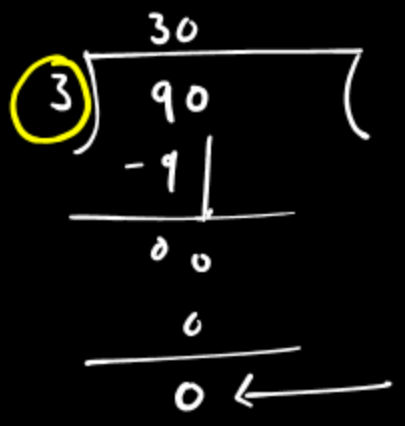
Factors and Multiples

Grade 4: Number System

Factors and Multiples

Factors :

3 is factor of 90



when a number divides another number completely, then the divisor is called a factor of dividend

For example: ① Factor(s) of 12

6 is a factor of 12.
3 is a also a factor of 12.
4 is another factor of 12
2 is also a factor. of 12
1 is also a factor of 12

All factors of 12 are: 1, 2, 3, 4, 6, 12

(ii) 7 divides 49 completely.,

7 is a factor of 49

(iii) $3 \times 4 \times 5 = 60$

$\therefore$ 3, 4 and 5 are factors of 60.

Q. Is 11 a factor of 1034?

Lets divide 1034 by 11

$$\begin{array}{r} 94 \\ 11 \overline{)1034} \\ \underline{1034} \\ 0 \end{array}$$

∵ 11 divides 1034 completely

∴ 11 is a factor of 1034

Q. check if 15 is a factor of 1309

$$15 \overline{)1309}(87$$

Remainder: 4

$\because$ ⇒ Since or Because

$\because$ 15 does not divide 1309 completely.

$\therefore$ 15 is not a factor of 1309.

Q Find all the factor of 24.

$24 = 1 \times 24$

$24 = 2 \times 12$

$24 = 3 \times 8$

$24 = 4 \times 6$

∴ all factors of 24 are : 1, 2, 3, 4, 6, 8, 12, and 24

Factor(s) can never be greater than the given no.

Q. Find all the factors of 48.

$24 \times 2 = 48$

$12 \times 4 = 48$

$48 \times 1 = 48$

$3 \times 16 = 48$

$6 \times 8 = 48$

$48 \overline{)0}$

$$3 \overline{)48} \quad 16$$
$$-3\downarrow$$
$$18$$

∴ All the factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

$48 \div 0$ Not defined in math.

$2 \div 0$ = infinite

$5 \div 0$ = infinite

Division by zero is not defined.

Properties of factors

① Every non-zero number has a limited number of factors.

eg. • factors of 1 ⇒ 1

• factors of 18 ⇒ 1, 2, 3, 6, 9, 18

• factors of 24 = (1), 2, 3, 4, 6, 8, 12, (24)

Imp: 1 is a factor of every number & it is the smallest factor of every number.

2 | 12

1 × 24 = 24
2 × 12 = 24
3 × 8 = 24
4 × 6 = 24

(2) Every non-zero number is the largest factor of itself.

eg: 24 is the largest factor of 24.

(3) Every non-zero number is a factor of zero.

eg: 1 is a factor of 0

3 is also a factor of 0

Imp: Zero (0) has unlimited factors

(4) **Zero**: Division by zero is not-defined.

Therefore, zero is not a factor of any number.

(5) Every factor of a non-zero number is either less than or equal to the given number.

Eg: factors of 25 are : 1, 5, 25

factors of 19 are : 1, 19

factors of 11 are : 1, 11

7 : 1, 7

29 : 1, 29

9
+9

9.5
+9.5

Highest Common Factors (HCF)

Eg: Factors of 6 : 1, 2, 3, 6

Factors of 9 : 1, 3, 9

Common factor of 6 and 9 are : 1, 3

$\Rightarrow$ Highest common factor of 6 and 9 is 3.

$\Rightarrow$ HCF of 6 and 9 is 3.

Q. Find the HCF of 18 and 27

Sol:

Factors of 18 : 1, 2, 3, 6, 9, 18

Factors of 27 : 1, 3, 9, 27

HCF

HCF of 18 and 27 is 9

"The HCF of two of more numbers is the largest number that divides each of the given numbers completely."

$1 \times 18 = 18$

$2 \times 9 = 18$

$3 \times 6 = 18$

&

$1 \times 27 = 27$

$3 \times 9 = 27$

Find HCF of 16 and 23.

Sol: factors of 16 : 1, 2, 4, 8, 16

factors of 23 : 1, 23

HCF of 16 and 23 is 1

Q. Find HCF of 8 and 12.

factors of 8 are : 1, 2, 4, 8

factors of 12 : 1, 2, 3, 4, 6, 12

HCF of 8 and 12 is 4.

Symbol for Zero in Roman numerals:

(a) 0 (b) X−X (c) Does not exist (d) None of these.

Find the HCF of 27 and 36.

Factors of 27 : 1, 3, 9, 27

Factors of 36 : 1, 2, 3, 4, 6, 9, 12, 18, 36

HCF of 27 and 36 is 9.

$$\begin{array}{r} 17 \\ \times\ 2 \\ \hline 34 \\ \hline \end{array}$$

$3 \times 9 = 27$

$$\begin{array}{r} 13 \\ \times\ 2 \\ \hline 26 \\ \hline \end{array} \qquad \begin{array}{r} 14 \\ \times\ 2 \\ \hline 28 \\ \hline \end{array}$$

$2\overline{)36}$

$2 \times 18 = 36$

$3 \times 12 = 36$

$4 \times 9 = 36$

Multiples

"When a number divides another number completely, then the dividend is called a multiple of the divisor."

For examples

⇒ ∵ 4 divides 8 completely

⇒ ∴ 8 is a multiple of 4

Eg. Multiples of 2 : 2, 4, 6, 8, 10, 12, ... ∞

multiples of 3 : 3, 6, 9, 12, 15, ... ∞

Multiples of 4 : 4, 8, 12, 16, ... ∞

Multiples of 9 : 9, 18, 27, 36, ... ∞

Multiples of 9 (∞)

1, 3, 9, 18, 27, 36, 45, ...

factors of 9

(finite)

(limited)

Q. Is 1378 a multiple of 13?

$13 \times 106 = 1378$

$\because$ 13 divides 1378 completely

$\therefore$ 1378 is a multiples of 13.

[$\because$ since / because
$\therefore$ therefore

Also, 13 is a factor of 1378.

Q. Find sixth multiple of 18!

Sol. $18 \times 6 = 108$

18
× 6

Properties of multiples

(i) Every non-zero number has unlimited number of multiples.

eg. multiples of 10 : 10, 20, 30, ... ∞

(ii) Every number is a multiple of 1.

eg: Multiples of 1 : 1, 2, 3, 4, 5, 6, ... ∞

(iii) Every non-zero number is a multiple of itself.

eg. 3 is a multiple of 3.

H.W.

Q1: Find the HCF of:

(a) 16 and 21

(b) 20 and 28

(c) 45 and 75

(d) 54 and 81

(e) 77 and 121

Q2: Is 3016 a multiple of 29?

Q3: Write first 9 multiples of 17.

(iv) Zero is a multiple of every non-zero number.

eg. multiples of 3: $3\times0=\boxed{0}$, $3\times1=3$, $3\times2=6$, ⋮

multiples of 7: $7\times0=\boxed{0}$, $7\times1=7$, ⋮, 14

(v) Every multiple of non-zero number is either greater than or equals to the given number.

(vi) First multiple of every number is the number itself.

Lowest Common Multiple (LCM)

Find LCM of 6 and 8.

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, –, –, –, –

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, –, –, –

Lowest common multiple

"The LCM of two or more numbers is the smallest common multiples of those numbers."

Q: Find the LCM of 9 and 12.

Sol:

Multiples of 9 : 9, 18, 27, 36, 45, 54, 63, 72, ...

Multiples of 12 : 12, 24, 36, 48, 60, 72, 84,

$\Rightarrow$ LCM of 9 and 12 is 36

eg: LCM of 2 and 6

LCM of 2 and 6 is 6

[Note: LCM is either greater than or equal to the highest given nos.]

Find LCM of 2, 3 and 4

Multiples of 2 : 2, 4, 6, 8, 10, 12, 14, ...

Multiples of 3 : 3, 6, 9, 12, 15, 18, ...

Multiples of 4 : 4, 8, 12, 16, 20, 24, 28, 32, ...

LCM of 2, 3, and 4 is 12

Find LCM of 3, 4 and 6

multiples of 3 : 3, 6, 9, 12, ...

multiples of 6 : 6, 12, 18, ...

multiples of 4 : 4, 8, 12, ...

LCM of 3, 4 and 6 is 12

H.W.

Find LCM of:

(i) 8 and 12

(ii) 10, 15, 20

(iii) 2, 6, 8.

Prime numbers and Composite Numbers

2, 3, 5, 7, 11, 13, 17, 19. → Prime numbers.

factors of 2 = 1, 2

factors of 3 = 1, 3

5 = 1, 5

7 = 1, 7

11 = 1, 11

13 = 1, 13

17 = 1, 17

19 = 1, 19

Prime numbers are the counting numbers that has got only-and-only two factors, the number itself and 1.

eg: 2, 3, 5, 7, 11, 13, 17, 19, 23, ..., 29

Composite Numbers:

A counting number having more than two factors is called a composite number.

Examples: 4, 6, 8, 9, 10, 12, 14, 15, 16, ...

Imp. Note:

(i) The smallest prime no. is 2

(ii) 2 is the only even prime number.

(iii) 4 is the smallest composite.

(iv) 1 is neither prime nor composite

Method to find all the prime numbers upto 100.

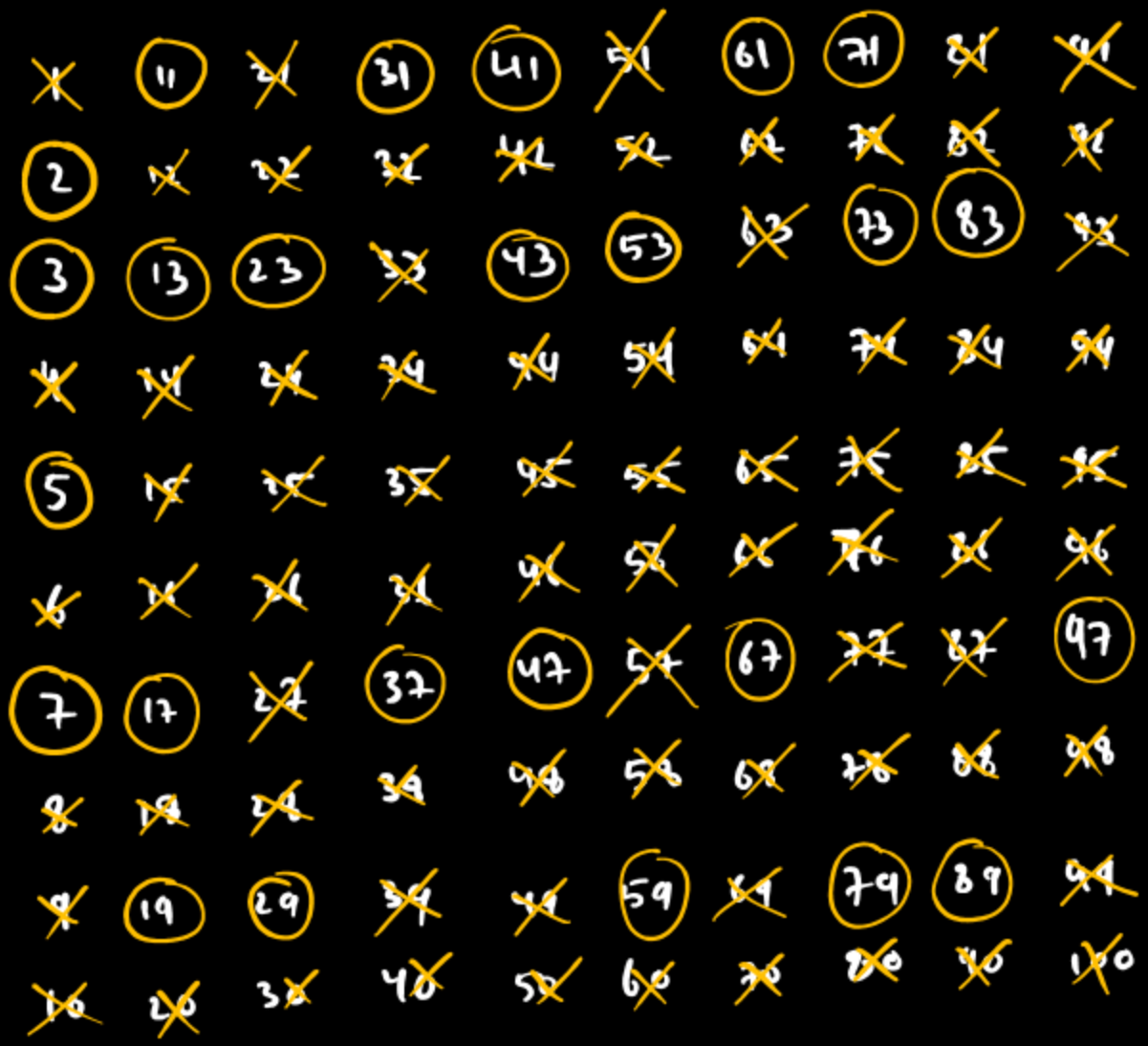
Method:

Step 1: Cross out 1.

Step 2: "Cross out all the multiples of single digit prime numbers, except the numbers itself."

* Cross out multiples of 2 except 2.
* Cross out multiples of 3 except 3.
* Cross out multiples of 5 except 5.
* Cross out multiples of 7 except 7.

$7 \overline{)91}$ (



Prime factorisation

eg: factrs of 12 : 1, 2, 3, 4, 6, 12

3 × 2

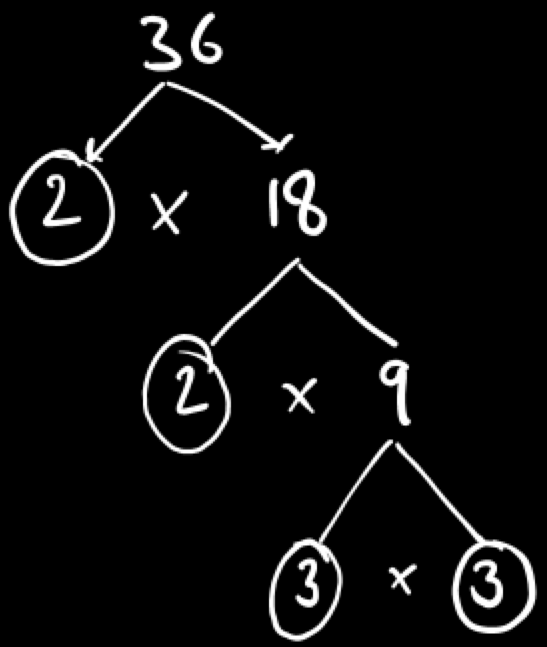
2 × 2

Every composite number can be written as the product of prime factors.

eg:
- 4 ⇒ 2 × 2
- 6 ⇒ 3 × 2
- 8 ⇒ 2 × 2 × 2
- 9 ⇒ 3 × 3

} "Prime factorisation"

eg. Find prime factorisation of : 36



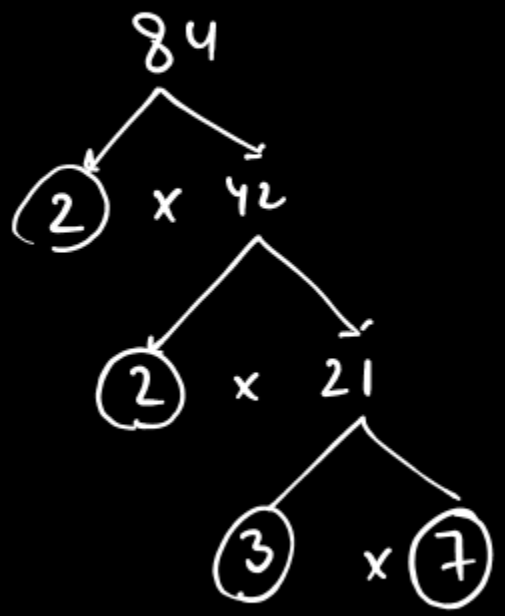
Factor tree
(Prime factor tree)

2	36
	18

$$2\overline{)36}$$
18
−2
16

$$36 = 2 \times 2 \times 3 \times 3$$

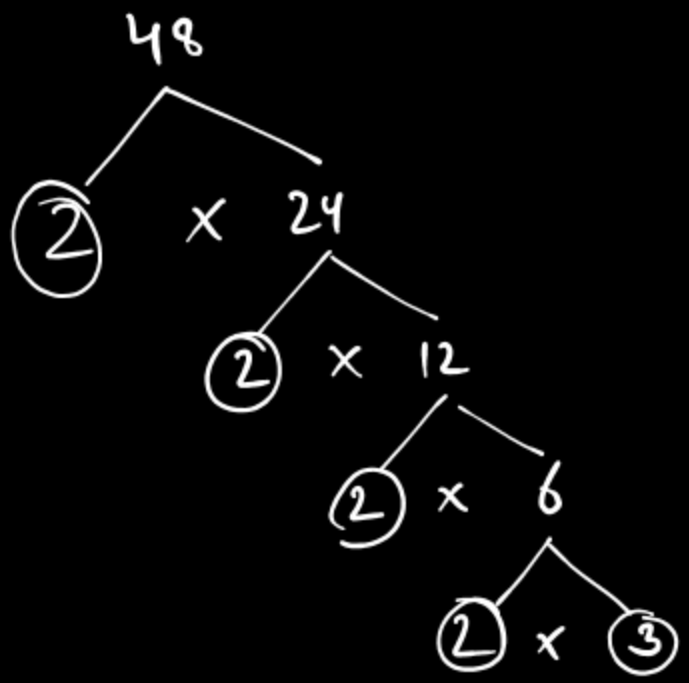
Prime factorisation of 84.



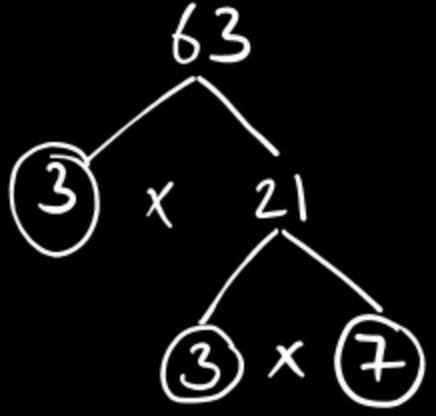
Factor tree
or
Prime factor tree

$84 = 2 \times 2 \times 3 \times 7$

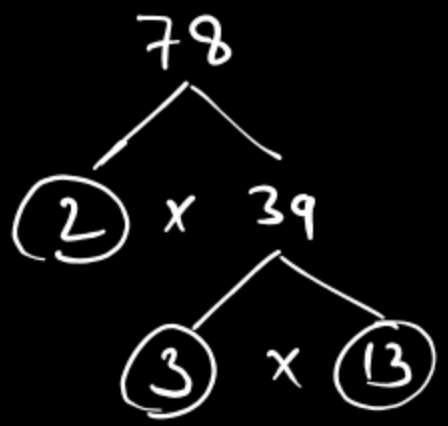
Prime factorisation :



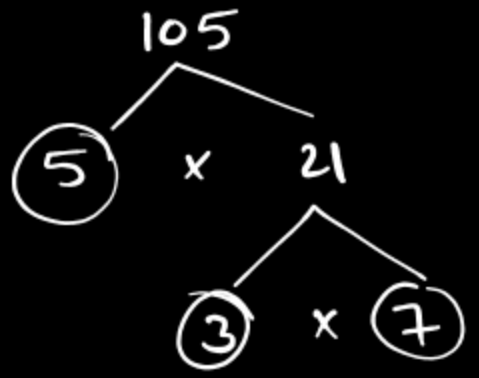
$48 = 2 \times 2 \times 2 \times 2 \times 3$



$63 = 3 \times 3 \times 7$



$78 = 2 \times 3 \times 13$



$105 = 3 \times 5 \times 7$

Test of Divisibility

① Test of divisibility by 2:

eg 8 is divisible by 2
10 is divisible by 2

150

163

123578

by 2

A given number is divisible by 2, if its unit digit is 0, 2, 4, 6, or 8.

H.W.

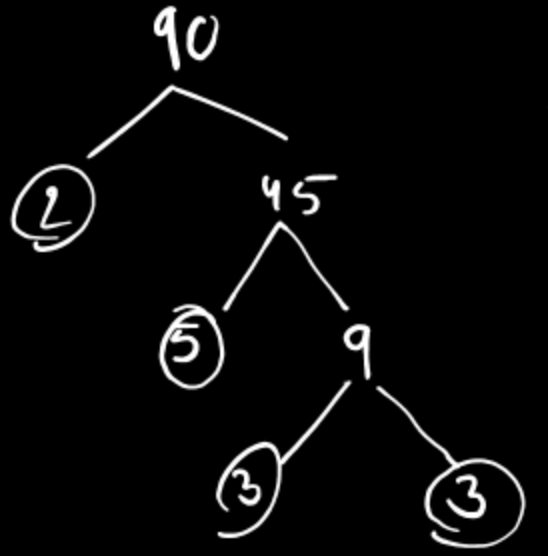
Find the prime factorisation of the following numbers by building factor trees.

(i) 90 = ~~2x5x5~~ 2x3x3x5

(ii) 54 = 2x3x3x3

(iii) 144

(iv) 196

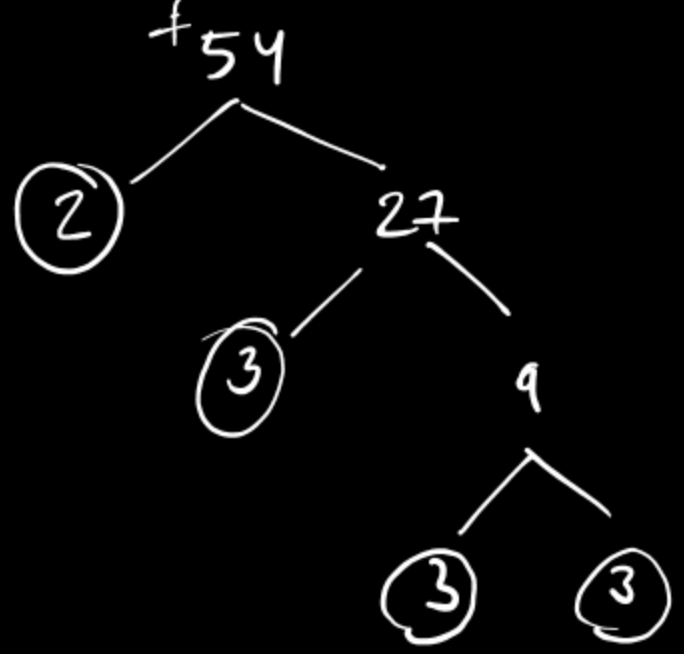


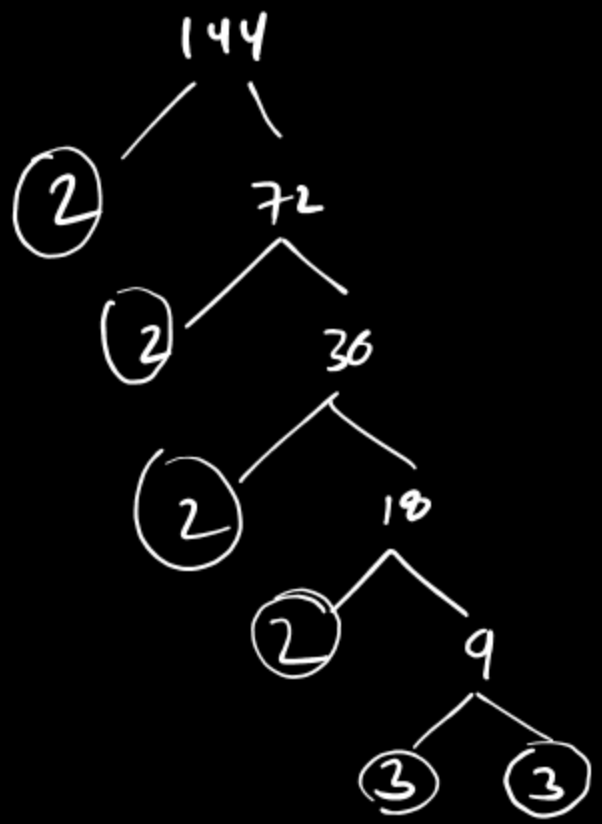
5x7 = 35

3x7 = 21

4x5 = 20

4x9 = 36





2	144
2	72
2	36
2	18
3	9
3	3
	1

Division method

$2 \overline{)144}$

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$144 \div 2 = \frac{144}{2}$

$$\frac{144}{2}$$

$\frac{4}{2} = 2$

$\frac{10}{2} = 5$

$\frac{12}{2} = 6$

$\frac{24}{2} = 12$

$\frac{34}{2} = 17$

$\frac{58}{2} = 29$ R 1

$2 \overline{)34}$

-2

14

-14

0

28

$\frac{\cancel{516}}{2}$

37

$\frac{714}{2}$

54

$\frac{\cancel{108}}{2}$ =

18

$\frac{\cancel{54}}{3}$

$\frac{5}{3}$

$3\overline{)54}$

21

$\frac{\cancel{63}}{3}$

327

$\frac{\cancel{981}}{\cancel{3}}$

238

$\frac{7\overset{2}{1}4}{3}$

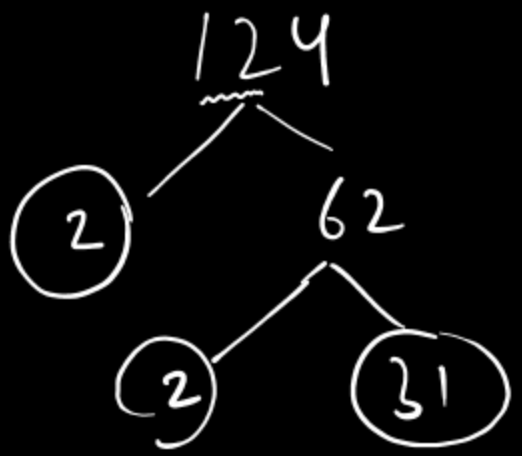
$196 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$ ✗

$196 = 2 \times 2 \times 7 \times 7$

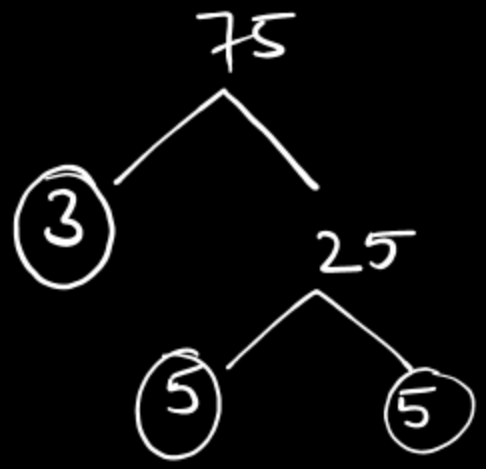
2	196
2	98
7	49
7	7
	1

→ Division method

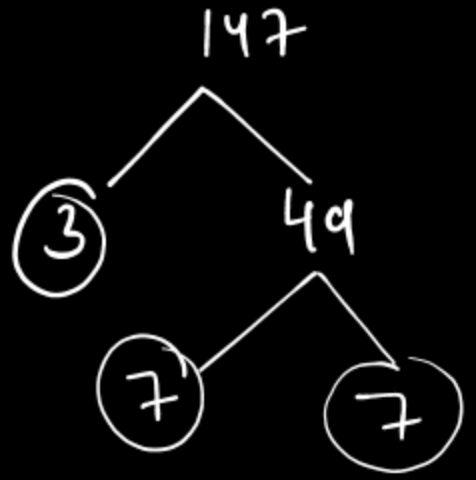
✓ $\left(\frac{196}{2}\right)$ $\left(2\overline{)196}\right)$



$124 = 2 \times 2 \times 31$



$75 = 3 \times 5 \times 5$



$147 = 3 \times 7 \times 7$

$$\frac{147}{3}$$

$$\begin{array}{r} 49 \\ +\cancel{147} \\ \hline 3 \end{array}$$

Find Prime factors of:

(I) $81 = 3 \times 3 \times 3 \times 3$

(II) $100 = 2 \times 2 \times 5 \times 5$

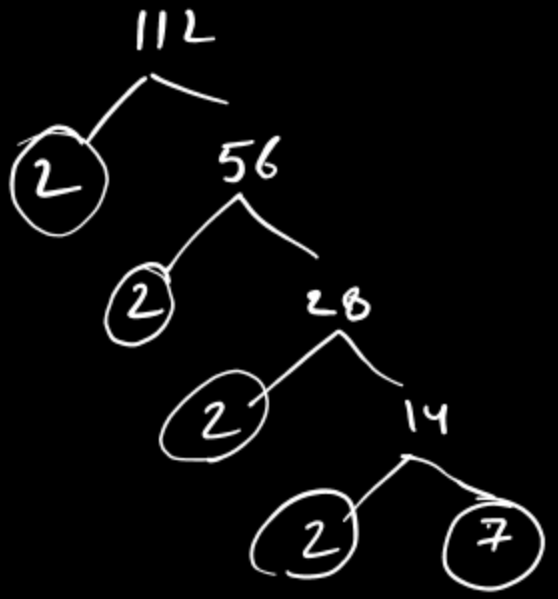
(III) $225 = 3 \times 3 \times 5 \times 5$

(IV) $242 = 2 \times 11 \times 11$

Find prime factors of 112.

$$112 = 2 \times 2 \times 2 \times 2 \times 7$$

56
2 | 112
2



Find LCM of 8 and 12.

8 = 8, 16, 24, 32, ...

12 = 12, 24, 36, ...

Find HCF of 16 and 21.

16 = 1, 2, 4, 8, 16

21 = 1, 3, 7, 21

HCF(16, 21) = 1

Tests of Divisibility

① Test of Divisibility by 2

1562

A given number is divisible by 2, if its unit digit is 0, 2, 4, 6, or 8.

Eg: 3490, 4672, 725890 divisible by 2

35291, 497, 599 Not divisible by 2

(11) Test of divisibility by 3.

"A given number is divisible by 3, if the the sum of its digits is divisible by 3."

eg. (i) 61702 ⇒ 6+1+7+0+2 = 16

16 is not divisible by 3

∴ 61702 is not divisible by 3.

(ii) 57102 ⇒ 5+7+1+0+2 = 15 ⇒ divisible by 3

∴ 57102 is divisible by 3.

(iii) 56022 divisible by 3

(iv) 64285 ⇒ 6+4+2+8+5 = 25

✗ ↳ not divisible by 3

(v) 64284 ✓

64287 ⇒ ✓

Even Numbers :

Numbers which are divisible by 2 are called even numbers.

eg: 0, 2, 4, 6, 28,

1000, 1056, 2489 Not even

Odd Numbers :

⇒ Numbers which are not divisible by 2 are odd numbers.

eg: 47, 59, 3, 9, 11

even + even = even

even + odd = odd

odd + odd = even

even x even = even

odd x even = even

odd x odd = odd

$3 \times 5 = \underline{\underline{15}}$

$\underline{3} \times \underline{7} = \underline{\underline{21}}$

(3) Test of divisibility by 4.

"A number is divisible by 4, if its last two digits is divisible by 4 or if its last two digits are 00."

Eg: 1236 ✓ 36 is divisible by 4

56944 ✓ 44 is divisible by 4.

$4 \overline{)44}$

$\frac{44}{4}$

26270000 ✓

(4) Test of divisibility by 5.

A given number is divisible by 5, if its unit digits is 0 or 5.

Eg: 20215 ✓

4L910 ✓

5521 ✗

5
10
15
20
25
30
35
40

5. Test of divisibility by 6

A given number is divisible by 6, if it is divisible by both 2 and 3.

36

eg: (i) 13074

div. by 2 ✓
div. by 3 ✓
} div. by 6

(ii) 53204

→ div. by 2.
→ not div. by 3
} Not. div. by 6.

(iii) 53304

div. by 2
div. by 3
} div. by 6

6. Test of divisibility by 9. (Similar to test of div. by 3)

⇒ A given number is divisibly by 9, if the sum of its digits is divisible by 9.

eg: (i) 33714 ⇒ 3+3+7+1+4 = 18 → div. by 9
↳ divisible by 9

(ii) 14607 → 1+4+6+0+7 = 18 ✓
↳ div. by 9.

81
54

(iii) 32604 → 3+2+6+0+4 = 15 ✗
✗ → do not div. by 9.

(7) Test for divisibility by 10

A given number is divisible by 10, if its unit digit is a zero.

$10 \times 1 = 10$

$= 10$

30

40

eg: 23610 → ✓

64900 ✓ div. by 10 [also div. by 4]

Select prime no.

a) 91 ✗

b) 93 ✗

c) 97 ✓

d) 99 ✗

Q. Which of the following is a common factor of 44 and 121

(a) 3
(b) 11
(c) 5
(d) 9

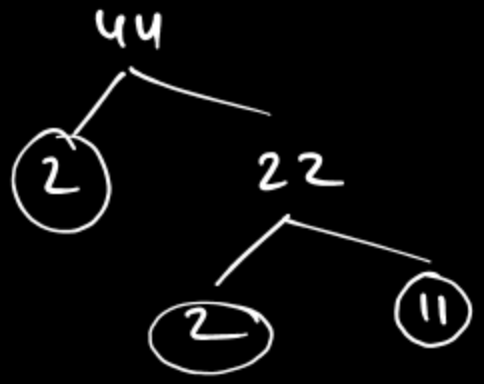
Factor of 44 = 1, 2, 4, 11,
44 = 2×2×11

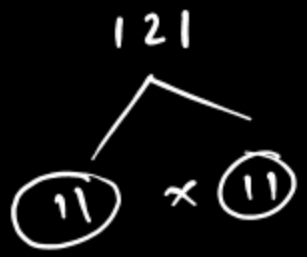
factors of 121 = 11×11

11×11 = 121

12×12 = 144

13×13 = 169





Q. Product of second multiple of 5 and 4 is :

× 0

a) 20

b) ~~40~~ ✓

c) 50

d) 24

10 × 4 = 4

Q. Find the value of X and Y in the given multiplication problem:

$$\begin{array}{r} 3\ 5\ 6 \\ \times\quad X\ 6 \\ \hline 2\ 1\ 3\ 6 \\ +\ 7\ Y\ 2\ 0 \\ \hline 9\ 2\ 5\ 6 \end{array}$$

(a) 2, 3

(b) ~~2, 1~~

(c) Y, 1 ×

(d) Y, 2 ×

Q. If $\Delta + \square + \square + \square + \square = 120$ and

$\square + \square + \square - \Delta = 55$, then $\square$ = ___?

a) 25

b) 30

c) 20

d) 35

$\square = 20$

$\Delta = 40$

$\boxed{\Delta} = 25$

$\Delta\Delta = 20$

Q. The common factors of 30 and 45 which are also multiples of 5.

(a) 5, 10, 15 ×

(b) 5, 15 ✓

(c) 1, 5, 10 ×

(d) 1, 3, 5, 15 · ×

- factors of 30 : 1, 2, 3, 5, 6, 10, 15, 30

- factors of 45 : 1, 3, 5, 9, 15, 45

common factors : 1, 3, 5, 15

5, 15

If $\bigcirc + \bigcirc + \bigcirc + \bigcirc + \bigcirc = 125$ and $\bigcirc = \triangle + 4$, then find the value of $\triangle + \triangle + \bigcirc + \bigcirc$.

(a) 25

(b) 45

(c) 50

(d) 92

$\bigcirc = 25$

$\triangle = 21$

Find HCF of 15 and 25

⇒

factors of 15 = 1, 3, 5, 15

25 = 1, 5, 25

HCF of 15 and 25 is 5

Q. If M is the 5th multiple of 10 and N is the 4th multiple of 5, then find the HCF of M and N.

2014 JIMO

(A) 8 ×

(B) 10 ✓

(C) 5

(D) 20 ×

M = 50

N = 20

HCF of 50 and 20

factors of 50 = 1, 2, 5, 10, 25, 50

20 = 1, 2, 4, 5, 10, 20

Q. Find the estimated sum of the smallest and greatest 4-digit number that can be formed from the digits 7, 2, 5 and 3 (using each digit only once), when each number is rounded off to the nearest hundreds / tens place.

Smallest 4 digit no = 2357 ⇒ 2400

Greatest 4 digit no = 7532 ⇒ 7500

(A) 9900

(B) 9000

(C) 8500

(D) 9800

2357 ⇒ 2360 ⇐ 2000

7532 ⇒ 7530 ⇐ 8000

③ If A + B + B + C = 189 ; A + B + C = 147 and B − C = 12, Then find the value of C.

; → Semicolon

(a) 30
(b) 42
(c) 35
(d) 47

$O - \uparrow = 12$

$42 - \uparrow = 12$

$\uparrow = 42 - 12 = 30$

$\square + O + O + \uparrow = 189$

$\Rightarrow 147 + O = 189$

$\square + O + \uparrow + O = 189$

147

$147 + O = 189$

$O = 189 - 147 = 42$

$O = 42$

$A + B + B + C = 189$

$A + B + C = 149$

$B - C = 12$

$C = ?$

$149 + C = 189$

Given

□ + ○ + ○ + ⇧ = 189

Given

□ + ○ + ⇧ = 147

147 + ○ = 189

○ = 189 − 147

○ = 42

Given.

○ − ⇧ = 12

To find

⇧ = ?

42 − ⇧ = 12

⇧ = 42 − 12

= 30

Given

→ $B + T = 1300$

⇒ $C + C + C + C + C = 3500$

⇒ $C + T = 1200$

B → Bike

C → Car

T → Truck

Find $B + C = ?$

$B = ?800$

$C = ?700$

$B+C = 800 + 700 = 1500$

Given, $C+C+C+C+C = 3500$

$5C = 3500$

$C = \frac{3500}{5} = 700$

$C = 700$

$C + T = 1200$

$700 + T = 1200$

$T = 1200 - 700$

$T = 500$

$B + T = 1300$

$B + 500 = 1300$

$B = 1300 - 500$

$B = 800$

Find the value of P and Q respectively.

$$\begin{array}{r} 41742 \\ +\ 32465 \\ \hline 74207 \end{array}$$

(A) 41742, 74147

(B) 4645, 74147

(C) 41742, 74207

(D) 4645, 78257

Q. Sumit gave 50 stickers to each of his students. He then has 20 stickers left. If he has bought 80 packets of 9 stickers each, then to how many students did he give the stickers?

(A) 10

(B) 14

(C) 12

(D) 15

He had, Total stickers $= 80 \times 9 = 720$

Total stickers given $= 720 - 20 = 700$

Total no. of ~~stickers~~ student $= \frac{700}{50} = \frac{70}{5}$

$= 14$

Q. Box X contained 1230 cards and box Y contained twice as many cards as box X. How many total cards do the two boxes contain?

A) 3675

~~B)~~ 3690

C) 3540

D) None of these

X = 1230

Y = 2460

Q. If $X = 475 \times 29$; $Y = 49582 + 3781$ and $Z = 54100 - 10973$, then find the estimate value of $X+Y+Z$, when rounded off to nearest hundreds place.

(A) 110265

(B) 100000

(C) 110300

(D) 110200

$X = 475 \times 29$ → 13800

$Y = 49582 + 3781$ → 53400

$Z = 54100 - 10973$ → 43100

$X+Y+Z$
$= 13800 + 53400 + 43100$
$= 110300$.

475
× 29
13775

13800

49582
+ 3781
53363
53400

54100
-10973
43127

43100

End of the Chapter