}{2}$ (of sphere)

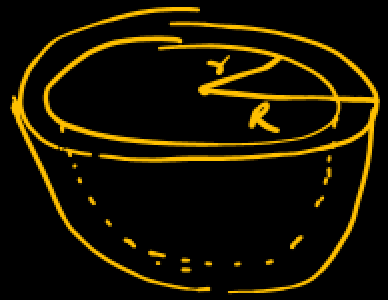
$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{4}^{2}}{3}\pi r^3\right)$$

$$= \frac{2}{3}\pi r^3$$



Volume of hemispherical shell $= \frac{2}{3}\pi\left(R^3 - r^3\right)$

Volume of the material of hemispherical shell.



Q. Find the volume of a sphere of diameter 14 cm.

Sol. diameter of sphere = 14 cm.

radius(r) of sphere = 7 cm.

volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 7$

$= \frac{88 \times 49}{3}$

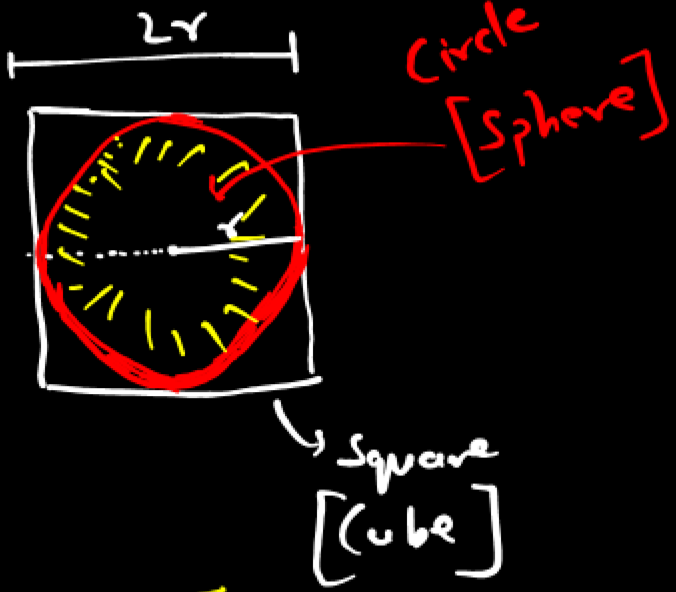
$= \frac{4312}{3}$

$= 1437.33 \text{ cm}^3$

$88\cancel{x}$
$\times\ 49$

49
$\times\ 88$

Q. If a sphere is inscribed in a cube, then find the ratio of the volume of the cube to the volume of the sphere.



Sol: Let the radius of sphere be r unit
$\therefore$ length of cube $= 2r$ unit.

Volume of cube $= (2r)^3$ unit^3 $= 8r^3$ unit^3

Volume of sphere $= \frac{4}{3}\pi r^3$

$$\therefore \frac{\text{volume of cube}}{\text{volume of sphere}} = \frac{8\cancel{r^3}}{\frac{4}{3}\pi \cancel{r^3}} = 8 \div \frac{4\pi}{3} = \overset{2}{\cancel{8}} \times \frac{3}{\cancel{4}\pi} = \frac{6}{\pi}$$

Ratio of vol. of cube to the vol. of sphere will be $\underline{6 : \pi}$

Q. What quantity of milk can a hemispherical bowl of diameter 10.5 cm hold?

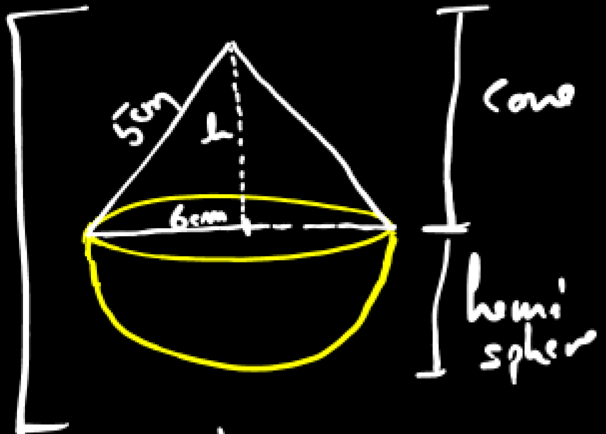
$\boxed{\sim 303.03 \text{ cm}^3}$

$\boxed{30\pi}$

Q. In the fig. a hemisphere is surmounted by a conical block of wood. The diameter of their base is 6 cm each. and the slant height of the cone is 5 cm. Calculate:

(i) the height of the cone = 4 cm

(ii) the volume of the solid.



Solid ⇓

vol. (cone + hemisphere)

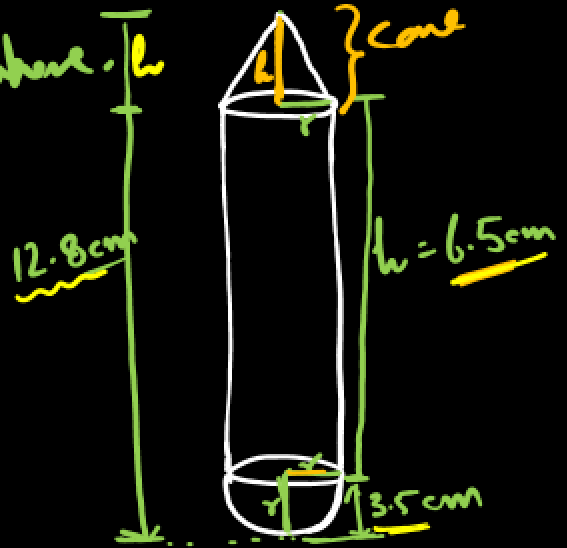
$30(\pi)$ $30 \times 3.14 = 94.2 \text{ cm}^3$

Q. A solid consist of a cylinder surmounted by a cone at one end and a hemisphere at the other end. Given that common radius = 3.5 cm, the height of the cylinder = 6.5 cm and the total height = 12.8 cm. Calculate the volume of solid.

Vol. of solid = vol. of cone + vol. of cylinder + vol. of hemisphere.

$$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= 119.65\,\pi \text{ cm}^3$$



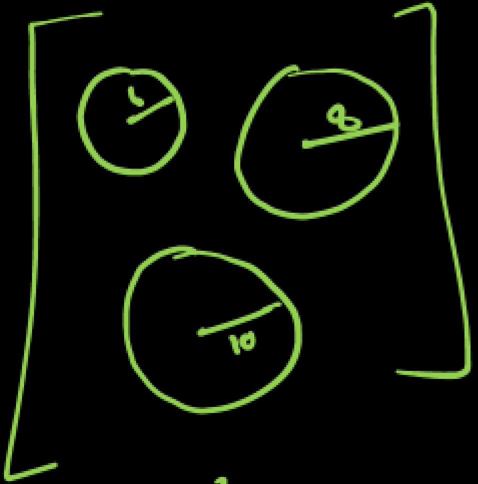
$r = 3.5$ cm

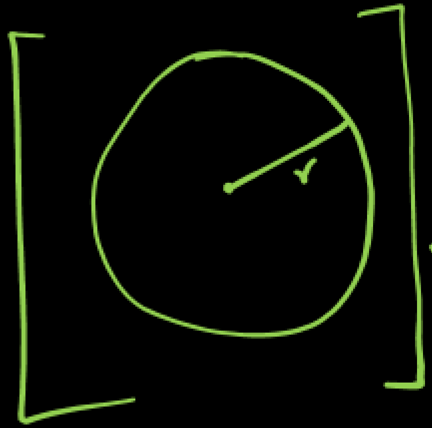
$$h = 12.8 - (6.5 + 3.5)$$
$$= 12.8 - 10$$
$$= 2.8 \text{ cm}$$

Q. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

[Total volume before melting = total volume after melting]

old shape ... new shapes

vol. 

= vol. 

$$\frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3 = \frac{4}{3}\pi r^3$$

$$\cancel{\frac{4}{3}\pi}\left(6^3 + 8^3 + 10^3\right) = \cancel{\frac{4}{3}\pi} r^3$$

$$\begin{array}{r} 36 \\ 6 \\ \hline 216 \\ \hline \end{array}$$

$$6^3 + 8^3 + 10^3 = r^3$$

$$216 + 512 + 1000 = r^3$$

$$r^3 = 1728$$

$$r = \sqrt[3]{1728}$$

$$r = \sqrt[3]{12^3}$$

$$\boxed{r = 12 \text{ cm}}$$

Q. Two solid spheres of radii 2cm and 4cm are melted and recast into a cone of height 8cm. Find the radius of the cone so formed.

$$\underbrace{\frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi(4)^3} = \frac{1}{3}\pi r^2(8)$$

$$\frac{4}{\cancel{3}}\cancel{\pi}\times(2^3+4^3) = \frac{1}{\cancel{3}}\cancel{\pi} r^2 8$$

$$\boxed{\frac{\cancel{1}}{\cancel{3}}\times 4(2^3+4^3) = \frac{\cancel{1}}{\cancel{3}} r^2 8}$$

$$4(2^3+4^3) = r^2 8$$

$$4(8+64) = r^2 8$$

$$r^2 8 = 4\times(72)$$

$$r^2 = \frac{4(8+64)}{8}$$

$$r^2 = \frac{\cancel{4}\times\cancel{72}^{36}}{\cancel{8}_{\cancel{2}}}$$

$$r^2 = 36$$

$$\boxed{r = 6\text{ cm}}$$

Q. 504 cones, each of diameter 3.5cm (height 3 cm) are melted and recast into a metallic sphere. Find the diameter of the sphere.

$$\frac{1}{3}\,\underline{168} \times (1.75)^2 \times 3 = \frac{4}{3} r^3$$

$$\left(\frac{\cancel{175}^{\,7}}{\cancel{100}_{\,4}}\right)^2$$

$$\frac{\cancel{24} \times \cancel{\pi} \times 7^2}{\cancel{24}} = \frac{\cancel{24} \times \cancel{\pi}\, 6^2}{\cancel{24}}$$

$$\boxed{r = 10.5 \text{ cm}} \; \times$$

$$\boxed{r = 21 \text{ cm}} \; \checkmark$$

$$3.5 = \frac{35}{10} \quad \frac{1}{2} \times \frac{\cancel{35}^{\,7}}{\cancel{10}_{\,2}} = \left(\frac{7}{4}\right)$$

Q. A wooden article was made by scooping out a hemispher from each end of the cylinder. If the height of the cylinder is 12 cm and its base is of radius 4.2 cm, find the volume of the wood left in the article.

End of the chapter