Factors and Multiples

Grade 5

Factors and Multiples

Factors

factors of 0 = 1, 2, 3, 4, 5, ..., ∞

- 12 = 1, 2, 3, 4, 6, 12
- 16 = 1, 2, 4, 8, 16
- 20 = 1, 2, 4, 5, 10, 20

Properties of factor :

* 1 is factor of every number.
* Every number (except zero) is a factor of itself.
* Every number (except zero) has limited number of factors.
* A factor of non-zero number is less than or equal to the number.
* Every non-zero number is a factor of zero.

* 1 is the only number having one factor, namely 1 itself.

factors of 1 = 1

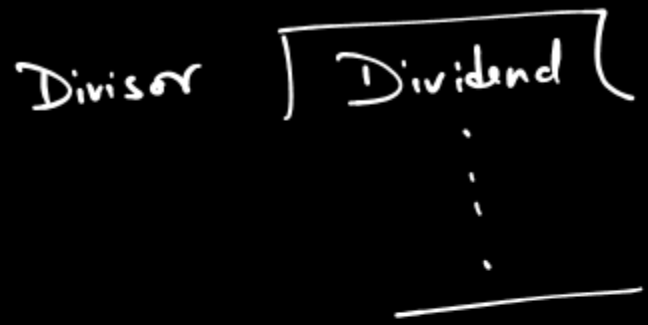
factors of 2 = 1, 2

3 = 1, 3

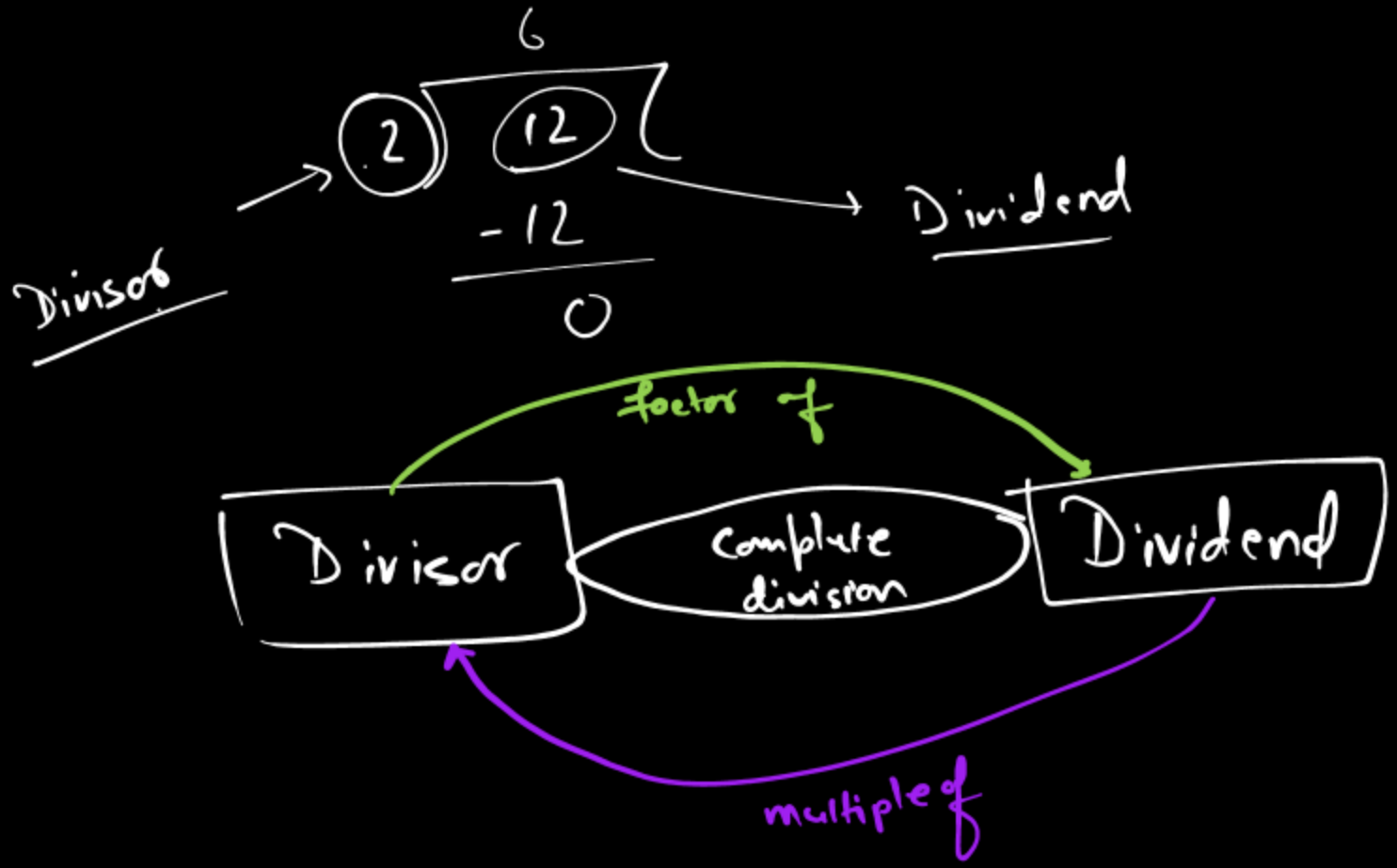
4 = 1, 2, 4

25 = 1, 5, 25

Multiples



When a number divides another number completely, then dividend is called multiple of divisor. and the divisor is called a factor of dividend.



- Factors of 12 : 1, 2, 3, 4, 6, 12
- Multiples of 6 : 6, 12, 18, 24, 30, 36, ...
- Factors of 19 : 1, 19
- Factors of 13 : 1, 13

Properties of multiples :

* Every number is a multiple of 1.
* Every number is a multiple of itself.
  eg. multiple of 6 is 6, 12 ...
* Zero is a multiple of every number.

eg. multiples of 6 : 0, 6, 12, 18, ...
multiples of 8 : 0, 8, 16, 24, ...

* Every non-zero multiple of a non-zero number is greater than or equal to the number.
* There are infinite multiples of a non-zero number.

> Non-zero numbers are all the numbers except zero.
> Example: 1, 2, 3, 4, 5, 6, ... ∞

Even Numbers :

Numbers which are completely divisible by 2.

eg. 0, 2, 4, 6, 8, 10, ...

Odd numbers : Numbers which are not divisible by 2 completly.

eg: 1, 3, 5, 7, 9, 11, ...

Solve:

$$\left(\frac{5}{6} \times \frac{3}{9}\right) + \left(\frac{13}{9} \times \frac{7}{6}\right)$$

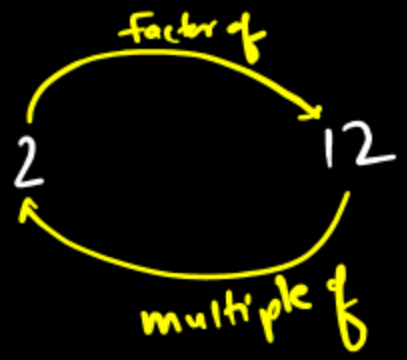
$$\left(\frac{5}{18}\right) + \left(\frac{91}{54}\right)$$

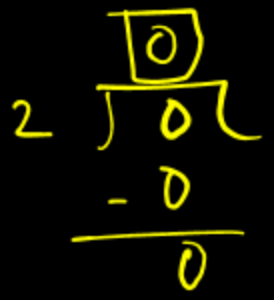
$$\Rightarrow \frac{5}{18} + \frac{91}{54}$$

$$\Rightarrow \frac{15}{54} + \frac{91}{54}$$

$$\Rightarrow \frac{106}{54} = \frac{53}{27} = \boxed{1\frac{26}{27}}$$

Solve: $\frac{7}{16} \div \frac{49}{2}$





0 → is an even number

Test of Divisibility.

① Test of divisibility by 2.

A number is divisible by 2, if its unit digit is any one of 0, 2, 4, 6 or 8.

eg: 342 unit place has 2

∴ 342 is divisible by 2.

[351, 444, 249, 250, 671, 849, 972, 555, 328]

546290

$2 \times 1 = 2$
$2 \times 2 = 4$
6
8
10
12
14
16
18
20
22
24
26
⋮

(11) Test of divisibility by 3.

A number is divisible by 3, if the sum of its digits is divisible by 3.

For eg: 16701

Sum of digits = 1+6+7+0+1 = 15, which is divisible by 3

∴ → Therefore

∴ 16701 is divisible by 3.

78421, 542301, 240

78421 ✗

(iii) Test of divisibility by 4

A number is divisible by 4, if the number formed by last two digits is divisible by 4 or the number ends with 00 (zero zero).

eg. 58236

← last two digit i.e. 36 is divisible by 4

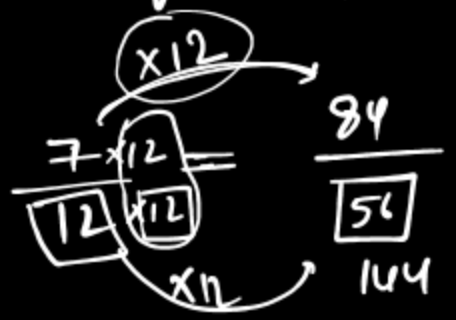
∴ 58236 is also divisible by 4.

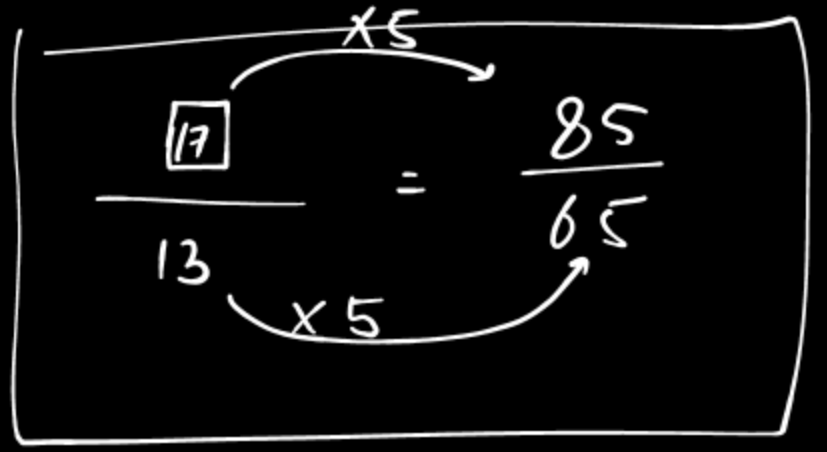
i.e. ⇒ that is

459106 →

Revision of fractions

LCM of 4, 10 and 12 //





$$\frac{7}{12} \times \frac{2}{2} = \frac{14}{24}$$

$5\overline{)85}$

Q. The difference of two numbers is $\frac{5}{24}$. If the larger number is 18 , find the smaller number.

$\frac{7}{8}$

A − B = $\frac{5}{24}$

$\frac{7}{8} - \square = \frac{5}{24}$

$18 - 6 = 12$

$\frac{7}{8} - \square = \frac{5}{24}$

$\frac{7}{8} - \frac{5}{24}$

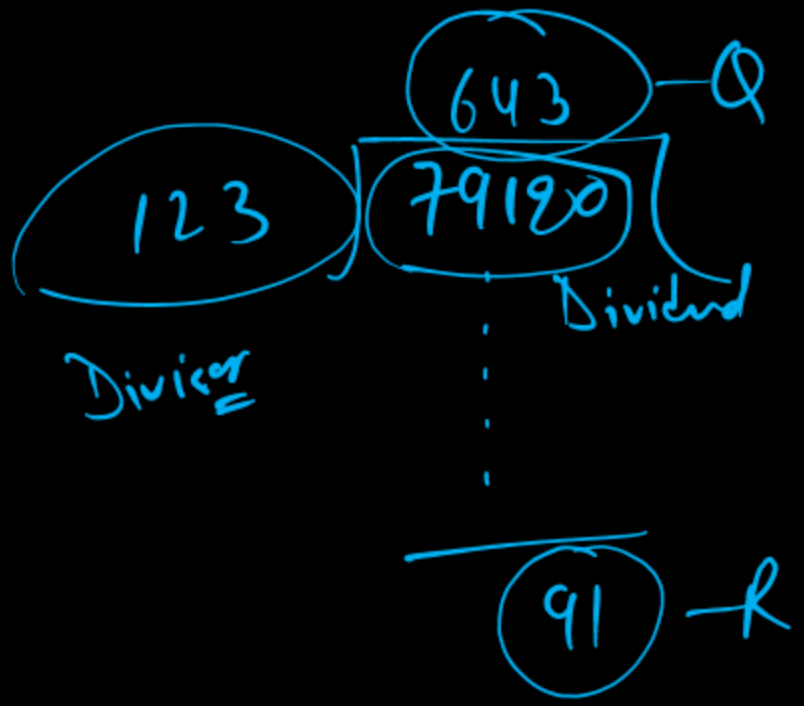
$\frac{21}{24} - \frac{5}{24} = \frac{16}{24} : \frac{8}{12}$

$\frac{2}{3}$

$\frac{4}{6} = \frac{2}{3}$

2 | 6 (

$79180 \div 123$



Dividend = Divisor × Quotient + Remainder

$\Rightarrow 79180 = 123 \times 643 + 91$

$= 79180$

LHS = RHS

Q. What must be added to $7\frac{1}{6}$ to get $9\frac{1}{9}$?

$$7\frac{1}{6} + \square = 9\frac{1}{9}$$

$$= 9\frac{1}{9} - 7\frac{1}{6}$$

$$= \frac{82}{9} - \frac{43}{6}$$

$$= \frac{164}{18} - \frac{129}{18}$$

$$= \frac{35}{18} = 1\frac{17}{18}$$

$$\frac{134}{36} = \frac{67}{18}$$

Q. What must be subtracted from $\frac{6}{7}$ to get $\frac{1}{5}$?

Fraction →

$\frac{3}{27}$ → Numerator (Dividend)

$\frac{3}{27}$ → Denominator (Divisor)

$$\frac{3 \div 3}{27 \div 3} = \frac{1}{9}$$

$$\begin{array}{r} 123 \\ \hline 2\,46 \\ \hline 3,2 \\ \hline 16 \end{array} = \frac{123}{16}$$

$$2\frac{3}{4} \div 145\frac{1}{5}$$

$$\Rightarrow \frac{11}{4} \div \frac{726}{5}$$

$$\Rightarrow \frac{\cancel{11}^{1}}{4} \times \frac{5}{\cancel{726}_{66}}$$

$$\Rightarrow \frac{1 \times 5}{4 \times 66} = \frac{5}{264}$$

$66 \times 4 =$

$$11\overline{)726}\ (66$$
$$-66\downarrow$$
$$66$$

(i) $\frac{\cancel{261}^{3}}{\cancel{4}\ 1} \times \frac{\cancel{16}^{4}}{\cancel{87}\ 3} = \frac{3 \times 4}{1 \times 3}$

(ii) $\frac{\cancel{81}^{9}}{16} \times \frac{\cancel{25}^{5}}{\cancel{9}} \times \frac{\cancel{125}^{25}}{8}$

$\frac{\cancel{81}^{9}}{\cancel{16}\ 2} \times \frac{\cancel{25}^{1}}{\cancel{9}\ 1} \times \frac{\cancel{8}^{1}}{\cancel{125}\ 5}$

$\frac{9}{10}$

$\cancel{5} \times \cancel{10}^{2} = 2$

$\frac{\cancel{5}}{2} \times \frac{\cancel{10}}{25}$

$$12\frac{3}{4} \div 7\frac{2}{3}$$

$$\Rightarrow \frac{51}{4} \div \frac{23}{3}$$

$$\frac{51}{4} \times \frac{3}{23} = \frac{153}{92} = 1\frac{61}{92}$$

$$\frac{54}{\cancel{162}} \times \frac{\cancel{126}\ 42}{\cancel{630}\ \cancel{378}\ \boxed{189}\ 63} = \frac{\cancel{26}\ \cancel{18}}{54_2} \times \frac{\cancel{14}\ 2}{42} = \frac{2}{1} \times \frac{1}{7} = \frac{4}{7}$$

$$\cancel{324}\ 945_4 \quad \cancel{189}\ 63$$

$$\frac{864}{1152} \div \left(\frac{1980}{2376} \div \frac{165}{220} \right)$$

$$\Rightarrow \frac{864}{1152} \div \left(\frac{1980}{2376} \times \frac{220}{165} \right)$$

165, 330, 990, 1980 — 2376, 1188, 594

4, 44, 220 — 165, 33, 3, 1

$$\Rightarrow \frac{846}{1152} \div \left(\frac{165}{594} \times \frac{4}{1} \right)$$

55, 165 — 594, 198

$$= \left(\frac{846}{1152} \times \frac{99}{220} \right)$$

99, 198 — 220, 110, 10

9

47, 141, 423 — 846, 1152, 576, 192, 64

$$= \frac{846}{1152} \times \frac{9}{10}$$

$$= \frac{47}{64} \times \frac{9}{10}$$

$$= \frac{423}{640}$$

$\frac{18}{27} \times \frac{42}{63}$

$\frac{3}{7}$ of $\frac{44}{12} \Rightarrow \frac{3}{7} \times \frac{44^{7}}{12_{4}} = \frac{7}{4} = 1\frac{3}{4}$

$\frac{3}{7}$ of $63 \Rightarrow 63 \times \frac{3}{7} \Rightarrow 27$

$$\frac{\cancel{18}^{\,2}}{\cancel{27}_{\,3}} \times \frac{\cancel{42}^{\,14}}{\cancel{63}_{\,3}} = \frac{4}{9}$$

$$\Rightarrow \frac{\cancel{25}}{\cancel{30}_{\,3}} \times \frac{\cancel{18}^{\,6}}{27} \times \frac{\cancel{20}^{\,2}}{\cancel{24}_{\,4_2}} = \frac{25}{54}$$

$$1\frac{8}{9} \times 1\frac{47}{65} \times 2\frac{1}{12}$$

$$\frac{17}{9} \times \frac{\cancel{56}^{\,2}\,{}^{8}}{\cancel{65}_{\,13}} \times \frac{5}{\cancel{12}_{\,\cancel{6}_{\,3}}}$$

$$\frac{17 \times 28 \times 5}{9 \times 13 \times 3} =$$

$$2\overline{)112}$$

(i) $\frac{\cancel{13}^{1}}{\cancel{26}_{2}} \times \frac{\cancel{39}^{13}}{\cancel{52}_{4}} \times \frac{\cancel{65}^{5}}{\cancel{78}_{2}}$

$= \cancel{1} \times \frac{1}{2} \times \frac{1}{4} \times \frac{5}{2}$

$= \cancel{5} \quad \boxed{\frac{5}{16}}$

Q. $13\frac{2}{9} \times 7\frac{4}{7} \times 5\frac{1}{3}$

$\frac{17}{\cancel{119}} \times \frac{\cancel{53}}{\cancel{7}} \times \frac{16}{3} = \frac{14416}{27}$ ✓

$37\frac{2}{27}$

$\frac{17 \times 53 \times 16}{9 \times 3}$

$$\begin{array}{r} 53 \\ \times\ 17 \\ \hline 371 \\ 530 \\ \hline 901 \end{array}$$

$$\begin{array}{r} 901 \\ \times\ 16 \\ \hline 5406 \\ 9010 \\ \hline 14416 \end{array}$$

$$\begin{array}{r} 901 \\ \times\ 16 \\ \hline \\ \hline \end{array}$$

Q. Arrange the following fractions in a descending order.

(Ascending order)

$\frac{2}{5}, \frac{3}{10}, \frac{7}{15}, \frac{1}{2}$

$LCM(5, 10, 15, 2) = 30$

$\frac{2 \times 6}{5 \times 6} = \frac{12}{30}$

$\frac{3 \times 3}{10 \times 3} = \frac{9}{30}$

$\frac{7 \times 2}{15 \times 2} = \frac{14}{30}$

$\frac{1 \times 15}{2 \times 15} = \frac{15}{30}$

$\frac{9}{30} < \frac{12}{30} < \frac{14}{30} < \frac{15}{30}$

$\frac{3}{10} < \frac{2}{5} < \frac{7}{15} < \frac{1}{2}$

$\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{9}{16}$

$LCM(2, 4, 8, 16) = 16$

$\frac{1 \times 8}{2 \times 8} = \frac{8}{16}$

$\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$

$\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$

$\frac{9}{16} = \frac{9}{16}$

$\frac{12}{16} > \frac{10}{16} > \frac{9}{16} > \frac{8}{16}$

$\frac{3}{4} > \frac{5}{8} > \frac{9}{16} > \frac{1}{2}$

Q. $7\frac{4}{9} - 3\frac{5}{12}$

$\frac{4}{36}$ $4\frac{1}{36}$

$\frac{67}{9} - \frac{41}{12}$

$\frac{268 - 123}{36}$

$\frac{67 \times 4}{9 \times 4} = \frac{268}{36}$

1. Simplify:

(i) $4\frac{1}{3} - 2\frac{3}{4} + 5\frac{1}{6}$

(ii) $10 - 3\frac{1}{5} - 5\frac{3}{10}$

2. Subtract the difference of $6\frac{2}{3}$ and $5\frac{1}{6}$ from the difference of 4 and $2\frac{1}{3}$.

Fractions and Multiples continued...

4. Test of Divisibility by 5.

A number is divisible by 5, if its unit digit is 0 or 5.

1600

2300

Eg: 2045 → divisible by 5.

30001 → Not divisible by 5.

5. Test of divisiblity by 6.

⇒ If a number is divisible by both 2 and 3, then it is divisible by 6.

Eg: (i) 75864 ⇒ ∵ 75864 is divisible by 2 and 3.
∴ It is divisible by 6.

(ii) 27568 ⇒ ∵ Number is divisible by 2, but not by 3.
∴ It is not divisible by 6.

6. Test of divisibility by 7.

Step 1: Double the digit at unit place

Step 2: Subtract the doubled value from rest of the number.

(rest of the number: the number formed by digits excluding unit place)

Step 3: if the number so obtained is divisible by 7, then the actual no. is also divisible by 7.

Eg. ① 934 is divisible by 7 or not.

Step 1: $4 \times 2 = 8$

Step 2: $93 - 8 = 85$

$\because$ 85 is not divisible by 7.

$\therefore$ 934 is not divisible by 7.

eg 2, 6895

$\Rightarrow$ 689 − (5×2) = 689 − 10 = 679

679

$\Rightarrow$ 67 − (9×2) = 67 − 18 = 49

∵ 49 is divisble by 7

∴ 6895 is also divisible by 7.

Is 2455 is divisible by 7?

2455

$(245 - 10) = 235$

235

$(23 - 5 \times 2) = \boxed{15}$

Q. 458409 is divisible by 7 or not?

$\Rightarrow 45840 - (9 \times 2) = 45840 - 18 = 45822.$

$\Rightarrow 4582(2)$

$4582 - (2 \times 2) = 4582 - 4 = \boxed{4578}$ ✓

$4582 - 4$

$\Rightarrow 457(8)$

$457 - 8 \times 2 = 457 - 16 = 441$

$\Rightarrow 44(1)$

$44 - 1 \times 2 = 44 - 2 = \boxed{42}$

$\because$ 42 is divisible by 7

$\therefore$ 458409 is also divisible by 7.

Q. $3\frac{5}{7}$ of 49

Q. Is 1001 is divisible by 7?

Yes

Test of divisibility by 8

⇒ A number is divisible by 8, if its last three digits are divisible by 8.

eg. 6753104

104

104 is divisible by 8.

∴ 6753104 is also divisible by 8.

13
104
8

H.W.

Is 5978164 divisible by 8?

164

↓

is not divisible by 8

$$8 \overline{)164} \quad 2$$
$$-16$$
$$\overline{4}$$

$$\frac{164}{8}$$

∴ 5978164 is not divisible by 8.

Test of divisibility by 9 (Similar to divisibility by 3)

"A number is divisible by 9, if sum of its digits is divisible by 9".

eg. . 867105 $\Rightarrow 8+6+7+1+0+5 = 27$

27 is divisible by 9

$\therefore$ 867105 is divisible by 9.

. 5632104 $\Rightarrow 5+6+3+2+1+0+4 = 21$ not divisible by 9.

Not divisible by 9

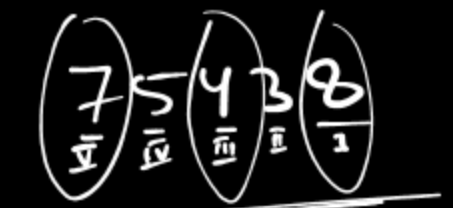
Test of divisibility by 10

⇒ A number is divisible by 10, if its unit digit is 0.

Test of divisibility by 11

A number is divisible by 11, if the difference between the sum of digits at odd places and the sum of digits at even places, is either zero or a multiple of 11.

eg.



- Sum of the digit at odd places: 7+4+8 = 19
- Sum of the digits at even places: 5+3 = 8

The difference: 19-8 = 11 ✓

eg. 4 9 7 3 6 5 ✓ (Divisible)

VI V IV III II I

eg. 6 2 3 4 11 ✗ Not divisible

Q. First 4 multiples of 12.

Q. First 6 odd multiples of 3.

Q. ~~Find~~ Write all the factors of 60.

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Find all the factors 72.

72 ⇒ 1, 2, 3, 4, 6, 9, 12, .., 72

Prime Numbers and Composite Numbers

Prime Numbers :-

Numbers having exactly two distinct factors, i.e 1, and the number itself.

Smallest prime no.

eg: (2), 3, , 5, 7, 11, 13, 17, 19, 23, etc....

Composite Numbers :→ Numbers having ~~having~~ more than two distinct factor.

Smallest composite no.

eg: (4) → 1, 2, 4
(6) → 1, 2, 3, 6
(8), (9), 10, 12, 14, 15, 16, 18, 20, 21, 22, ...etc.

[∵ 1 has only one factor

∴ 1 is neither prime nor composite.]

Sieve of Eratosthenes (Method to find all prime nos. upto 100)

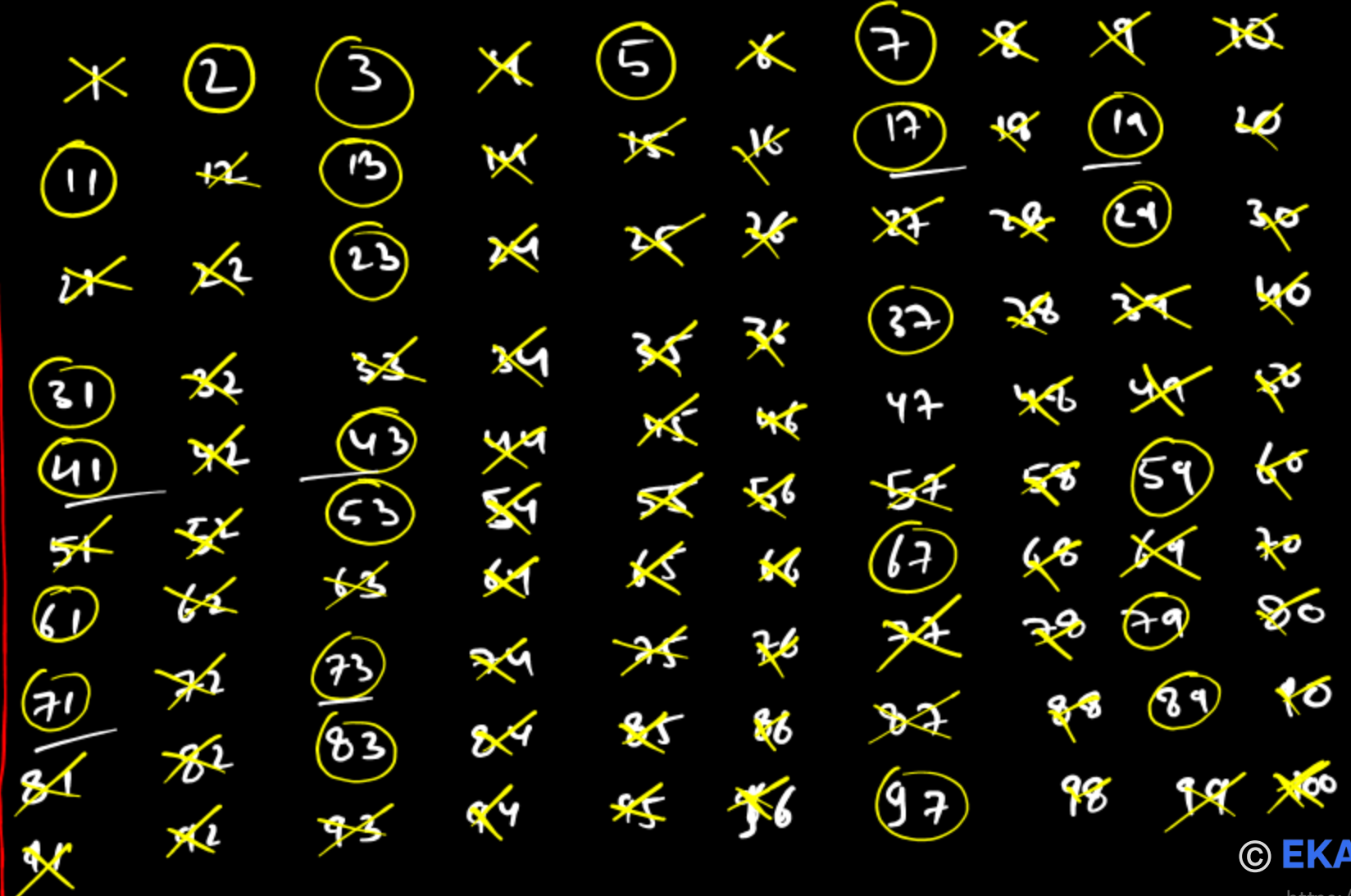
(i) Cross out 1

(ii) Circle (2) and cross at all the multiples of 2

(iii) Circle (3) and cross at all the multiples of 3.

(iv) Circle (5) and cross out all the multiples of 5.

(v) Circle (7) and cross out all the multiples of 7.



Twin Primes :→

⇒ Prime number which differ by 2.

eg: 3 and 5 are twin prime

5 and 7
11 and 13
17 and 19
etc.
} are twin prime.

Co-primes :

Two numbers are said ~~Sonia~~ to be coprime if they do not have any common factor except 1.

3 and 4

2 and 3

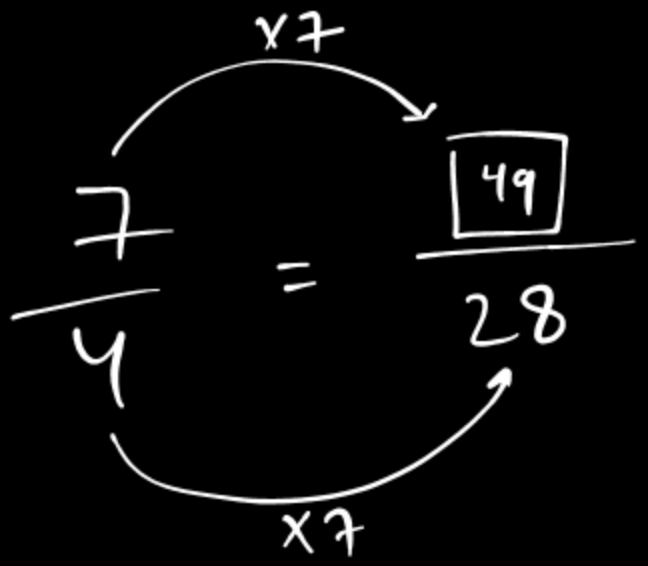
8 and 17

21 and 25

Equivalent fraction : (from class-test)

$\frac{7}{4} \xrightarrow{\times 7} \frac{49}{28}$

$\frac{7}{4} = \frac{14}{8} = \frac{21}{12}$



$\frac{7 \times 2}{4 \times 2} = \frac{14}{8}$

$\frac{7 \times 3}{4 \times 3} = \frac{21}{12}$

$$\frac{3976}{28}$$

Prime Factorisation

Prime factors of 30

Factors of 30 : 1, (2), (3), (5), 6, 10, 15, 30

Prime factors of 30 are 2, 3 and 5.

Every number can be written as product of its prime factors.

$30 = 2 \times 3 \times 5$

Q. Find ~~find~~ prime factors of 72.

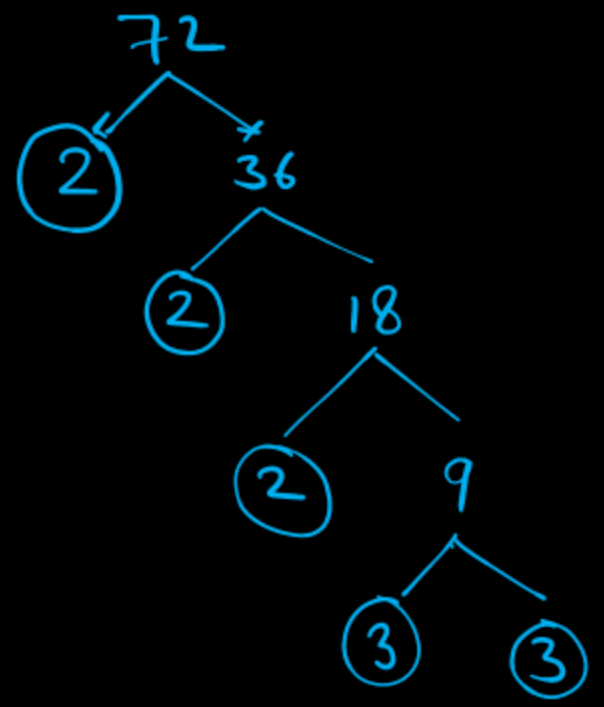
Division Method

2	72
2	36
2	18
3	9
3	3
	1

$72 = 2 \times 2 \times 2 \times 3 \times 3$

Prime factorisation of 72

Factor tree method



Q. Using division method find prime factors (P.F.) of 210 and write down prime-factorisation of 210.

$$210 = 2 \times 3 \times 5 \times 7$$

prime factorisation of 210

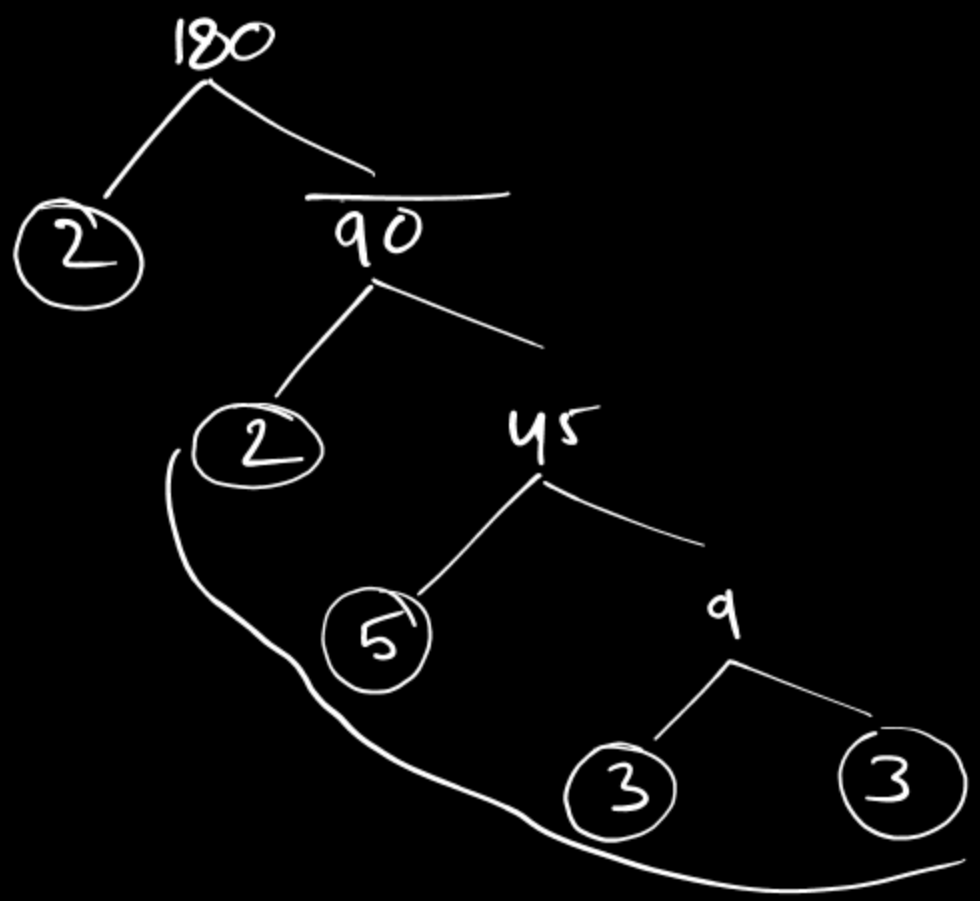
Q. Write the prime factorisation of 468 using division method.

$$468 = 2 \times 2 \times 3 \times 3 \times 13$$

Q. P.F. of 180 using factor-tree method.

$180 = 2 \times 2 \times 5 \times 3 \times 3$ ✓

$180 = 2 \times 2 \times 3 \times 3 \times 5$



Write Prime factorisation of 630. by division method.

2	630

$630 = 2 \times 3 \times 3 \times 5 \times 7$

Find prime factorisation of 90.

$$90 = 2 \times 3 \times 3 \times 5$$

2	90
3	45
3	15
5	5
	1

H.C.F.

• Prime factorisation of 315 by division method:

5	315
3	63
7	21
3	3
	1

$315 = 3 \times 3 \times 5 \times 7$

315 → Composite

$3 \times 3 \times 5 \times 7$ → Product of prime factors

Highest Common Factor (H.C.F.) or Greatest Common Divisor (G.C.D.)

HCF (or GCD) of two or more number is the greatest / largest number that divides each of the given numbers completely (No remainder).

Method 1: HCF by factorisation method

eg: Find the HCF of (24) and (32).

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 }

Factors of 32: 1, 2, 4, 8, 16, 32 }

Common factors: 1, 2, 4, (8)

HCF (Highest common factor) is (8).

Q. Find the HCF of 18, 24 and 30, using factorisation method.

18: 1, 2, 3, (6), 9, 18

24: 1, 2, 3, 4, (6), 8, 12, 24

30: 1, 2, 3, 5, (6), 10, 15, 30

HCF : 6

Method 2: HCF by Prime factorisation method

Eg: Find HCF of 72 and 90

Prime factorisation of 72 : $2 \times 2 \times 2 \times 3 \times 3$

Prime factorisation of 90 : $2 \times 3 \times 3 \times 5$

H.C.F of 72 and 90 = product of common prime factor.

$= 2 \times 3 \times 3$

$= 18$

2	72
2	36
2	18
3	9
3	3
	1

2	90
3	45
3	15
5	5
	1

Q. Find HCF of 32, 80 and 96 by prime factorisation method.

Prime factorisation of $32 = 2 \times 2 \times 2 \times 2 \times 2$

Prime factorisation of $80 = 2 \times 2 \times 2 \times 2 \times 5$

Prime factorisation of $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

(2) (2) (2) (2)

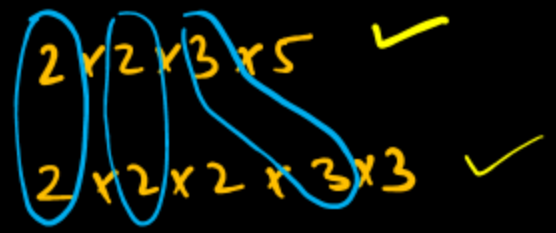
32

HCF of 32, 80 and 96 = Product of common prime factors

$= 2 \times 2 \times 2 \times 2$

$= 16$

Q. Find HCF of 60 and 72. using prime factorisation method



HCF of 60 and 72 = 2 x 2 x 3

= 12

① Factor method.

② Prime factorisation method.

Q. Find HCF of 135 and 180 by prime factorisation method.

$135 = 3 \times 3 \times 3 \times 5$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

5	135
3	27
3	9
3	3
	1

2	180
2	90
3	45
3	15
5	5
	1

⇒ HCF of 135 and 180 $= 3 \times 3 \times 5$

$= 45$

Method 3: Common Division Method

eg. Find the HCF of 60 and 72

2	60, 72
2	30, 36
3	15, 18
	5, 6

HCF = 2 x 2 x 3

= 12

Q. Find HCF of 56, 84 and 154, using common division method.

=

2	56, 84, 154
7	28, 42, 77
	4, 6, 11

HCF: $2 \times 7 = 14$

Method 4: HCF by Division Method (Long Division Method)

⇒ Used for find HCF of large/big numbers.

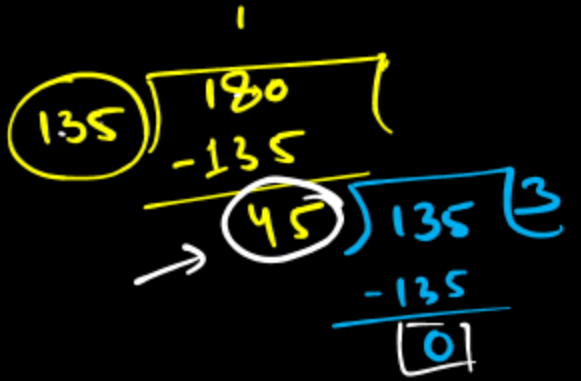
eg: Find the HCF of 135 and 180 ; using division method.

Step 1: Divide the larger number by the smaller number.

Step 2: Divide the divisor by the remainder.

Step 3: Repeat the process of dividing the previous divisor by the remainder obtained till 0 is obtained as remainder

⇒ Then, the last divisor is the required HCF.



HCF of 135 and 180 is 45.

Q. Find the HCF of 217 and 385, using division method.

$$
\begin{array}{r}
1 \\
217 \overline{)\,385\,(} \\
-\,217 \\
\hline
168 \overline{)\,217\,(} \; 1 \\
-\,168 \\
\hline
49 \overline{)\,168\,(} \; 3 \\
-\,147 \\
\hline
21 \overline{)\,49\,(} \; 2 \\
-\,42 \\
\hline
7 \overline{)\,21\,(} \; 3 \\
-\,21 \\
\hline
0
\end{array}
$$

HCF of 217 and 385 = 7

Q. HCF of 594, 792 and 1848, using division method.

Sol: first find HCF of 594 & 792

$$\begin{array}{r} 594 \overline{)\,792\,(1} \\ -594 \\ \hline 198 \overline{)\,594\,(3} \\ -594 \\ \hline 0 \end{array}$$

HCF of 594 and 792 is 198

Now find HCF of 198 and remaining number (1848).

$$\begin{array}{r} 198 \overline{)\,1848\,(9} \\ -1782 \\ \hline 66 \overline{)\,198\,(3} \\ -198 \\ \hline 0 \end{array}$$

HCF of 594, 792 and 1848 is 66.

Q. $\frac{2}{4} \div 3\frac{3}{4}$

Find HCF of 60, 96 and 150 using division method.

$$
\begin{array}{r|l}
60 \overline{)96}\,(1 & \\
-60 & \\
\hline
36 \overline{)60}\,(1 & \\
-36 & \\
\hline
24 \overline{)36}\,(1 & \\
-24 & \\
\hline
12 \overline{)24}\,(2 & \\
-24 & \\
\hline
0 & \\
\end{array}
$$

HCF of 60 and 96 is 12

$$
\begin{array}{r|l}
12 \overline{)150}\,(12 & \\
12 & \\
\hline
30 & \\
-24 & \\
\hline
6 \overline{)12}\,(2 & \\
-12 & \\
\hline
0 & \\
\end{array}
$$

HCF of 60, 96 and 150 is 6

Q. Find HCF of 75, 100 and 140. using division method.

H.C.F = 5

Q. H.C.F. of 144, 180 and 192 using division method.

$180 \overline{)192}$ (

1

12

$12 \overline{)144}$ (12

-144

HCF of 144, 180 and 192 is 12

LCM (Least Common Multiple)

eg: LCM of 4 and 6

4 : 4, 8, 12, 16 ....

6 : 6, 12, 18

LCM is completly divisible by the given nos.

Method 1: LCM by Prime-factorisation method.

eg: find LCM of 48 and 72

Prime factors:

2	48
2	24
2	12
2	6
3	3
	1

2	72

$\Rightarrow 48 = 2 \times 2 \times 2 \times 2 \times 3$

$\Rightarrow 72 = 2 \times 2 \times 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 3$

$= 144$

[LCM = Product of common and non-common prime factors.]

Short-Division Method

2	48, 72

LCM of 90 and 108, using prime-factorisation method.

$90 = 2 \times 3 \times 3 \times 5$

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$\text{LCM} = 2 \times 3 \times 3 \times 5 \times 3 \times 2$

$= 18 \times 3 \times 10$

$= 54 \times 10$

$= 540$

5	90

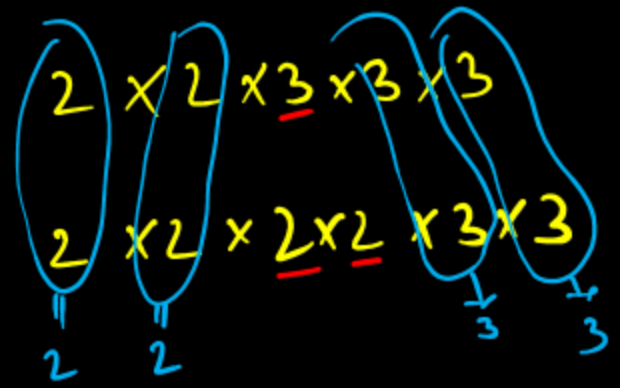
Q. Find the LCM of 108 and 144 by prime factorisation method.

| 108

| 144

108: $2 \times 2 \times 3 \times 3 \times 3$

144: $2 \times 2 \times 2 \times 2 \times 3 \times 3$



16
27
432

LCM of 108 and 144 = $2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 2$

= 432

8 mins.

Find LCM of 24 and 36 using prime factorisation method.

24 : $2 \times 2 \times 2 \times 3$

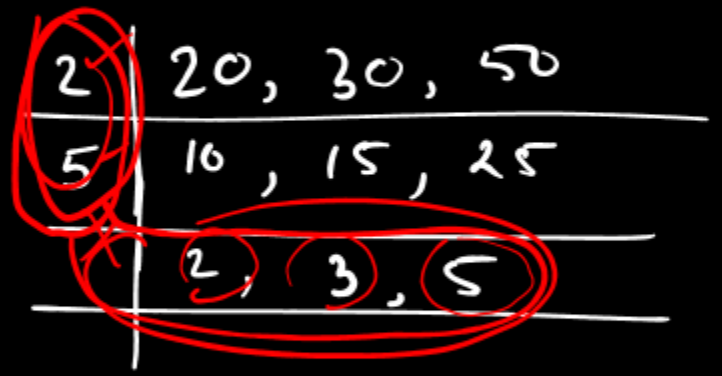
36 : $2 \times 2 \times 3 \times 3$

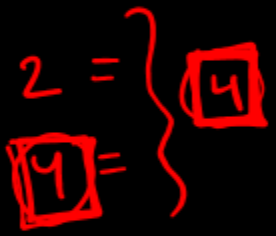
LCM = $2 \times 2 \times 2 \times 3 \times 3 = 72$

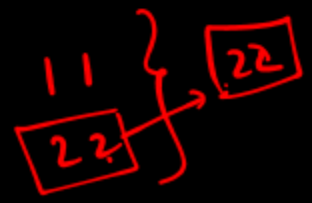
72

LCM by Short Division method (short-cut method)

eg LCM of 20, 30 and 50.



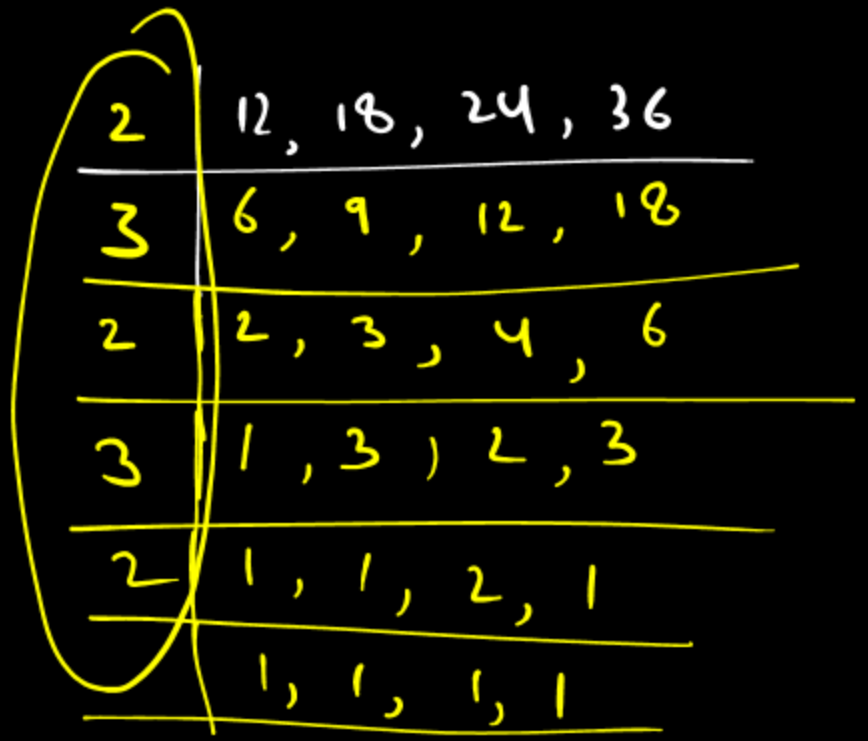




LCM = $2 \times 5 \times 2 \times 3 \times 5$

$= 300$

Find the LCM of 12, 18, 24 and 36, using short division method.



LCM = 2 x 2 x 2 x 3 x 3

= 72

Find LCM of 96, 108 and 180, using short-division method.

	96, 108, 180

4320

4:50

Some facts about HCF and LCM.

(i) If HCF of two numbers is 1, then they are coprime.

(ii) LCM of two co-prime numbers is equal to their product.

Eg: 2 and 5

HCF = 1

LCM = 2 × 5 = 10

(iii) HCF of two numbers is always smaller than their LCM.

HCF (3 and 9) = 3

LCM (3 and 9) = 9

3 (HCF) divides 9 (LCM) completely.

(iv) HCF divides LCM completely.

(v) The product of two given numbers is equal to the product of their HCF and LCM. in the next class

Eg: 6 and 9

HCF(6, 9) = 3

LCM(6, 9) = 18

HCF(6,9) × LCM(6,9) = 3 × 18 = 54

6 × 9 = 54

First number (I no.)

Second number (II no.)

For any two numbers

$$\text{I no.} \times \text{II no.} = \text{HCF}(\text{I \& II no.}) \times \text{LCM}(\text{I, II no.})$$

$$\text{LCM of two given nos.} = \frac{\text{I no.} \times \text{II no.}}{\text{HCF of two given nos.}}$$

$$\text{HCF of two given nos.} = \frac{\text{I no.} \times \text{II no.}}{\text{LCM of two given nos.}}$$

Q. Find the HCF and LCM of 680 and 816

HCF of 680 and 816

$$680 \overline{)816}\ (1$$
$$-680$$
$$136 \overline{)680}\ (5$$
$$-680$$
$$0$$

2	680, 816
2	340, 408
2	170, 204
	85, 102

HCF of (680 and 816) = 136

$$LCM(680 \text{ and } 816) = \frac{680 \times 816}{136}$$

$$= \frac{5 \times 816}{1}$$

$$= 4080$$

Q. The HCF of two numbers is 144 and their LCM is 2880. If one of the numbers is 720, find the other number.

(x)

$$720 \times x = 144 \times 2880$$

$$\text{Missing number} = \frac{\text{HCF} \times \text{LCM}}{\text{given number}}$$

$$\text{Missing no./Other no.} = \frac{144 \times 2880}{720} = 2 \times 288$$

$$= 2 \times 288$$

$$= 576$$

Q. Find HCF and LCM of 87 and 147.

Sol: Find HCF first using division method.

$$87 \overline{)147} (1$$
$$-87$$
$$60 \overline{)87} (1$$
$$-60$$
$$27 \overline{)60} (2$$
$$-54$$
$$6 \overline{)27} (4$$
$$-24$$
$$3 \overline{)6} (2$$
$$0$$
$$0$$

H.C.F is 3 ✓

$$LCM = \frac{87 \times 147}{3}$$

$$LCM = \frac{\text{Multiply both the nos.}}{\text{HCF}}$$

$$LCM = \frac{\overset{29}{\cancel{87}} \times 147}{\cancel{3}_{1}}$$

$$= 29 \times 147$$

$$LCM = 4263$$

$$\begin{array}{r} 147 \\ \times\ 29 \\ \hline \end{array}$$

Find HCF and LCM:
270 and 450

HCF (270 and 450) = 90

$$LCM = \frac{3}{\cancel{270} \times 450}{\cancel{90}_{1}}$$

$$= 3 \times 450$$

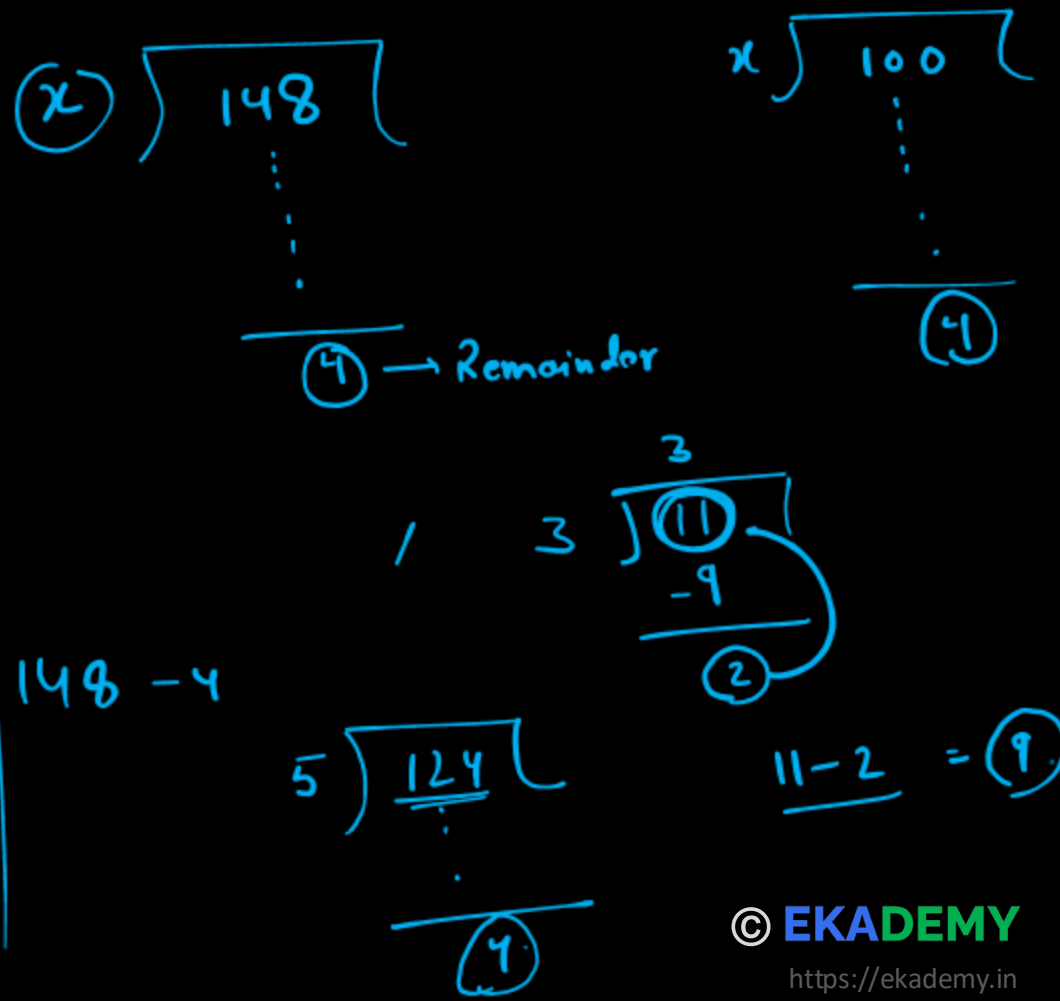
$$= 1350$$

Q. Find the greatest number which divides 148 and 100 leaving remainder 4 in each case.

Sol: Here, we need to find out, the greatest number completely divides $(148-4)=144$ and $(100-4)=96$.

$\therefore$ Required number = HCF of 144 and 96.

$= 48$

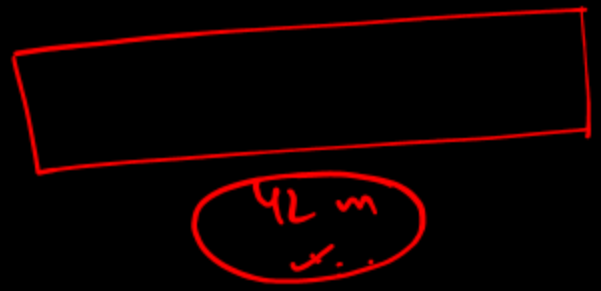


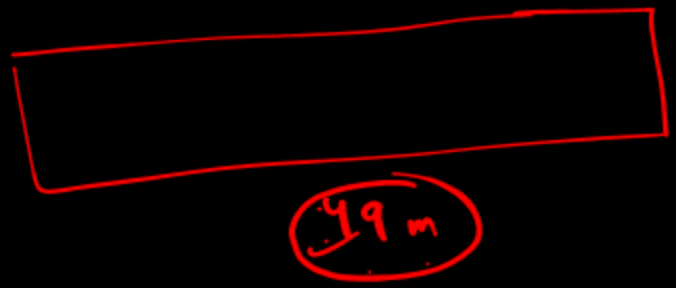
Q. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?

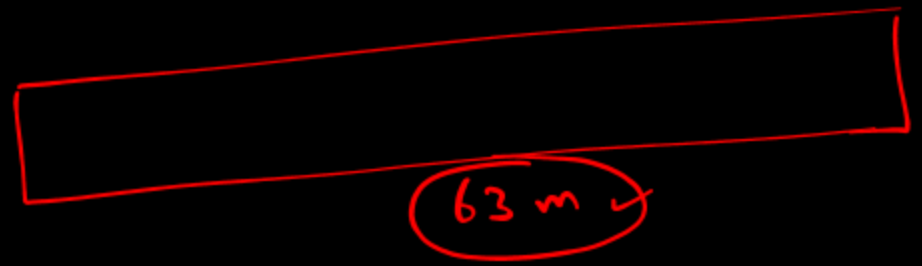
H.C.F

we need to find the highest/ greatest no. that divides, 42, 49 and 63 completely.

i.e., HCF of 42, 49 and 63







7	42, 49, 63
	6, 7, 9

HCF = 7

The required length of each plank = 7m.

Q. Two ropes 12 m and 18 m long are to be cut into small pieces of equal lengths. What will be the maximum (greatest) length of each piece?

[for max. length we find H.C.F.]

Sol: HCF of 12 and 18.

2	12, 18
3	6, 9
	2, 3

HCF = 2 × 3 = 6

Therefore, the maximum length of each piece = 6 m.

Q. Find the least number of marbles so that the heaps of 12, 15 or 20 marbles can be made.

[For least / minimum no. we find LCM]

Sol. We find LCM of 12, 15, 20.

2	12, 15, 20
2	6, 15, 10
5	3, 15, 5
3	3, 3, 1
	1, 1, 1

LCM = $2 \times 2 \times 3 \times 5 = 60$

∴ The least number of marbles = 60.

Q. Find the greatest number which divides 18 and 24 exactly.

Sol: HCF of 18 and 24.

18, 24	

HCF = 6

∴ The greatest number = 6.

Q. Find the least number which is exactly divisible by 9, 12 and 18.

LCM

Sol. LCM of 9, 12 and 18.

9, 12, 18

LCM = 2 x 2 x 3 x 3 = 36

∴ the least no. = 36.

Q. A shopkeeper sold mathematics books for ₹108 on Monday and for ₹84 on Tuesday. What can be the maximum price of each book?

H.C.F.

Sol: HCF of ₹108 and ₹84

$$
\begin{array}{r|l}
84 & 108 \\
 & -84 \\
\hline
24 & 84 \\
 & -72 \\
\hline
12 & 24 \\
 & -24 \\
\hline
 & 0
\end{array}
\qquad
\begin{array}{l}
(1 \\
(3 \\
(2
\end{array}
$$

∴ the max. price of each book = ₹12.

Q. What is the least number of bananas a teacher should have so that, when he distributes equal number of them to his three groups of 10, 15 or 25 students, no banana is left with him?

Sol: LCM of 10, 15, 25.

5	10, 15, 25
	2, 3, 5

LCM: $2 \times 3 \times 5 \times 5$

$= 150$

∴ The least no. of bananas = 150.

Q. An

Q. Four bells begin to toll together and toll respectively at intervals of 6, 7, , 8 and 12 seconds. After how much time will they toll together again?

time	A	B	C	D
0	•	•	•	•
	6	7	8	12

Sol:- LCM (6, 7, 8, 12)

= 168

| 6, 7, 8, 12

⇒ After 168 s they will toll together again.

↳ 2 min 48 sec

Q. Find the least number which is exactly divisible by 12, 16 and 24.

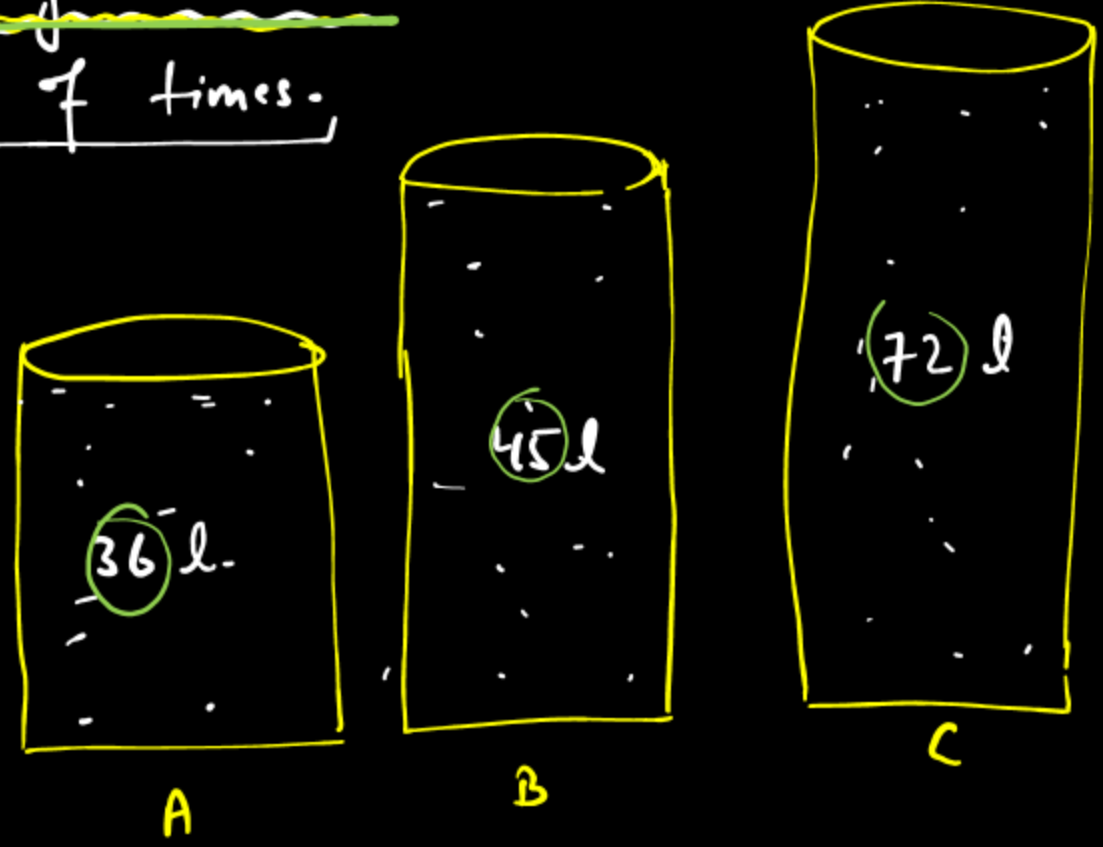
Sol: LCM of 12, 16, 24. = 48

Q. Three drums contain respectively 36 litres, 45 litres and 72 litres of oil. Find the capacity of the largest container which can measure the content of each drum an exact no. of times.

Sol: HCF of 36, 45 and 72

= 9

Ans: Capacity of largest container will be 9 litres.



Q. Find the largest number which divides 209 and 260 leaving remainder 5 in each case.

Sol: HCF of (209-5) and (260-5)

HCF of 204 and 255 = 51

$$204\overline{)255}\,(1$$
$$-204$$
$$51\overline{)204}\,(4$$
$$-204$$
$$0$$

$$51 \times 4 = 204$$

Q. The product of two numbers is 3750 and their LCM is 150. Find the HCF of these numbers.

Q. Six bells starts tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. After how much time will they toll together again?

H.W.

End of the chapter.